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MATTER WAVES : A NEW TOOL FOR THE DETECTION OF GRAVITATIONAL WAVES ?

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Abstract. Matter wave interferometers are used today as high precision inertial sensors. Inertial sensors are sensitive to gravitational waves. We have already calculated their sensitivity (Delva et al. 2006). Here we recall some of the results that have been obtained and we present an original idea for gravitational wave detection based upon matter wave cavities.

1 Introduction

The wave behavior of a massive particle, as predicted by De Broglie (1925), was first shown by a diffraction experiment of electrons (Davisson & Germer 1927). Thirty-five years later the first neutron interferometer was built by Maier-Leibnitz & Springer (1962). In 1991, the first signals of four atomic wave interferometers (hereafter called Matter Waves Interferometers MWIs) were detected (Carnal & Mlynek 1991; Keith et al. 1991; Riehle et al. 1991; Kasevich & Chu 1991). Since then, MWIs have been used for a large variety of studies (Miffre et al. 2006) thanks to the improvements of sensitivity, stability and compactness.

In 2004, Chiao & Speliotopoulos suggested to use a MWI as a detector for Gravitational Waves (GWs). They proposed an estimation of the device sensitivity. Their results were discussed by Foffa et al. (2006) and Roura et al. (2006). They all found different results because they studied different physical experiments. Due to the various interpretations of the coordinates systems and of the boundary conditions, it was not obvious from the beginning that the experiments were different. In a special case, Roura et al. (2006) demonstrated explicitly the equivalence of two different descriptions of the same experiment, which is a basic principle of general relativity. In 2006, we discussed two different experiments, where fixed and free interferometers were considered in comoving coordinates, and calculated the corresponding sensitivities to GWs. Here we recall the results already obtained and we present an original idea for the detection of GWs.

2 Sensitivity to inertial effects

Matter wave interferometers are used in high precision experiments such as accelometer or gyrometer. Indeed, atoms are much more sensitive than light to inertial effects. Unfortunately the low atom flux in MWIs compared to the photon flux in Light Wave Interferometers (LWIs) limits their sensitivity. Inertial effects and gravitational effects (e.g. GWs) are summarized in the perturbed metric

$$\mathrm{d}s^2 = (\eta_{\mu\nu} + K_{\mu\nu})\,\mathrm{d}x^\mu \mathrm{d}x^\nu \ , \ K_{\mu\nu} \ll 1 \tag{2.1}$$

where $\eta_{\mu\nu}$ is the Minkowsky metric and $K_{\mu\nu}$ the perturbation of the metric due to the considered effects. The phase difference $\Delta\phi$ between the two arms of an interferometer can be calculated with the method of Linet & Tourrenc (1976). The equation reads

$$\Delta\phi \sim \frac{c^2}{\hbar} \int K_{\mu\nu} p^\mu p^\nu \frac{\mathrm{d}t}{E} \tag{2.2}$$

where p^μ is the 4-impulsion of the atoms, E their energy, $\hbar$ the Planck constant and c the light velocity.

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An acceleration a can be modeled to the first order by $K_{00} \sim aL/c^2$, with L the MWI's arm length. Then formula (2.2) gives

$$\Delta\phi \sim \frac{ma}{\hbar} \cdot \frac{\mathcal{A}}{v} \tag{2.3}$$

where $\mathcal{A}$ is the MWI's area and v and m are the atom velocity and mass. One sees immediately that for a good sensitivity to the accelerations, the area needs to be as large as possible and the atoms must be as slow as possible. In a Ramsey-Bordé interferometer (Fig. 1), atoms are separated by a laser pulse. The separation angle θ is larger when the atom are slow. Then, for the same arm length, the area $\mathcal{A}$ is larger when the atoms are slow, and the sensitivity of the device is better. This conclusion is also valid for an atom gyrometer. A rotation Ω is modeled to the first order by $K_{0i} \sim \Omega L/c$. Then formula (2.2) gives

$$\Delta\phi \sim \frac{m\Omega}{\hbar} \cdot \mathcal{A} \tag{2.4}$$

One sees that in order to have a large area the atoms have to be slow. This is the main idea of the MWI described in Canuel et al. (2006). But this recommendation is no longer valid for GWs. Indeed, for $K_{ij} \sim h^{\mathrm{TT}}$, where h^{TT} is the GW amplitude, formula (2.2) gives

$$\Delta\phi \sim \frac{mh^{\mathrm{TT}}}{\hbar} \cdot \frac{\mathcal{A}}{T} \tag{2.5}$$

where T is the atom flight time in one arm. Here, for a given area, the flight time must be as low as possible, and thus the atom velocity must be as high as possible. From this point of view, the MWI from Gustavson et al. (2000) is more suitable for the GW detection than the MWI described in Canuel et al. (2006). However its sensitivity remains too low for the detection of GWs.

We will see now what would be the characteristics of a MWI sensitive enough to detect GWs.

3 Comparison between matter wave and light wave interferometers

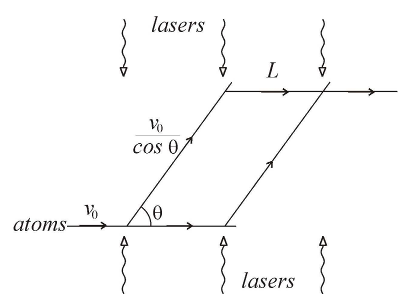

Fig. 1. The Ramsey-Bordé interferometer (Bordé 1989).

The results of this section are based upon the article of Delva et al. (2006). For a rigid Ramsey-Bordé configuration (see Fig. 1), the phase difference due to a periodic GW of frequency Ω and wavelength Λ is

$$\Delta\phi = \Delta\phi_+ + \Delta\phi_\times \tag{3.1}$$

with

$$\Delta\phi_+ = -4\pi h_+ \frac{L}{\lambda} \sin\psi \tan^2\theta \left[\cos(\Omega t + \psi) + \frac{\sin\psi}{2\psi}\cos(\Omega t)\right]$$

$$\Delta\phi_\times = -4\pi h_\times \frac{L}{\lambda} \cos\psi \tan\theta \left[\sin(\Omega t + \varphi_\times - \psi) - \frac{\sin\psi}{\psi}\tan\psi\cos(\Omega t + \varphi_\times)\right]$$

where $h_{+,\times}$ are the amplitude of the two polarizations of the GW, λ is the de Broglie wavelength of the atoms, θ is the separation angle (see Fig. 1) and $\psi = \Omega L/v_0$.

In the first regime where $L \ll \frac{v_0}{c} \cdot \Lambda$ $\left(\text{i.e. } \psi = \frac{\Omega L}{2v_0} \ll 2\pi\right)$ and for $\theta \simeq \pi/4$, the amplitude $\widetilde{\Delta\phi}$ of the phase difference is

$$\widetilde{\Delta\phi} \simeq 4\pi h_\times \cdot \frac{L}{\lambda} \tag{3.2}$$

From formula (3.2), one can compare the sensitivities of MWIs and LWIs when they are limited by the shot noise. On Fig. 2 and 3 are represented the required characteristics[2] of a MWI necessary to reach the sensitivity of Virgo (Fig. 2) and that of LISA (Fig. 3). The cross on each figure corresponds to the MWI described in Gustavson et al. (2000).

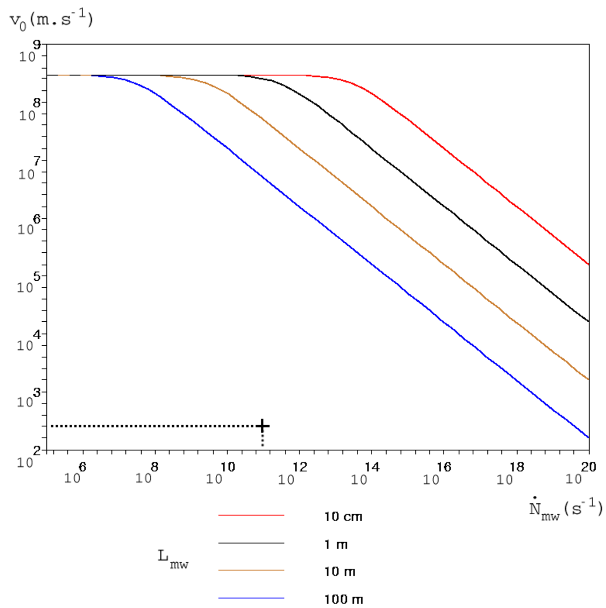

Fig. 2. Required characteristics of a MWI necessary to reach the sensitivity of Virgo.

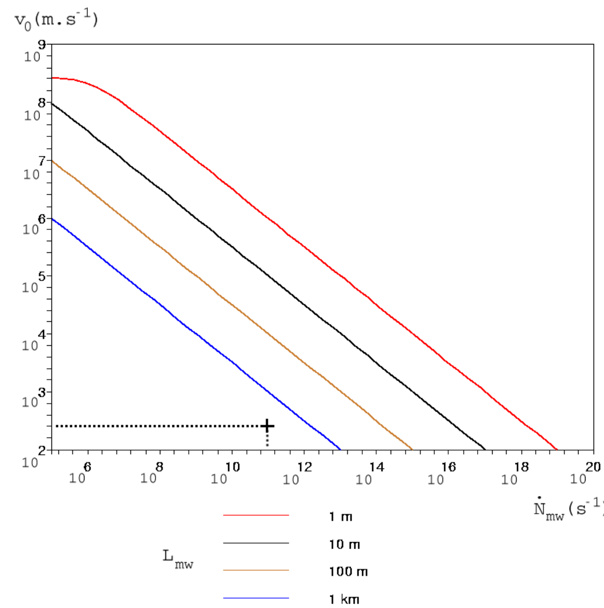

Fig. 3. Required characteristics of a MWI necessary to reach the sensitivity of LISA.

One can see on Fig.2 that in order to reach the sensitivity of Virgo with the actual atomic fluxes, the atoms need to be relativistic. It would be a real challenge to keep the coherence of the atoms at such a speed. The situation is better for low frequency GWs. Indeed, the photon flux in LISA will be very low compare to the atom flux in the Gustavson et al.'s MWI. On Fig. 3 one can see that a kilometric MWI can reach the sensitivity of LISA at a thermal velocity. It is to be compared with the five billions kilometers arms of LISA. The kilometric MWI could even be reduced to a one meter MWI with a matter wave cavity, similar to the Fabry-Perot cavity in Virgo. Cavities where atoms are trapped between two mirrors have been described theoretically by Balykin & Letokhov (1988); but atomic Fabry-Perot cavities coupled to a MWI have never been built (for different views on matter wave cavities see Balykin et al. 2000; Law & Bigelow 1998; Impens et al. 2006).

The main interest of a compact MWI is that it can be cooled to very low temperatures to reduce the thermal noise.

4 Matter wave cavities and Bose-Einstein condensates

A Bose-Einstein condensate (BEC) trapped in an electromagnetic field is a quasi perfect matter wave cavity, where atoms have all the same energy : the ground state of the electromagnetic trap. It can act as a new GW detector : when a GW pass through the cavity, the trapped BEC energy has a certain probability to change, depending on the GW frequency. This probability is very small, but there can be a resonance between the GW

[2] v_0, $L_{\rm mw}$ and $\dot{N}_{\rm mw}$ are respectively the initial atom wave group velocity, the MWI arm length and the atom flux.

and the energy levels of the trap. Therefore the energy left by the GW can be stored in the trap for a ”long while”.

One difficulty is to describe the BEC in general relativity. Some work has been done in this direction for a static background (Al’taie 1978; Kirsten & Toms 1995; Smith & Toms 1996). The next step is the introduction of a GW background and the determination of the final state of the trapped BEC. The detailed calculation will be available in a future article where the resonance above will be put forward.

5 Conclusion

Today MWIs are not sensitive enough to detect GWs. Thanks to the rapid progress in the field one can believe that the required experimental capabilities could be achieved not too far in the future. In this context, we think that it is interesting to study possible new detectors of GWs. In this talk, we emphasized the different experimental characteristics to be improved : high atom velocity, atom cavities, high flux, good separation . . . , and we presented a new type of GW detector which takes advantage of the development of matter wave cavities.

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