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REVIEW OF ANALYTICAL STUDIES OF RELATIVE MOTIONS IN FLIGHT FORMATIONS

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Abstract. This paper deals with the dynamics of flight formations, their characteristics and their study, at past and at present. The review would be exhaustive and present the different type of developments that have been done on the topic. The author also express his personal view of which should be the most interesting ways for future researches.

1 Introduction

The interest of reviewing the analytical works about relative motions is to acquire a global view of the situation and frame future works. As it will be shown, there have been two major approaches, that depend on the used variables: local position and velocity, or differences of orbital elements.

The study of relative motion from an analytical point of view has two major interests. The first one is the analysis of new missions. From this point of view, an analytical approach is a strong tool to find the most stable configurations. The second interest of the analytical approach is the modeling of perturbations which affect intersatellite links. The goal of observing the Earth or other planets by flight formations consists in using such links (e.g. GRACE) as data sources. The inversion of such data forms the basis of the modeling of geodynamical parameters via the perturbation method.

The first section is dedicated to the definition and the classification of flight formations. Hill's equations and further developments are explained in the second section, as they have been historically the main tool to study the dynamics of flight formations. The third section is devoted to new methods that have appeared in the last two decades. They are based essentially on the use of differential orbital elements. Finally, conclusions and recommendations about future work on this topic are presented.

2 What is a flight formation?

In the last decades, a large number of space missions used the concept of flight formation. Sometimes, there is the belief that a formation is the same as a constellation. It is important to understand the differences between both concepts. The common point of constellations and formations is that both use a certain number of satellites with the same goal. Constellations usually use a large number of satellites (30 in the GALILEO constellation). In the case of formations, the number of satellites is usually lower, sometimes just two satellites. There is a second characteristic that, usually, makes a difference between formations and constellations: the relative distances between satellites. In constellations, the satellites are moved apart by long distances, usually thousands of kilometers. In formations, they are moved apart by middle (some kilometers) or low distances (hundreds of meters). The definition criteria is the control of the system. The control of the constellation is done individually, satellite by satellite, whereas the control of the formation is done through the relative distances and velocities of satellites. The advantages of flight formations are: more possibilities, bigger security, and smaller prize. Flight formations will enable spatial interferometry, bigger spatial telescopes, or the exploration of Earth and planets.

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2.1 Classification of flight formations

There are many ways of classifying flight formations. From a dynamical point of view, the most interesting is the following one:

- around a central body: some examples are: GRACE (to study the Earth's gravity field), the A-train (Earth observation), or TanDEM-X/ TerraSAR-X (a high precision radar interferometer for terrestrial altimetry). The difficult point in modeling these formations is the control of secular drifts induced by gravitational and non-gravitational perturbations.

- around a Lagrange point: the environment at Lagrange points is quite. The interest of sending formations to Lagrange points is to avoid perturbations and to improve the control and the stability. Interferometry missions are specially interested in these orbits. Some examples of this kind of missions are LISA or DARWIN. As in the Lagrange points the influence of the Sun is of the same order as the one due to the Earth, satellites have no more a central body dynamics. This paper does not deals with this dynamics problem.

3 Working with Hill equations

Relative motion is not a new problem. In the fifties and sixties, relative motions were studied for the spatial rendezvous problem: the final approach of a spatial spacecraft respect to another. The problem was studied using Hill equations. They have rested as the most common tool during forty years. In the last two decades that new approaches have appeared in the literature.

It is important to precise which are the variables used to represent the relative motion. Different possibilities are found in literature. The first point is the position and the velocity of each satellite projected in an inertial reference frame IJK: $\overrightarrow{x}_i|_{IJK} = (\overrightarrow{r}_i|_{IJK}, \overrightarrow{v}_i|_{IJK})^T$. One of them is chosen as the reference one, and we will indicate it by the indice $_{ref}$. All the others will be referenced generically by the indice $_i$.
A first way of expressing relative motion is the use of difference of vectors: $\Delta\overrightarrow{x}_i|_{IJK} = \overrightarrow{x}_i|_{IJK} - \overrightarrow{x}_{ref}|_{IJK}$. Such a representation is not very useful because of lack of physical sense. On the other hand, it is much more useful to use the projection of this vectors in the RTN reference frame $\Delta\overrightarrow{x}_i|_{RTN}$. The RTN reference frame is defined by the radial direction of the reference orbit(R), by its out of plane direction(N), and by a third direction perpendicular to the others (T).

It is important to note that, even if the relative positions ($\overrightarrow{\delta} = \delta_R, \delta_T, \delta_N$) and $\Delta\overrightarrow{r}_i|_{RTN}$ are equivalent, it is not the same case for velocities ($\Delta\overrightarrow{v}_i|_{RTN}, \dot{\overrightarrow{\delta}}$), the differences between both velocities being given by Coriolis terms.
A third way of expressing relative motions is through the differences of orbital elements $\Delta EO = EO_i - EO_{ref}$, where EO are the usual keplerian elements $a, e, i, \Omega, \omega, M$.

3.1 The actors of relative motion

Hill equations were first derived in Hill (1878). They are a set of differential equations that give the temporal evolution of a body relative motion on respect to a rotating reference frame. Initially, they were deduced using relative position and velocity ($\overrightarrow{\delta}, \dot{\overrightarrow{\delta}}$).
It is possible to linearize the system with respect to the distance between the reference point and the body, if they are close enough. After linearization, the equations have an analytical explicit solution without perturbations:

$$
\begin{aligned}
\delta_R(t) &= \frac{\dot{\delta}_R(0)}{n_0}\sin n_0 t - \left(2\frac{\dot{\delta}_T(0)}{n_0} + 3\delta_R(0)\right)\cos n_0 t + \left(2\frac{\dot{\delta}_T(0)}{n_0} + 4\delta_R(0)\right) \\
\delta_T(t) &= 2\frac{\dot{\delta}_R(0)}{n_0}\cos n_0 t + \left(4\frac{\dot{\delta}_T(0)}{n_0} + 6\delta_R(0)\right)\sin n_0 t + \left(-2\frac{\dot{\delta}_R(0)}{n_0} + \delta_T(0)\right) - \left(3\dot{\delta}_T(0) + 6n_0\delta_R(0)\right)t \\
\delta_N(t) &= \delta_N(0)\cos n_0 t + \frac{\dot{\delta}_N(0)}{n_0}\sin n_0 t
\end{aligned}
$$

These equations give the temporal evolution of a relative motion in the local reference frame (RTN), parametrized by initial conditions, and using time as independent variable. Three major hypothesis have been done to reach

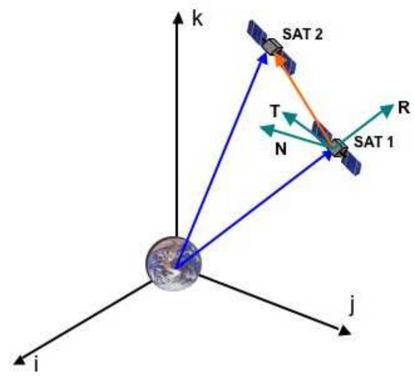

Fig. 1. *IJK and RTN reference frames*

this solution: short intersatellite distances, circular reference orbit, and no perturbation.
Each time that a new perturbation is introduced, it is necessary to solve the new resulting differential equations system. Usually, it is quite complicated to find analytical explicit solutions. Good examples of this procedure are Schweighart & Sedwick (2001) and Tsoi et al. (2005).
There have been also a number of publications dealing with the non-circular reference orbit. The first of them, is Lawden (1963). On this paper, Lawden presents the form of the relative position using the true anomaly (f) as independent variable:

$$\begin{aligned}\delta_R(t) &= A\cos f + Be\sin f + CI_2 \\ \delta_T(t) &= -A\sin f + B(1+e\cos f) + \frac{D - A\sin f}{1+e\cos f} + CI_2 \\ \delta_N(t) &= \frac{1}{1+e\cos f}(E\cos f + F\sin f)\end{aligned} \tag{3.1}$$

where A, B, C, D, E, F are constants, e is the eccentricity, and I_1, I_2 are integrals that have no explicit solution. Major inconvenient of this solution is the impossibility to present the solution in an explicit way because of the two integrals I_1, I_2. Anyway, for control purposes they rest very interesting because they are more accurate than Hill's equations. Lawden's equations have also been used by some authors in particular for control problems (Carter et al. 1987; Carter 1990; Tillerson et al. 2001, 2002).

Gómez et al. (2006) have also done an interesting work trying to find high order solutions of Hill equations in order to avoid the effects of the linearization.

4 Using differences of orbital elements

Another approach to study the relative motion has been the use of orbital elements differences: ΔEO. The temporal evolution of the relative motion is given by the evolution of the orbital elements. We propose the following method:

1. conversion of initial conditions expressed in terms of position and velocity to differences of orbital elements: $\Delta EO_0 = [\mathcal{M}(EO_0)]\Delta\vec{x_0}|_{RTN}$
2. extrapolation of differences of orbital elements $\Delta EO(t) = f(EO(t), \Delta EO_0)$
3. conversion of differences of orbital elements to position and velocity $\Delta\overrightarrow{x(t)}|_{RTN} = [\mathcal{M}^{-1}(EO)]\Delta EO(t)$

The method combines the advantages of using the position and velocity to express the relative motion, with the advantages of extrapolating orbital elements. As example, it is possible to find the homogeneous solution

of Hill's equations, or the equivalent of Lawden's homogeneous solution, in a complete explicit way.
Just some steps of this method can be found in the literature.
In Casotto et al. (1993), it is possible to find the third step of the method. Casotto finds linear relation between the differences of position and velocity projected to the RTN reference frame ($\overrightarrow{x}_i|_{RTN}$), and the differences of orbital elements (ΔEO) .
Later on, Alfriend & al. introduced their 'geometrical method', which consists in the combination of steps two and three of our method. For the third step, the results of Alfriend are slightly different from Casotto one's because Alfriend uses the relative position and velocity ($\overrightarrow{\delta}$, $\dot{\overrightarrow{\delta}}$).The whole of 'geometrical method' and some applications can be found in Schaub et al. (2002), Alfriend et al. (2000), Schaub et al. (2003), and Vadali et al. (2001).
Even if in precedent literature the first step is not used, it is necessary to understand the effect of a variation of orbital elements in terms of position and velocity.

5 Conclusions

In the past, relative motions were studied using relative Cartesian coordinates. This representation is still nowadays interesting. The use of differences of orbital elements will enable us to introduce easily new perturbations. Anyway, it will always be necessary to combine both representations in order to assure physical understanding of the motion. There is also a lack of publications dealing with the topology of the non-perturbed and perturbed motion. This interesting topic is therefore to be developed. From a general point of view, the dynamics of relative motions has not yet been studied in depth. Although this is a key feature for future missions, there is still a lot of research to be done.

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