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D. Barret, F. Casoli, T. Contini, G. Lagache, A. Lecavelier, and L. Pagani (eds)

DE HAERDTL INEQUALITY: AN INEQUALITY OF GREAT DYNAMICAL INTEREST IN THE GALILEAN SYSTEM

B. Noyelles[1] and A. Vienne[1]

Abstract.
The quasi-commensurability 7:3 between the mean motions of the Galilean satellites Ganymede and Callisto, known as De Haerdtl inequality, has never been taken into account in the scenarii of dynamical evolution of the Galilean satellites.

We used numerical tools like frequency maps to detect the chaos induced by the inequality. We showed that it induces chaos, more particularly because of Callisto's eccentricity. Anyway, even if the orbits were nearly circular, Callisto's inclination is strong enough to induce chaos. This inclination is forced by the Sun, this means that Jupiter's obliquity is partly responsible of the chaos induced by De Haerdtl inequality. This chaos is due to Chirikov diffusion, say to overlaps of resonances.

1 Introduction

As we saw in (Noyelles & Vienne 2005), De Haerdtl great inequality between the Galilean satellites Ganymede and Callisto is of high dynamical interest. This 7 : 3 mean-motion inequality induces chaos because of overlaps of lots of resonances that can be associated to a 4th-order mean-motion commensurability.

In this proceeding, we go further in the study of De Haerdtl inequality. We determine the conditions on eccentricities and inclinations that lead to chaos, and we identify some resonances responsible for this chaos.

2 A numerical study

We use a numerical system of 21 variables simulating the motion of the 4 Galilean satellites. For each satellite, we use 5 classical variables a_i (semimajor axis), h_i, k_i (related to eccentricity and pericentre), p_i and q_i (related to inclination and ascending node), i varying from 1 to 4 and being related to the satellite involved (1 stands for J-1 Io, 2 for J-2 Europa, 3 for J-3 Ganymede and 4 for J-4 Callisto). The 21^{st} variable is $3\lambda_3 - 7\lambda_4$, where λ_i is the mean longitude. This last variable is the argument of De Haerdtl inequality.

We performed long-term numerical simulations with a 10^{th} order Adams-Bashforth-Moulton integrator. The dynamical effects we took into account are Jupiter's oblateness, secular perturbations of the four Galilean satellites and the Sun until 4^{th} order in eccentricity/inclination, and of course De Haerdtl inequality. Our initial conditions are summarized Table 1 and have been obtained from L1 ephemerides (Lainey et al. 2004).

We used Laskar's algorithm (see for instance Laskar 1993) to extract the nine main proper modes of the system ν_0 to ν_8. ν_0 is related to the Laplacian plane of the system, while ν_1 to ν_4 are near the pericentres of the 4 Galilean satellites, and ν_5 to ν_8 near their ascending nodes.

3 Apparition of chaos

3.1 The method

We detected chaos using frequency maps, as did Laskar (1993). More precisely, we plotted fundamental frequencies of the system on two-dimensional maps, for different values of eccentricities and inclinations of the

[1] LAL/IMCCE, Observatoire de Paris and Université de Lille 1, CNRS UMR 8028, 1, impasse de l'Observatoire, 59000 LILLE, FRANCE, e-mail: (noyelles,vienne)@imcce.fr

Table 1. Initial conditions of the system. They have been obtained from L1 ephemerids at their origin after having dropped the short-period terms. Some of these initial conditions (more particularly a_3 and a_4) change in some simulations, this will be detailed when needed.

	J-1 Io		J-2 Europa
a_1	422029.9575713722 km	a_2	671261.1707586185 km
h_1	$2.211748005524882 \times 10^{-5}$	h_2	$-1.012497227930342 \times 10^{-4}$
k_1	$-5.275314833959475 \times 10^{-5}$	k_2	$-1.113739262971603 \times 10^{-4}$
p_1	$-1.582110263967550 \times 10^{-4}$	p_2	$-4.005213221974558 \times 10^{-3}$
q_1	$-2.925173017648710 \times 10^{-4}$	q_2	$-4.801551767300432 \times 10^{-5}$
	J-3 Ganymede		J-4 Callisto
a_3	1070621.015486640 km	a_4	1883133.519125367 km
h_3	$4.036767745745128 \times 10^{-4}$	h_4	$7.373394680171398 \times 10^{-3}$
k_3	$-1.467743466069845 \times 10^{-3}$	k_4	$-6.832780074387413 \times 10^{-4}$
p_3	$5.104532163169044 \times 10^{-4}$	p_4	$-1.577330958427725 \times 10^{-3}$
q_3	$1.686910272912197 \times 10^{-3}$	q_4	$5.738930348850733 \times 10^{-4}$
$3\lambda_3 - 7\lambda_4$	0.1508363483032813		

Table 2. Dynamical parameters related to Ganymede and Callisto. e_i are the eccentricities and γ_i the sines of the semiinclinations of the satellites on Jupiter's equatorial plane. These values have been obtained with our system of equations and initial conditions obtained from L1 ephemerides imputed from terms that do not appear in our system.

Parameter	present value	"forced" part	"free" part
e_3	1.53×10^{-3}	9.6×10^{-4}	5.7×10^{-4}
γ_3	1.93×10^{-3}	7.9×10^{-4}	1.14×10^{-3}
e_4	7.37×10^{-3}	2.1×10^{-4}	7.16×10^{-3}
γ_4	3.93×10^{-3}	3.93×10^{-3}	small

satellites. An irregular map is associated to chaos. On every map, each point has been obtained after a numerical simulation whose initial values of a_3 and a_4 (Ganymede's and Callisto's semimajor axes) vary respectively between 1070150 and 1070700 km and 1883080 and 1883200 km. Their current mean values are respectively 1070621.016 and 1883133.154 km among Lainey's last ephemerides (Lainey et al. 2006).

3.2 Significant parameters

We homothetically changed (h_i, k_i) and (p_i, q_i) to change respectively the eccentricities and inclinations of the satellites from the values indicated Table 1, in order to check their influences on the chaos. It is interesting to notice that these parameters have a part forced by the other parameters, that is the reason why we cannot change these parameters freely. Table 2 summarizes these free and forced parts.

3.3 Results

Figure 1 shows two examples of frequency maps. They can be considered as projections on the plane $(\dot{\nu_1}, \dot{\nu_4})$ of an application that associates (a_3, a_4) to the fundamental frequencies of the system. These frequencies have been obtained after 1638.4 years, with a step of 0.1 year, so with 16384 values. They show the frequency $\dot{\nu_1}$ vs. $\dot{\nu_4}$ of the system with (left) and without (right) De Haerdtl inequality with current values of eccentricity and inclinations.

When the system is regular, the maps show parallelograms because the fundamental frequencies we plot are "near" pericentres' velocities. These velocities depend on semimajor axes, which are different for every point.

Looking at the maps lets us conclude that

- At low e_4, strong chaos appears when e_3 becomes higher than 3×10^{-3}.

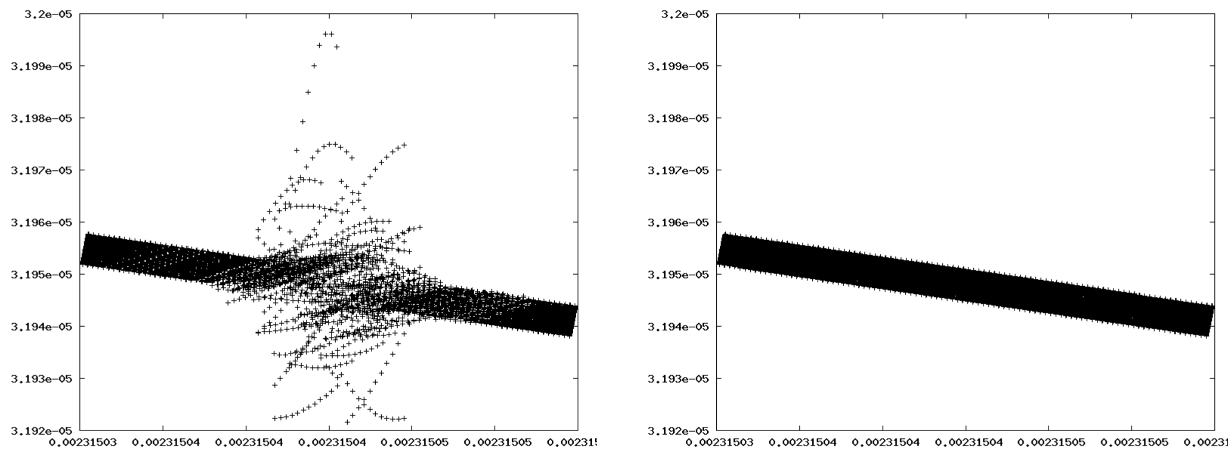

Fig. 1. Frequency maps $(\dot{\nu_1}, \dot{\nu_4})$ with (left) and without (right) De Haerdtl inequality, with current values of eccentricities and inclinations. The units are day^{-1}. These 2 plots illustrate clearly how to distinguish a regular map from an irregular one. This is a way to "see" the chaotic behaviour induced by De Haerdtl inequality.

- With e_3's present value, De Haerdtl inequality induces less chaos if e_4 is lower than 3×10^{-3}.
- If e_3 and e_4 are very low, strong chaos appears when γ_3 is higher than 2.5×10^{-3}.

4 Identification of chaos

In a planetary system, chaos is often due to Chirikov diffusion (1979), say to overlaps between resonances. It is possible to identify this chaos by plotting the possible resonances, whose arguments are like $3\lambda_3 - 7\lambda_4 + \Sigma q_i \nu_i$, with $\Sigma q_i = 4$ among D'Alembert rule, and to see an "interesting" behaviour (say, stable resonances, oscillations between several resonances, etc ...).

4.1 The method

The frequency analysis algorithm gives us the phase of every proper mode at a given time. We used this algorithm to plot the terms likely to be interesting along several numerical simulations with current values of eccentricities and inclinations, and with 400 different values of $\alpha = \frac{a_3}{a_4}$, between 0.56830 and 0.56855, current mean α being 0.56852.

4.2 Exploring some arguments

Plotting every argument of these 400 simulations shows us several possible resonances, as seen Fig. 2. This work is still in progress but we can right now give a short list of possible stable resonances :

- $3\lambda_3 - 7\lambda_4 + 2\nu_0 + \nu_3 + \nu_4$ (Fig. 2)
- $3\lambda_3 - 7\lambda_4 + 2\nu_4 + \nu_6 + \nu_7$
- $3\lambda_3 - 7\lambda_4 + \nu_0 + \nu_3 + \nu_4 + \nu_6$
- $3\lambda_3 - 7\lambda_4 + 3\nu_3 + \nu_4$

This list is not exhaustive and will be in a next future completed by other terms as well as overlaps leading for instance to stable chaos.

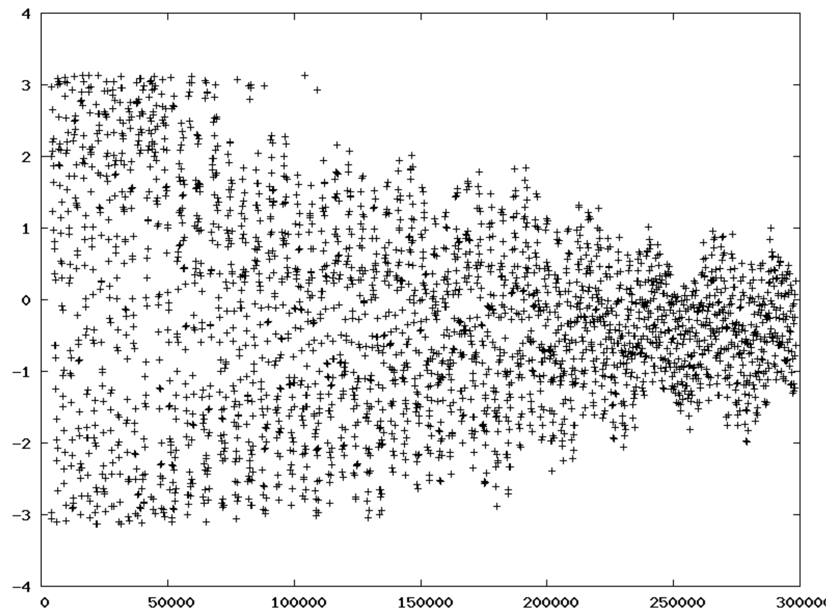

Fig. 2. An example of stable resonance. Here is plotted $3\lambda_3 - 7\lambda_4 + 2\nu_0 + \nu_3 + \nu_4$ vs time (in years). The system is trapped in a resonance at about 50000 years, this can be seen by the libration of this argument around 0. The amplitude of librations decreases with time, what lets us infer that this resonance is quite stable. So it is worth to compute this argument on a longer timespan.

5 Conclusion

This study shows that De Haerdtl inequality induces chaos for convenient values of eccentricies, inclinations and semimajor axes. We can infer that the semimajor axes have had these values in the past, the satellites migrating under tidal effects. This means that every scenario of dynamical evolution of the Galilean satellites must take De Haerdtl inequality into account. We also see that the inclinations have significant effects. These inclinations are forced by the Sun, say by Jupiter's obliquity. So, the evolution of Jupiter's obliquity should also be taken into account in a dynamical study of the Galilean satellites.

This work is still in progress and should lead to a new scenario of the formation of the Laplacian resonance between Io, Europa and Ganymede. This scenario must include De Haerdtl inequality as well as the evolution of Jupiter's obliquity, responsible of Callisto's inclination, and consequently of chaos.

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