*SF2A 2006*
*D. Barret, F. Casoli, T. Contini, G. Lagache, A. Lecavelier, and L. Pagani (eds)*

# CONVERGENCE STUDY OF THE MAGNETOROTATIONAL INSTABILITY IN A SHEARING BOX WITH A MEAN FIELD

Silvers, L. J.[1] and Balbus, S. A.[1]

**Abstract.** For more than a decade the so-called shearing box model has been used to study the fundamental dynamics of accretion discs. Such an approach is particularly useful when numerical simulations at high resolutions are needed. However, it is often difficult to know how reliable a numerically obtained quantity is, especially if the underlying flow is turbulent. Here we present some results from a resolution study, which is designed to address this issue directly.

## 1 Introduction

Linear stability analysis has shown that a differentially rotating disc is unstable in the presence of a weak magnetic field (see, for example, Balbus & Hawley 1991). This magnetororational instability (MRI) gives rise to turbulence within the disc and is a plausible way that efficient outward transport of momentum in a disc can be realized (and so permitting accretion onto the central object in a realistic timescale). It is crucially important, due to the non-linear form of the governing equations, to investigate the non-linear evolution and saturation of this instability.

The equations that govern this system are computationally demanding to evolve. Therefore, in order to study the evolution and saturation of key quantities associated with this instability, it is useful to employ a localized 'shearing-box' model. This approach has proved useful as it has enabled higher local resolutions to be achieved for the same computational cost as that for a full disc simulation. However, many of the key numerical calculations relating to this instability are over a decade old and therefore, even with a local model, authors were forced to publish results at a stated resolution (a typical resolution used was 32:64:32 points). Thus basic resolution tests were not conducted in great depth. Further, such results are published using a 1:2:1 ratio of grid points and, while it is plausible that the number of points may need to be less in the y direction, no numerical evidence exists in the literature to support this. Without *a priori* evidence to the contrary, one would choose to take a similar resolution per unit length in each of the three directions. Therefore, it is not clear if old calculations were conducted at sufficient resolution or even if they were run for sufficient duration and so it is important to return to this subject so that key statistical quantities (that are important to those who model discs) can be ascertained.

## 2 Formulation

The standard equations for a compressible isothermal fluid (in dimensional form), including magnetic diffusivity and viscosity are evolved. The parameters that can be specified in this problems are: the sound speed, $c_s$; angular velocity, $\Omega$; $q = d\ln\Omega/d\ln R$; the plasma beta, $\beta$; dynamic viscosity, $\mu$ and magnetic diffusivity, $\eta$. Here we consider the case where $c_s = \Omega = 0.001$, $\beta = 800$. Also, the dynamic viscosity and physical resistivity) are such as to give rise to an initial Reynolds number of 1000, equal to the initial magnetic Reynolds number, and an initial Lundquist number of 50.

The domain is taken to be periodic in both the $y$ and $z$ direction while it is shearing periodic in the $x$ direction (see for further details, for example, Hawley, Gammie & Balbus 1995). Simulations start from a small perturbation of the equilibrium solution for the hydrodynamics problem with a Keplarian shear velocity field

---

[1] Laboratoire de Radioastronomie, Ecole Normale Superieure, 24 rue Lhomond, 75231 Paris CEDEX 05 FRANCE

($v_y = -3/2\Omega x\hat{\mathbf{y}}$) and with a uniform magnetic field in the z direction. The perturbation used in this case is not composed of a random forcing at each grid point (as in many earlier simulations) but is a velocity perturbations that is comprised of smooth functions that are small in amplitude.

## 3 The Key Results

This work has so far shown that, while it was useful to return to this topic, there is still a lot that needs to be examined and clarified in this area and work is ongoing. Due to the required brevity of this article, we present here only a few the figures from the poster, which illustrate these points. These results on this topic have shown that:

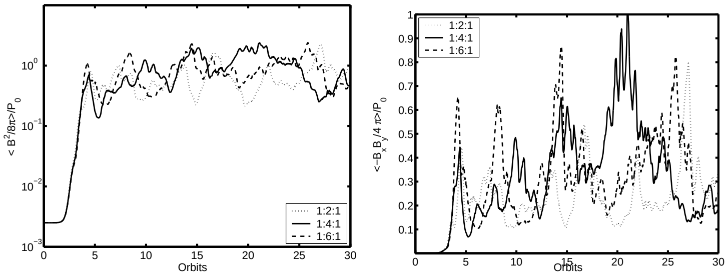

**Fig. 1.** (a) [left] The evolution of magnetic energy for different y resolution (fixed x & z resolutions). (b) [right] The evolution of the Maxwell stress for different y resolution (fixed x & z resolutions).

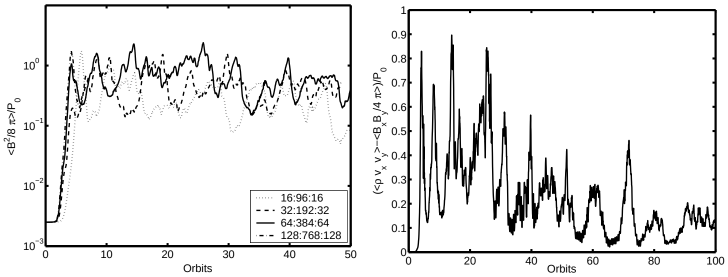

**Fig. 2.** (a) [left] The evolution of magnetic energy for different resolutions but with a fixed ratio of resolutions. (b) [right] Temporal evolution of the difference of the Reynolds stress and Maxwell stress for the 64:384:64 case.

1. There is a difference between cases with a 1:2:1 ratio of grid points and a 1:6:1 ratio. However, what is most important is that increasing the *x* resolution, from that typically used in early simulations, changes the results and we found very good agreement between the 64:384:64 and the 128:768:128 cases for over ten orbits.
2. Figure 2b shows that large numbers of orbits (considerably larger than in earlier works) are required to determine the average Reynolds and Maxwell stresses accurately as there is an initial transient phase that persists for thirty or more orbits.

This work was funded via a Chaire d'Excellence award to SAB.

## References

Balbus, S. A., & Hawley, J. F. 1991, ApJ, 367, 214

Hawley, J. F., Gammie, C. F., & Balbus, S. A. 1995, ApJ, 440, 742