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RECONSTRUCTING DARK ENERGY USING DISTANCE-LUMINOSITY AND BARYONIC OSCILLATIONS DATA

S. Fay and R. Tavakol[1]

Abstract.
We briefly summarise and further discuss the observational implications of the model-independent scheme recently put forward by us for the reconstruction of dark energy. The scheme employs rather weak geometrical features of the luminosity-distance relation motivated by recent observations. Using the SNLS supernovae data, the reconstructed luminosity-distance curves best fitting the data are shown to favour a Universe with a varying dark energy density, expanding at a slower rate than the ΛCDM model. The ΛCDM model. however, cannot be ruled out at high significance levels, in line with other recent reconstructions. We highlight the degeneracy concerning the CDM density parameter Ω_{m_0}, which acts as a barrier against a conclusive determination of whether the dark agency is the cosmological constant.

1 Introduction

The evidence that our Universe is currently undergoing a phase of acceleration at the present epoch is now overwhelming. Not only is this accelerated dynamics measured directly in the spectral and photometric data from high redshift surveys Supernovae data (Riess et al. 2004; Astier et al. 2006), it is also independently confirmed by observations of the Cosmic Microwave Background (CMB) anisotropy power spectrum (Spergel et al. 2003; Spergel et al. 2006) as well as observations of large scale structure (Tegmark et al. 2004; Seljak et al. 2005). The favoured explanation for this behaviour is that a substantial proportion (70%) of the energy density of the Universe is presently in the form of an effective fluid – dark energy – which is smooth on cosmological scales and which possesses a negative pressure. A fundamental question at present is what is the nature of this dark energy and importantly whether it is different from the cosmological constant with an Equation Of State (EOS) equal to -1.

Given the absence of a truly satisfactory theoretical model that can be embedded within candidate theories of quantum gravity, an alternative path has been to take the inverse approach of reconstructing the properties of the dark energy, including its EOS, from the Supernovae (Astier et al. 2006), and more recently the Baryon Acoustic Oscillation (BAO) data (Eisenstein et al. 2005). Many attempts have recently been made to perform such reconstructions. These take a spectrum of forms, ranging from schemes that assume specific functional forms for the cosmological parameters (Sahni et al. 2003; Szydlowski & Czaja 2003; Elgaroy & Multamaki 2006) to schemes employing more general parametrised forms (Alam et al. 2004a; Alam et al. 2004b). There have also been attempts at constructing model-independent schemes which can recover the cosmological parameters directly from the data without making specific assumptions regarding their functional forms (Wang & Mukherjee 2004; Shafieloo et al. 2006).

Recently a new model-independent reconstruction scheme was put forward which relied on rather weak assumptions regarding the geometrical features expected to be satisfied by the luminosity-distance relation (Fay & Tavakol 2006). Using this scheme together with observational data, including the recently released Supernova Legacy Survey (SNLS) and BAO data, allowed the reconstruction of the models and the corresponding EOS best fitting the data.

Here we give a brief presentation of these results and further discuss their observational implications. Furthermore, employing the geometrical form of the luminosity-distance relation we explain why the Supernovae

[1] School of Mathematical Sciences,
Queen Mary, University of London, London E1 4NS, UK

data do not tightly constrain the cold dark matter density parameter Ω_{m_0} or set a sharp lower bound on the equation of state. We also use the baryon acoustic oscillation peak in the SDSS data to constrain Ω_{m_0} and find the best fit to be in agreement with the CMB data.

The paper is organised as follows. After a very brief account of our scheme in Sect. 2 we briefly summarise the reconstruction results in Sect. 3. This is done first assuming a reasonable value for the cold dark matter density parameter. Then after a discussion demonstrating the degeneracy of the luminosity distance with respect to this parameter, baryon acoustic oscillation data is used to constrain this parameter. Finally Sect. 4 contains a brief summary of the results and further discussions.

2 The Scheme

The model-independent reconstruction scheme proposed in Fay & Tavakol (2006) was based on employing rather weak constraints on the geometrical from of the luminosity-distance d_l curves which are motivated by observations. These constraints concern the first three derivatives of the luminosity-distance d_l in the form.

(I) $d_l' \geq 0$, where a prime denotes differentiation with respect to the redshift z. This condition is trivially satisfied by for any expanding Universe and is solidly supported by all observations.

(II) $d_l'' \geq 0$. This condition can be shown to be satisfied by any Universe which is currently accelerating and which in the past tended to an Einstein-de Sitter model.

(III) $d_l''' \leq 0$. This condition can be shown to be satisfied at all redshifts by both Einstein-de Sitter and ΛCDM models. This is important as these two models are those commonly accepted as representing the early and late dynamics of the Universe.

These conditions are fully compatible with all the current high resolution observations. This provides an important justification for their use in constraining the reconstructed luminosity-distance curves.

3 Results

The reconstruction employs constraints from the SNLS supernovae data. It can proceed in two ways. Either one assumes that a value is given for Ω_{m_0} from observations or specifies Ω_{m_0} from the baryon acoustic oscillation peak in the SDSS data. Both these approaches were attempted in Fay & Tavakol (2006). Here we briefly summarise each.

Concerning the first approach, we give a summary of the results given in (Fay & Tavakol 2006) by plotting in Fig. 1 the subset of reconstructed d_l curves (represented as black in the figure) which are close to the curve best fitting the data with the χ^2 values given in the range $110.67 < \chi^2 < 111.7$. We note that 110.67 is the smallest value for χ^2 obtainable using our reconstruction method. For comparison we have also plotted in this figure the corresponding ΛCDM model with $\chi^2 = 114$ (gray line on the figure). As can be seen all these reconstructed curves have a χ^2 slightly smaller than that corresponding to the ΛCDM model and thus fit the data better. They predict a slower expansion rate than the ΛCDM model whose d_l curve lies above this reconstructed set. We note, however, that even though the ΛCDM model would be ruled out at 68.3% confidence level (assuming that the best fitting d_l curve does not correspond to a cosmological theory with more than three free parameters), it cannot be ruled out at 95.4% confidence level.

From the reconstructed d_l curves it is possible to obtain their corresponding EOS, which are shown in Fig. 2. The reconstructed EOS all allow better fits to the data than the ΛCDM model and predict values close to the ΛCDM model of -1 up to the redshift of around $z = 0.45$. Above this redshift the EOS in general increase to take values slightly larger than zero.

3.1 Degeneracy and constraining of Ω_{m_0}

The above reconstruction scheme does not provide any information about the CDM density parameter Ω_{m_0} which has to be assumed a priori. This is a consequence of the fact that the luminosity-distance relation is determined purely by the Friedmann equation which is highly degenerate with respect to this parameter.

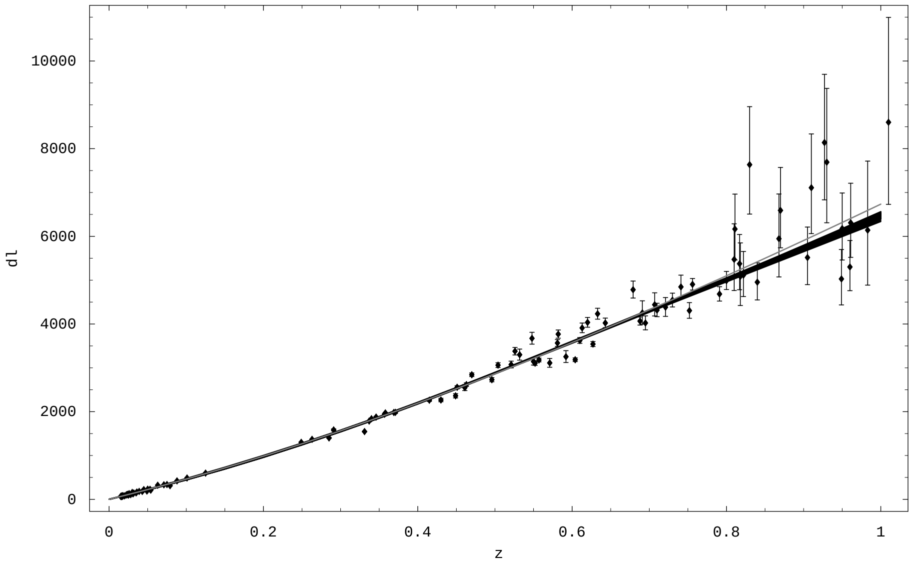

Fig. 1. Plot of a set of reconstructed luminosity-distance curves (lower blak lines) close to the best fitting curve with their χ^2 value lying in the range $110.67 < \chi^2 < 111.7$, together with the corresponding luminosity distance curve for the ΛCDM model (upper gray line), whose $\chi^2 = 114$.

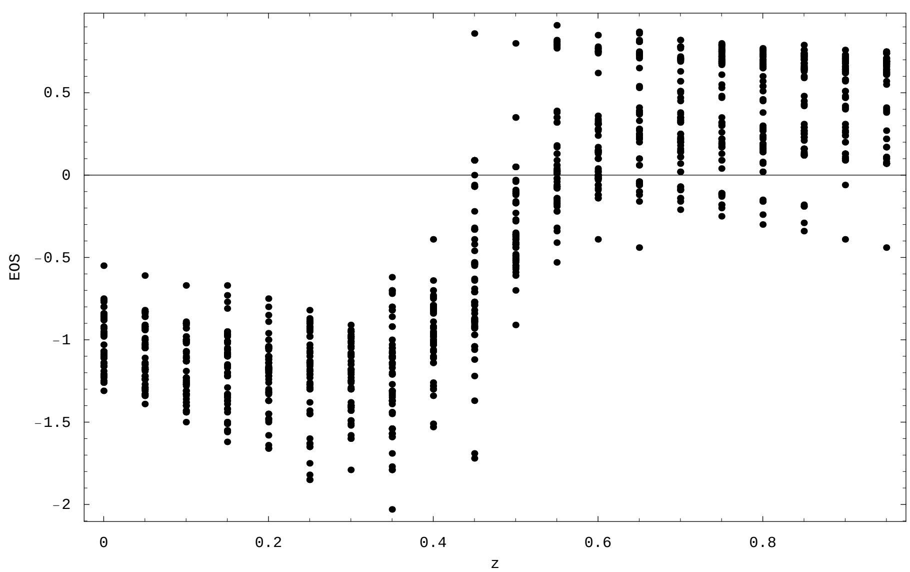

Fig. 2. Plot of the reconstructed EOS corresponding to the reconstructed luminosity-distance curves plotted in Fig. 1.

A transparent way of seeing this degeneracy is by considering a dark energy model with a constant EOS given by $(\mathrm{w} - 1)$. The corresponding Friedmann equation is then given by:

$$H^2 = H_0^2 \left[\Omega_{m_0}(1+z)^3 + \Omega_{DE_0}(1+z)^{3\mathrm{W}}\right]$$

Re-writing the CDM density parameter as $\Omega_{m_0} = \Omega_{1m_0} + \Omega_{2m_0}$, the Friedmann equation becomes:

$$H^2 = H_0^2 \left[\Omega_{1m_0}(1+z)^3 + \Omega_{2m_0}(1+z)^3 + \Omega_{DE_0}(1+z)^{3\mathrm{W}}\right]$$

where the subscript $_{DE0}$ indicates dark energy at z=0. Now this form of $H(z)$ can be viewed as a new dark energy model with a different CDM density parameter Ω_{1m_0}, and a different dark energy density ρ_{DE} represented by the last two terms in this expression. This demonstrates clearly that given a Hubble function we can construct different representations with different effective CDM density parameters and dark energy components, but with identical luminosity-distance functions d_l. Thus the luminosity-distance d_l is potentially highly degenerate with respect to the CDM density in Universe. This is important in determining the true nature of dark energy.

To constrain Ω_{m_0} one requires further observational information. Using the recent BAO data (Eisenstein et al. 2005) to constrain the reconstructed luminosity-distance curves, we found best fit for Ω_{m_0} to be $\Omega_{m_0} = 0.27$.

4 Discussion

We have briefly summarised a model-independent scheme for the reconstruction of dark energy, recently put forward in (Fay & Tavakol 2006). Using the SNLS supernovae data, we have reconstructed a set of luminosity-distance curves together with their corresponding reconstructed EOS. Confining ourselves to the neighbourhood of the best fitting curve provides a sharper representations of the results given in (Fay & Tavakol 2006). These reconstructions demonstrate that the luminosity-distance curves best fitting the data correspond to the Universe expanding slower than a ΛCDM model with a varying dark energy density. Despite this, however, the ΛCDM model cannot be ruled out.

We have highlighted the degeneracy concerning the CDM density parameter Ω_{m_0}. An important consequence of this degeneracy is that it acts as a barrier for a realistic observer (whose observations inevitably involves error bars) to determine conclusively whether the dark energy density is truly constant. This is significant in view of the fact that one of the urgent theoretical questions at present is to differentiate between the dark energy models and in particular to determine whether the dark agency is the cosmological constant.

We have also summarised how the recent baryon oscillation data can be used to give the best fit value $\Omega_{m_0} = 0.27$, in close agreement with CMB data (Spergel et al. 2003; Spergel et al. 2006). Future observations - particularly the expected SNLS data - will provide more accurate reconstructions which will in turn sharpen the bounds on the most likely value for Ω_{m_0} and the corresponding EOS. This will give a better indication as to whether the best fitting model is closer or further away from the ΛCDM model.

References

Riess, A. G., Strolger, L.-G., Tonry, J., et al. 2004, ApJ, 607, 665

Astier, P., Guy, J., Regnault, N., et al. 2006, A&A, 447, 31

Spergel, D.N., Verde, L., Peiris, H.V., et al. 2003, ApJS, 148, 175

Spergel, D.N., Bean, R., Doré, O., et al. 2006, astro-ph/0603449

Tegmark, M., Strauss, M.A., Blanton M.R., et al. 2004, Phys. Rev. D, 69, 103510

Seljak, U., Makarov, A., McDonald, P., et al. 2005, Phys. Rev. D 71, 103515

Eisenstein, D. J., Zehavi, I., Hogg, D.W., et al. 2005, ApJ, 633, 560

Sahni, V., et al. 2003, JETP Lett., 77, 201

Szydlowski, M., Czaja, M. 2004, Phys. Rev. D, 69, 023506

Elgaroy, O., Multamaki, T. 2006, astro-ph/0603053

Alam, U., Sahni, V., Deep Saini, T., Starobinsky, A.A. 2004a, MNRAS, 354, 275

Alam, U., Sahni, V., Starobinsky, A.A. 2004b, JCAP, 008, 0406

Wang, Y., Mukherjee, P. 2004, ApJ, 606, 654

Shafieloo, A., Alam, U., Sahni, V., et al 2006, MNRAS, 366, 1081

Fay, S., Tavakol, R. 2006, Phys. Rev. D, 74, 083513