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RECONSTRUCTION OF THE PRIMORDIAL DENSITY FIELD FROM REDSHIFT AND DISTANCE CATALOGS OF GALAXIES

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Abstract. Peculiar velocity reconstruction methods allow one to have a deeper insight into the distribution of dark matter: both to measure Ω_m and to characterize the primordial density fluctuations. We present here the Monge-Ampère-Kantorovitch method applied to redshift reconstruction. We show what are the limitations and the problems to overcome to prevent systematic errors. This leads for the first time to a good representation of the peculiar velocity field within a 8,000 km/s sphere. We measure $\Omega_m = 0.237^{+0.063}_{-0.056}$ using a catalog of distances within 3,000 km/s, which is in agreement with the analysis of WMAP and SDSS data.

1 Introduction

In the last 20 years, large surveys have given us the redshift positions of nearly a million galaxies. However, these differ from the real position by the radial peculiar velocities which are an important probe of the distribution of dark matter. Peculiar velocities are difficult to determine above 2000 km/s because measurement errors increase linearly with redshift. Here, we show how they can be accurately reconstructed from redshift positions and, in doing so, we also put constraints on the dark matter density. Our method also gives the primordial density fluctuations above 5 Mpc/h with no extra approximation. The reconstructed primordial universe corresponds exactly to our own, and not just in a statistical sense as is the case with CMB data.

To achieve our tasks we need find the mapping function between Lagrangian ($\vec{q}$) and Eulerian ($\vec{x}$) coordinates. The orbit of galaxies, taken as mass-tracers, between these coordinates are given by the stationary points of the Euler-Lagrange action. This problem, as formulated by Jim Peebles in 1989, does not have a unique solution, probably because of wrong choice boundary conditions and/or multistreaming. However above a few Mpc multistreaming is marginal and we shall see with the choice of proper boundaries, a unique solution can be achieved.

2 Reconstruction methods

We use the Monge - Ampère - Kantorovitch (MAK) method. It is a simplification of the exact problem proposed by Peebles and is based on the hypotheses that the displacement field is derived from a convex potential and constrained by the local mass conservation. These hypotheses are supported by simulations and arise naturally within Lagrangian perturbation theory. It has been proved (Brenier et al. 2003) that solving this problem is equivalent to solving a minimization problem:

$$I_\sigma = \min_{\sigma \in \mathbf{S}^N} \sum_{i=0}^{N} \left(\vec{x}_i - \vec{q}_{\sigma(i)}\right)^2 \tag{2.1}$$

where i runs over particles of the same mass, $\mathbf{S}^N$ is the group of permutations of rank N. Solving this problem directly is technically difficult. A fast algorithm, called the auction algorithm, has been invented by Bertsekas (1988), which has an effective complexity of $N^{2.25}$ where N is the number of particles.

This method has been tested directly on N-body simulations (Mohayaee et al. 2005). The difference between the reconstructed density field and the original one has a relative dispersion of 28% and a correlation coefficient of 96% when both are smoothed at 5 Mpc/h.

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3 Correction of errors through the analysis of mock catalogs

In general, catalogs of observations always have errors. We present here a few important errors and discuss in more detail three of them in the next subsections.

- **Boundary effects:** The *Lagrangian volume* ($\vec{q}$) needed for our reconstruction (see Eqn 2.1) is not given by catalogs. Determination of this unknown quantity can cause errors as discussed below.
- **Incompleteness:** Catalogs are built on survey based on the apparent luminosities of objects: farther intrisically more luminous objects will look fainter and hence be absent from the catalog (discussed below).
- **Redshift distortion:** In a catalog, distance of an object is generally expressed in redshift. This is not the true distance because it also includes the projection of its peculiar velocity along the line of sight. That is why we tend to use boundary constraints in redshift and not in real space. On the other hand, the reconstruction is expressed in terms of real coordinates and hence one needs make an appropriate correction for the effect.
- **Mass-to-light ratio:** Masses are not measured directly in the catalog. They are estimated either from the local dynamics of compound objects (such as groups) or from a general extrapolation of the mass to luminosity function applied to the luminosity of the object. We can cross-correlate the reconstructed peculiar velocities with the measured distances to determine the best mass-to-light ratio.
- **Zone of avoidance:** Catalogs generally do not have any data in the *galactic plane*. To fix this problem, we assume the general statistical properties are conserved across the boundary of the obscured region and hence fill it accordingly. This enables us to be able to keep the reconstruction of other parts of the catalog intact.
- **Malmquist bias:** To estimate the cosmological parameters, we compare the reconstructed displacement field to the measured distances which are affected by log-normally distributed errors. This effect is known as the volume malmquist bias, which we have taken into account by carefully computing the likelihood function for the parameters.
- **Modeling error:** The method uses an approximation of the dynamic to recover the peculiar velocity field. A measurement on mock catalogs show that the error on peculiar velocities has a Lorentzian distribution and we take this into account in the determination of Ω_m.

3.1 Redshift distortion

The easiest way to correct for redshift distortion is to assume the Zel'dovich approximation to compute the current peculiar velocity of catalog objects assuming some displacement vector. If we use this hypothesis we can build a new cost function from Eq. 2.1:

$$I_{\sigma,s} = \sum_{i=0}^{N} \left(\left(\vec{s}_i - \vec{q}_{\sigma(i)}\right)^2 - \frac{\beta(2+\beta)}{1+\beta} \frac{\left(\left(\vec{s}_i - \vec{q}_{\sigma(i)}\right)\cdot\vec{s}_i\right)^2}{||\vec{s}_i||^2} \right) \tag{3.1}$$

with $\beta = \frac{\dot{D}}{HD} \approx \Omega_m^{5/9}$, D the growth factor, H the hubble constant and $\vec{s}_i$ the redshift position of the i-th object.

The problem is that we make a linear approximation to recover the peculiar velocity and that we are again using this approximation to get the resulting predicted redshift distortion by MAK. This tends to sum up the errors and decrease the amount of useful signal.

We must be aware of two major problems. First, cluster fingers of god in redshift catalogs must be collapsed to get a good reconstruction. Second, redshift distortion generally introduces a loss of real space connectivity. This effect is particularly important on the boundaries where massive groups or clusters of galaxies may be separated from the main catalog in real space though they are connected in redshift space.

3.2 Lagrangian volume

It is possible to minimize the effect of our ignorance of the initial Lagrangian volume on the central part of the catalog. If we put the inhomogeneous catalog inside a bigger homogeneous cube and we do the reconstruction using a cubic Lagrangian volume of the same size, then we will exactly resolve the problem concerning the cube. However, as the boundaries for the catalog are not correct, the reconstruction will be fuzzy in the outer parts and at the same time the central part will be unaffected. This has been tested on mock catalogs and on average one recovers the density field with a relative dispersion of 42% and a correlation coefficient of 90%.

3.3 Catalog incompleteness

All catalogs suffer from incompleteness caused mainly by the decrease in luminosity of objects at large distances. To account for the missing luminosity at a given distance, we boost the luminosities of the object at that distance by an amount given by the Schechter luminosity function. This is not a trivial correction and probably will change the dynamics because too much mass can be assigned to the observed objects and thus these objects will be displaced less than the one whose luminosity has not been boosted. The conclusion of the study on mock catalogs is that the higher resolution part of the catalog is generally not affected by random uncertainties in the lower resolution part.

4 Application to real data

4.1 Velocity field

A direct application of the MAK reconstruction method on a redshift catalog gives us the displacement field which in turn yields the velocity field (Mohayaee et al. 2005). Here we use a catalog of 24,000 galaxies within a radius of 8,000 km s^{-1} we call 'Nearby Galaxies – 8,000 km s^{-1}', or NBG–8k (provided by Brent Tully and not yet public). We present in Fig. 1a the reconstructed velocity field for this catalog.

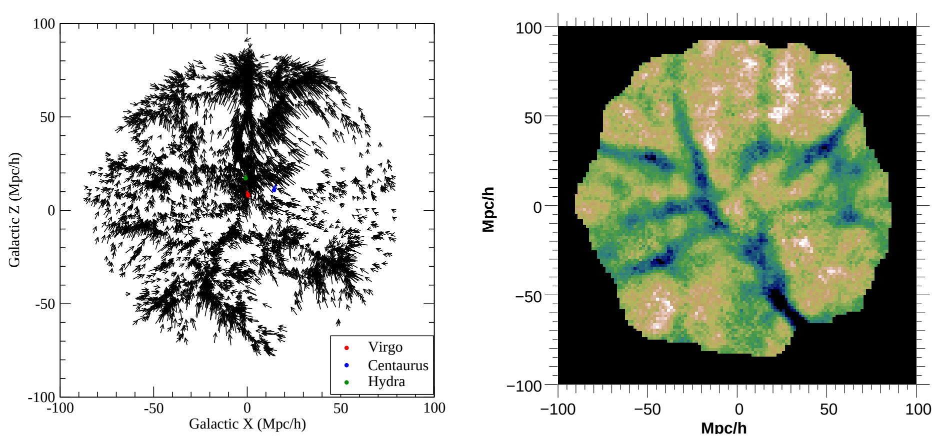

Fig. 1. (a) Left panel: Reconstructed velocity field using the NBG–8k catalog. The slice is 20 Mpc/h deep. (b) Right panel: Reconstructed primordial density field using the NBG–8k catalog. Finger-of-God effect is only partially corrected for.

4.2 Primordial density field

The primordial density field is linked to the linear displacement field. The displacement field we are reconstructing is non-linear but only to the second Lagrangian order (as with the Zel'dovich approximation). Thus, we can use the relationship between Zel'dovich displacement and the density field to obtain the primordial density field from the non-linear displacement field. The result is shown in Fig. 1b.

4.3 $\Omega_{\rm m}$ measurement

One of the applications of our method is a measurement of the local mean density of the universe. We may use the rules established in the former sections to build a correct set of initial data for the reconstruction. Then the likelihood function takes into account the special characteristics of the data in the catalog and the reconstructed velocity field. First, the error on distance modulus, and hence the absolute magnitude, is Gaussian distributed (e.g. Pizagno et al. 2007). Second, the reconstruction introduces a Lorentzian distributed error on the velocity field. The result of the measurement assuming these two distributions is shown on Fig. 2. We see that the result is in perfect agreement with other measurements from the SDSS (Tegmark et al. 2004) and WMAP3. Spergel et al. (2007) gives the measurement for WMAP3+SDSS: $\Omega_m = 0.266^{+0.026}_{-0.036}$. *Our likelihood function* is maximal at $(h, \Omega_m) = (0.805^{+0.005}_{-0.005}, 0.237^{+0.063}_{-0.056})$ at 68% of confidence level. The major axis of the ellipsoidal shape of the likelihood function can be fitted and we get $h = 0.81\left(\frac{\Omega_m}{0.3}\right)^{0.022}$. This formula can be compared to WMAP's relation: $\Omega_m h^2 = 0.127$ and SDSS's: $h = 0.7(\frac{\Omega_m}{0.3})^{-0.35}$.

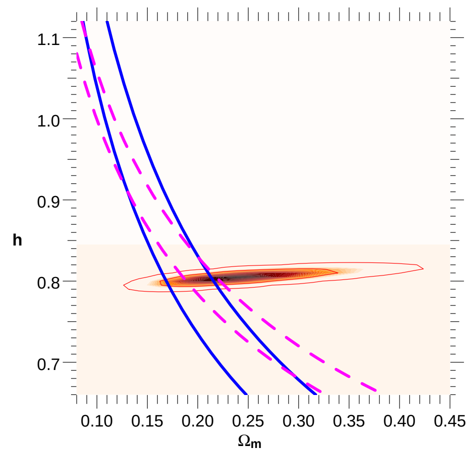

Fig. 2. Measurement of Ω_m based on NBG–3k catalog. The thin red lines are 68% and 95% of confidence limit for the measurement based on the catalog. The thick blue lines is coming from the WMAP3 experiment. The dashed magenta line is obtained by the SDSS project.

5 Conclusion

We have described how to use the MAK algorithm to accurately reconstruct the primordial density fluctuations above a few Mpc. The peculiar velocity field is accurately reconstructed provided one makes good assumptions on the mass-to-light ratio and takes into account the systematic errors. From it, we made a new apparently unbiased measurement of Ω_m in agreement with WMAP and SDSS results.

This method is better than a previous approach like POTENT (Bertschinger & Dekel 1989). As it uses Lagrangian approximations, it handles the first Eulerian non-linearities better. Studies on mock catalogs prove that our method works optimally with respect to the Zel'dovich approximation. Moreover, it only needs redshift positions to reconstruct the peculiar velocity field, contrary to POTENT.

As for the future, large statistical uncertainties remain and further work is needed to reduce them, e.g. by using bigger catalogs to have better constraints on the Lagrangian volume. We may also want to improve the reconstruction by dropping the hypothesis of convex potential on the displacement field and including higher order gravity effects along the trajectories.

Once the primordial density field is reconstructed we will use it to resimulate the local universe and determine if we recover the structures which are present in the catalog.

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