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ESTIMATION OF THE HUBBLE CONSTANT BY THE EFFECTS OF THE COSMOLOGICAL CONSTANT

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Abstract. We have modified the spherical infall model, first developed by Lemaître and Tolman, in order to include the effects of the cosmological constant. The resulting velocity-distance relation was evaluated numerically. This equation, when fitted to actual data, permits the simultaneous evaluation of the central mass and of the Hubble parameter. Application of this relation to the Local Group, to the Virgo cluster, to the M81 group, to the Sculptor group and to the IC342/Maffei group yields to a Hubble parameter $H_0 = 74 \pm 4\, kms^{-1}Mpc^{-1}$.

1 Introduction

New and high quality data on galaxies belonging to nearby groups have improved considerably estimates of masses and mass-to-light ratios (M/L) of these systems. Searches on the POSS II and ESO/SERC plates (Karachentseva & Karachentsev 1998; 2000) as well as "blind" HI surveys (Kilborn et al. 2002) lead to the discovery of new dwarf galaxies, increasing substantially their known population in the local universe. Moreover, in the past years, using HST observations, distances to individual members of nearby groups have been derived from magnitudes of the tip of the red giant branch (Karachentsev 2005 and references therein), which have permitted a better membership assignment and a more trustful dynamical analysis.

Previous estimates of M/L ratios for nearby groups were around $170 M_{\odot}/L_{B,\odot}$ (Huchra & Geller 1982). However, virial masses derived from the aforementioned data are significantly smaller, yielding M/L ratios around 10-30 $M_{\odot}/L_{B,\odot}$. If these values are correct, the local matter density derived from nearby groups would be only a fraction of the global matter density (Karachentsev 2005). However, for several groups the crossing time is comparable or even greater than the Hubble time and another approach is necessary to evaluate their masses, since dynamical equilibrium is not yet attained in these cases.

Lynden-Bell (1981) and Sandage (1986) proposed an alternative method to the virial relation in order to estimate the mass of the Local Group, which can be extended to other systems dominated either by one or a pair of galaxies. Their analysis is essentially based on the spherical infall model. If the motion of bound satellites is supposed to be radial, the resulting parametric equations describe a cycloid. Initially, the radius of a given shell embedding a total mass M expands, attains a maximum value and then collapses. At maximum, when the turnaround radius R_0 is reached, the radial velocity with respect to the center of mass is zero. For a given group, if the velocity field close to the main body, probed by satellites, allows the determination of R_0, then the mass can be calculated straightforwardly from the relation:

$$M = \frac{\pi^2 R_0^3}{8 G T_0^2} \tag{1.1}$$

where T_0 is the age of the universe and G is the gravitational constant.

Data on the angular power spectrum of temperature fluctuations of the cosmic microwave background radiation derived by WMAP (Spergel et al. 2003) and on the luminosity-distance of type Ia supernovae (Riess et al. 1998; Perlmutter et al. 1999), lead to the so-called "concordant" model, e.g., a flat cosmological model in which $\Omega_m = 0.3$ and $\Omega_v = 0.7$. The later density parameter corresponds to the present contribution of a

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cosmological constant term or a fluid with negative pressure, dubbed "quintessence" or dark energy. The radial motion leading to the aforementioned $M = M(R_0, T_0)$ relation neglects the effect of such a term, which acts as a "repulsive" force. This repulsive force is proportional to the distance and its effect can be neglected if the zero-velocity surface is close to the center of mass. Turnaround radii of groups are typically of the order of 1 Mpc (Karachentsev 2005), while the characteristic radius at which gravitation is comparable to the repulsion force is $R_* = 1.1M_{12}^{1/3}$ Mpc, where $M_{12} = M/(10^{12}M_\odot)$ and the Hubble parameter H_0 was taken equal to 70 $kms^{-1}Mpc^{-1}$. This simple argument suggests that the effect of the cosmological term can not be neglected when deriving masses from the $M = M(R_0, T_0)$ relationship.

Recently, Peirani & Pacheco 2006 have derived a new velocity-distance relation and a new estimation of the mass inside the zero-velocity radius R_0 by taking into account the effects of the cosmological term. They found:

$$M = 1.827\frac{H_0^2R_0^3}{G} = 4.1 \times 10^{12}h^2R_0^3 \ M_\odot \tag{1.2}$$

$$v(R) = -\frac{0.875H_0}{R^n}(\frac{GM}{H_0^2})^{(n+1)/3} + 1.274H_0R \tag{1.3}$$

where $h = H_0/(100 \ kms^{-1}Mpc^{-1})$, R_0 is in Mpc and $n = 0.865$.

Comparing with Eq. 1.1, the inclusion of the cosmological constant represents, for a given R_0, an *increase* of about 38% on the mass derived by such a procedure.

In this paper, we apply the new velocity-distance relation to the Local Group, to the Virgo Cluster, to the Sculptor group, to the M81 group and to the IC342/Maffei group. As we shall see, values of the Hubble parameter resulting from fits of the actual data to the velocity-distance relation including a cosmological term, are in better agreement with recent estimates then those derived from the relation obtained either by Lyndell-Bell (1981) or Sandage (1986).

2 Applications

2.1 The Local Group

An immediate application of the velocity-distance relationship is the determination of the Local Group mass, concentrated mainly on M31 and the Milky Way, as well as the Hubble parameter itself. Recent data on neighboring galaxies of the Local Group were summarized by Karachentsev et al. (2002), who have estimated $R_0 = 0.94 \pm 0.10$ and derived from Eq. 1.1 a total mass of 1.3×10^{12} $M_\odot$ for the M31/MW pair.

Here, Eq. 1.2 was fitted to the data by Karachentsev et al. (2002), but varying now both the mass and the Hubble parameter in order to minimize the velocity dispersion. We have obtained $h = 0.74 \pm 0.04$ and $M = (2.5 \pm 0.7) \times 10^{12}$ $M_\odot$, where the quoted errors are again estimates based on the uncertainties of the fitted parameters. Figure 1 shows data points and the velocity-distance relation defined by the previous parameters. Had we used Eq. 1.2 instead of Eq. 1.1 in the fitting procedure, a similar result for the mass would have been obtained, but with a *higher* Hubble parameter, e.g., $h = 0.87 \pm 0.05$.

2.2 The Virgo Cluster

Dynamical models for the Virgo cluster based on the Lemaître-Tolman model were, for instance, developed by Hoffman et al. (1980). They have modeled the projected velocity dispersion as a function of the angular distance and, from comparison with data, derived a mass of $(4.0 \pm 1.0) \times 10^{14}h^{-1}$ $M_\odot$ contained inside a sphere of 6^o radius, which corresponds approximately to the central relaxed core of the cluster. Using the virial relation, Tully & Shaya (1984) obtained a mass of $(7.5 \pm 1.5) \times 10^{14}$ $M_\odot$ for this central core. More recently, Fouqué et al. (2001) using the Lemaître-Tolman model derived a mass of 1.3×10^{15} $M_\odot$ inside a radius of 8^o.

Galaxies with Virgocentric distances higher than 1.7 Mpc, corresponding approximately to the core radius, and less than 15 Mpc, were selected from the list by Teerikorpi et al. (1992), constituting a sub-sample of 27 objects. The best fit of Eq. 1.3 to data gives $h = 0.65 \pm 0.09$ and $M = (1.10 \pm 0.12) \times 10^{15}$ $M_\odot$. Figure 1 shows the velocity-distance data for the galaxies of our sample and the theoretical v = v(R) relation computed with the derived parameters. The higher mass derived in the present analysis confirms some early results based on models of the velocity field in the vicinity of the Virgo cluster using the Lemaître-Tolman equations, as those performed by Tully & Shaya (1998) and Fouqué et al. (2001).

2.3 M81, Sculptor and IC342/Maffei

The same analysis have been performed for three nearby groups of galaxies. First, the group associated to M81 have been studied in detail by Karachentsev et al. (2002), where about 30 satellites have been considered. Figure 1 shows the best fit by the relation Eq. 1.2 to those data and we obtained $M = (9.7 \pm 3.4) \times 10^{11}\ M_\odot$ and $h = (0.69 \pm 0.05)$. This value of the mass is comparable to the one obtained by Karachentsev et al. (2002) $M = (1.6 \pm 0.3) \times 10^{12}\ M_\odot$ by using Eq. 1.1.

The Sculptor group is the closest group of the Local Group. Here again, data relative to this group can be found in Karachentsev et al. (2003a). The best fit by (1.2) to those data are represented in the Fig. 1. We found $M = (1.5 \pm 1.3) \times 10^{11}\ M_\odot$ and $h = (0.67 \pm 0.06)$. For comparaison, Karachentsev et al. (2003a) derived a mass slighty higher of $M = (5.5 \pm 2.2) \times 10^{11}\ M_\odot$ with Eq. 1.1.

Finally, we have considered the IC342/Maffei group. The data are available in Karachentsev et al. (2003b). We derived $M = (2.0 \pm 1.2) \times 10^{11}\ M_\odot$ et $h = (0.58 \pm 0.10)$ and the best fit to the data is shown in Fig. 1.

3 Conclusions

The contribution of a dark energy term in the mass-energy budget of the universe seems to be well established at the present time. In this study, the usual velocity-distance relation based on the Lemaître-Tolman model, was revisited in order to include effects due to such a cosmological term. The dynamical equations were solved numerically and the relation $M = M(H_0, R_0)$, defining the mass inside the zero-velocity surface was recalculated. For a given R_0, the resulting masses are about 38% *higher* with respect to the original relation derived from the Lemaître-Bondi model ($\Omega_v = 0$), if the dark energy is modeled by a cosmological constant. The resulting v = v(R) relation was applied to the Local Group, to the Virgo cluster and to three other nearby groups. From the best fitting procedure, the mass for each group and the Hubble constant value derived are:

Galaxy group	mass ($M_\odot$)	h
Local Group	$(2.5 \pm 0.7) \times 10^{12}$	0.74 ± 0.04
Virgo Cluster	$(1.10 \pm 0.12) \times 10^{15}$	0.65 ± 0.09
M81	$(9.7 \pm 3.4) \times 10^{11}$	0.69 ± 0.05
Sculptor	$(1.5 \pm 1.3) \times 10^{11}$	0.67 ± 0.06
IC342/Maffei	$(2.0 \pm 1.2) \times 10^{11}$	0.58 ± 0.10

If we take into account all the values relative to the Hubble constant, we obtain $h = 0.67 \pm 0.03$. The introduction of a cosmological constant term modifies the velocity-distance relation in comparison with that derived from the Lemaître-Tolman model. Nevertheless both descriptions of the velocity field near the Local Group or the Virgo cluster yield masses comparable to within a factor of two. This is probably due to the fact that errors still present in distance estimates mask differences between both models. However, when searching for a best fit of both models to data, there is a substantial difference in the resulting Hubble parameter. The Lemaître-Tolman model requires h in the range 0.87-0.92 in order to fit adequately the Local Group and Virgo data respectively, whereas Eq. 1.1 requires h in the range 0.65-0.74, more consistent with recent determinations and with the "concordant" model. In this sense, the inclusion of the cosmological constant in the v(R) relation seems to improve the representation of actual data.

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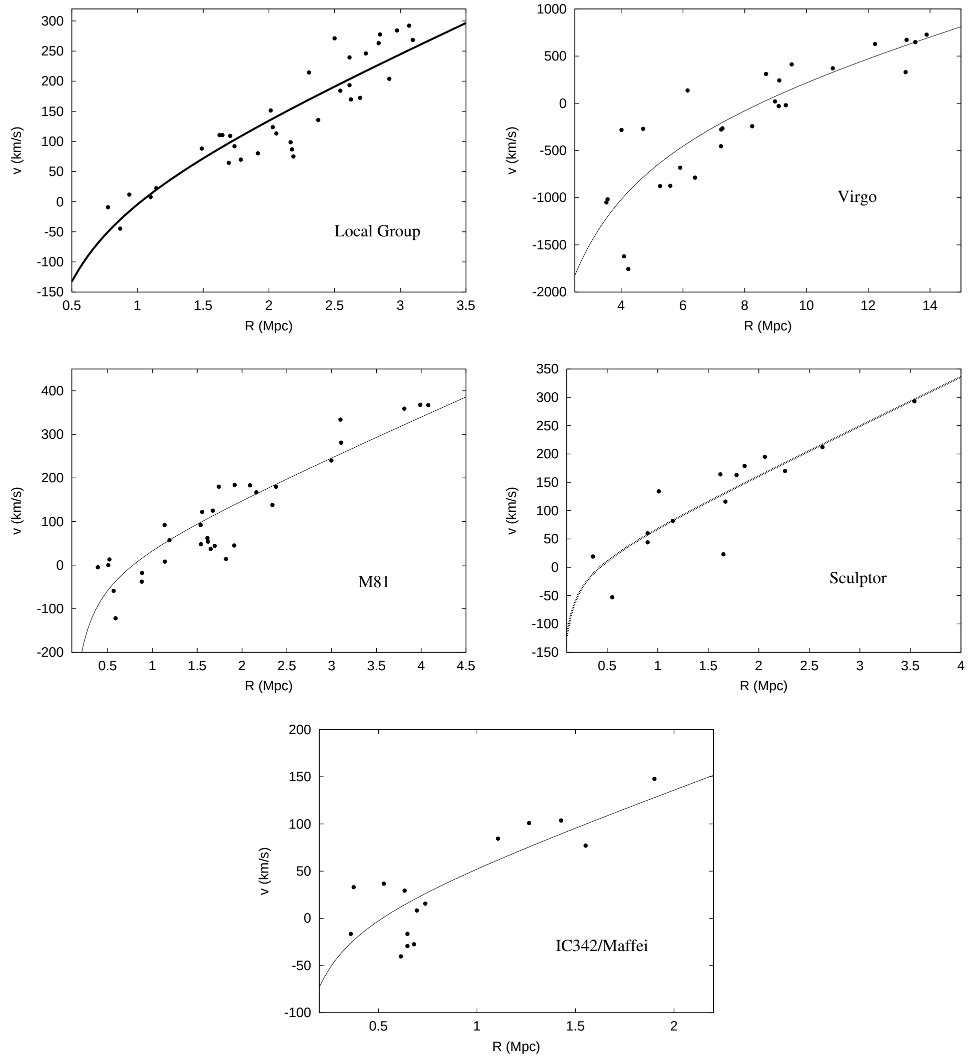

Fig. 1. Velocity and distance data for galaxies of the Local Group (upper left panel), Virgo cluster (upper right panel), M81 group (center left), Sculptor group (center right) and IC341/Maffei (lower panel). Each solid line represents the best fit to the $v = v(r)$ relation.

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