SF2A 2006
D. Barret, F. Casoli, T. Contini, G. Lagache, A. Lecavelier, and L. Pagani (eds)

TIDAL EFFECTS IN EXTRASOLAR PLANETARY SYSTEMS

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Abstract. With the discovery of new extrasolar planetary systems day after day, we have to understand the physical processes which are driving the dynamical evolution of such systems. We focus here on the tidal processes acting between stars and their giant fluid planets. We describe briefly our new theoretical results on the hydrodynamical tides (equilibrium and dynamical tides). In particular, we present the complete set of dynamical equations governing the tidal evolution which we have derived in a consistent way and for the most general case: eccentric orbits, non synchronized components, general inclination between the orbital spin and those of components.

1 Astrophysical motivation

The principal astrophysical motivation of this work is the study of the dynamics of planetary systems. In the solar system, the tidal effects acting on giant planets and their satellites are still not understood: at best, their modeling involves adjustable parameters. The same is true for the extra-solar planetary systems, of which we have now various examples. Many questions remain open concerning the physical processes that are responsible for the tidal dissipation, but these can now be constrained by the orbital properties of the extra-solar planetary systems (semi-major axis, eccentricity, obliquity and stability). The dynamical evolution of binary systems has received much attention already, but often some simplifying assumptions have been made, such as neglecting the relative inclinations of the orbital and rotational spins. That is why we consider here the most general case, with non-zero eccentricity and inclinations.

2 The cause of the dynamical evolution of binary systems: the energy dissipation

The dynamical evolution of binary systems can be summarized as follows. In the initial state, the keplerian orbits of the two components are elliptical, their rotation are not synchronized with the orbital motion and their spins are not aligned with the orbital spin. The final state is that of minimal energy, where the orbits are circularized, where the rotation of both components are synchronized with the orbital motion and where all spins are aligned. The rate at which the systems evolves toward this final state depends on the physical processes that are responsible for the conversion of kinetic energy into heat.

3 The tidal velocity field and the energy dissipation processes

The tidal potential induces flows inside giant planets (or stars) which are submitted to dissipation processes. Those can be classified in two types. The first one is the **equilibrium tide** which is a large-scale circulation induced by the hydrostatic adjustment of the planet (or of the star) to the perturbation of the tidal potential. On the other hand, tides excite the low frequencies eigenmodes of the planet (or of the star) such as gravity waves (in stably stratified regions), inertial waves and the associated mixed modes, namely the gravito-inertial waves (in stably stratified regions) (see Fig. 1); this is the **dynamical tide**.

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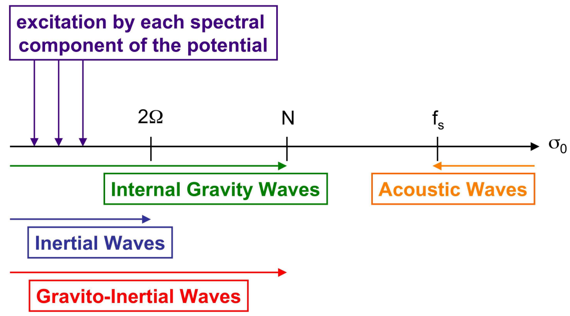

Fig. 1. The dynamical tide in a fluid planet: the eigenmodes are excited by the tidal potential (2Ω, N, $f_{\rm s}$ are respectively the inertial, Brunt-Väisälä and acoustic cut-off frequencies).

The two modes of dissipation of kinetic energy into heat in fluid planets and stars are the **viscous friction** due to the action of turbulence on tidal flows inside convective regions and the **radiative damping** in stably stratified zones. Viscous friction acts on the equilibrium tide and on the dynamical tide (the inertial waves) inside the convective envelope of giant planets and of solar-type stars, while radiative damping has a negligible effect on the equilibrium tide but a strong impact on the dynamical tide (the gravity and the gravito-inertial waves) (cf. Zahn 1966a,b-1975-1977, Mathis 2005b and references therein).

4 The equilibrium tide

In the most general case, the differential rotation of the components (cf. Mathis & Zahn 2005a) and their relative inclinations are taken into account (see Fig 2).

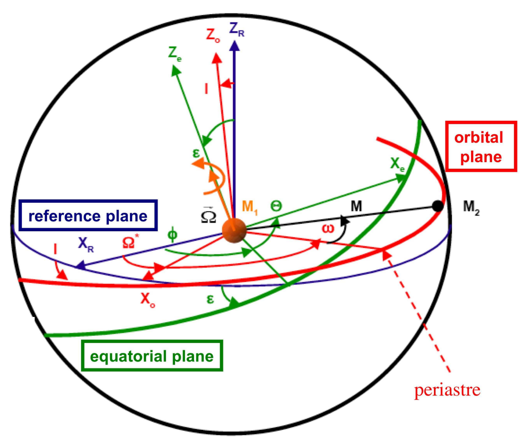

Fig. 2. Binary system in the general case where the relative inclinations are taken into account. The Euler angles of the orbit are its inclination I, the argument of the periastron ω, and the longitude of the ascending node Ω^*. Those of the components are the obliquity ε, the sideral angle Θ, and the precession angle ϕ.

The macroscopic velocity field is expanded as:

$$\vec{V} = r \sin\theta \, \Omega \left(r, \theta\right) \widehat{e}_{\varphi} + \vec{V}_{\rm T}, \tag{4.1}$$

where the first term on the right-hand side is the azimuthal field associated with the differential rotation, and Ω the angular velocity, while $\vec{V}_{\rm T}$ corresponds to the tidal flow. r, θ, φ are the spherical coordinates with respect to the equatorial plane, and $\widehat{e}_\varphi$ is the unit vector in the azimuthal direction.

We may assume that the equilibrium tide is a small perturbation to the hydrostatic equilibrium. Thus, we proceed with the following linear expansion for every scalar quantity, X:

$$X\left(r,\theta,\varphi,t\right)=X^{(0)}\left(r\right)+X^{(1)}\left(r,\theta,\varphi,t\right), \tag{4.2}$$

$X^{(0)}$ and $X^{(1)}$ being respectively associated with the hydrostatic configuration and with the tidal perturbation. Then, we split all equations and functions in two parts:

$$\vec{V}_{\rm T}=\vec{V}_{\rm I}+\vec{V}_{\rm II} \quad \text{and} \quad X^{(1)}=X_{\rm I}+X_{\rm II}\,; \tag{4.3}$$

- **System I** which describes the adiabatic response of the star to the tidal potential, and which is in phase with it: it is the *adiabatic tide*;
- **System II** which describes the response induced by the dissipative processes (here the viscous friction due to turbulence in the convective envelope), which is in quadrature with the perturbing potential: we call it the *dissipative tide*.

Those expansions are then introduced in the system formed by the equation of dynamics

$$\rho\left[D_t\vec{V}+2\vec{\Omega}\wedge\vec{V}+\vec{\gamma}\left(\vec{V}\right)\right]=-\vec{\nabla}P+\rho\vec{\nabla}U\left(\vec{r},t\right)+\rho\vec{\nabla}\phi+\vec{F}_{\rm V}\left(\nu_t,\vec{V}\right), \tag{4.4}$$

the equation for the mass conservation

$$D_t\rho+\vec{\nabla}\cdot\left(\rho\vec{V}\right)=0, \tag{4.5}$$

the entropy equation

$$D_tS+\vec{V}\cdot\vec{\nabla}S=0, \tag{4.6}$$

and the Poisson equation for the gravitational potential

$$\nabla^2\phi+4\pi G\rho=0. \tag{4.7}$$

Here $D_t=\partial_t+\Omega\partial_\varphi$ and $\vec{\gamma}(\vec{V})=r\sin\theta\,\vec{V}\cdot\vec{\nabla}\Omega\widehat{e}_\varphi$. ρ, P, S, ϕ are respectively the density, the pressure, the macroscopic entropy and the gravitational potential. $U\left(\vec{r},t\right)$ is the tidal potential while the action of turbulence on the tidal flow in convective envelopes is modeled through an eddy-viscosity, ν_t. The treatment of Eq. 4.4, 4.5, 4.6 and 4.7 yields the velocity field of the adiabatic tide, $\vec{V}_{\rm I}$, and that of the dissipative tide, $\vec{V}_{\rm II}$. Then, we get the perturbation of the gravific potential associated to the dissipative tide, $\phi_{\rm II}$, which drives the secular dynamical evolution of the binary system. We directly deduce the perturbing function which is the most useful to get the complete set of the dynamical evolution equations:

$$\mathcal{R}_{\rm II}=\frac{M_1+M_2}{M_1}\phi_{\rm II}\left(r\geq R\right)=-\sum_{\{m,j,p,q\}\in\mathcal{D}}\left\{\mathcal{R}_{{\rm II};m,j,p,q}\left(\Omega,\nu_t;\varepsilon,a,e,I\right)\right\}. \tag{4.8}$$

The sum over m,j,p,q corresponds to the sum over the different Fourier components of the tidal potential. Using the method given by Yoder (1995), we deduce the equations for the rotation rate and the obliquity of each component

$$\left\{\begin{array}{l} I\frac{{\rm d}\Omega}{{\rm d}t}=\Gamma_{\rm T}=-\sum_{\{m,j,p,q\}\in\mathcal{D}}\left\{m\mathcal{R}_{{\rm II};m,j,p,q}\left(\varepsilon,a,e,I\right)\right\}\\ I\Omega\frac{\rm d}{{\rm d}t}\cos\varepsilon=-\sum_{\{m,j,p,q\}\in\mathcal{D}}\left\{\left(j-m\cos\varepsilon\right)\mathcal{R}_{{\rm II};m,j,p,q}\left(\varepsilon,a,e,I\right)\right\}, \end{array}\right. \tag{4.9}$$

while the Lagrange equation leads us to the evolution equations for the semi-major axis, the eccentricity and the inclination of the orbital plane:

$$\left\{\begin{array}{l} \frac{1}{a}\frac{{\rm d}a}{{\rm d}t}=\frac{1}{a^2}\sum_{\{m,j,p,q\}\in\mathcal{D}}\left\{\left[2\left(1-p\right)+q\right]\mathcal{R}_{{\rm II};m,j,p,q}\left(\varepsilon,a,e,I\right)\right\}\\ \frac{1}{e}\frac{{\rm d}e}{{\rm d}t}=\frac{1}{2}\frac{1-e^2}{e^2}\frac{1}{a^2}\sum_{\{m,j,p,q\}\in\mathcal{D}}\left\{\left[2\left(1-p\right)\left(1-\frac{1}{\sqrt{1-e^2}}\right)+q\right]\mathcal{R}_{{\rm II};m,j,p,q}\left(\varepsilon,a,e,I\right)\right\}\\ \frac{\rm d}{{\rm d}t}\cos I=-\frac{1}{2}\frac{1}{\sqrt{1-e^2}}\frac{1}{a^2}\sum_{\{m,j,p,q\}\in\mathcal{D}}\left\{\left[2\left(1-p\right)\cos I-j\right]\mathcal{R}_{{\rm II};m,j,p,q}\left(\varepsilon,a,e,I\right)\right\}. \end{array}\right. \tag{4.10}$$

These equations may be integrated to describe the evolution of any particular system, from given initial conditions. Then one can simply compare the characteristic time-scales for the synchronization $t_{\rm sync}=-I\left(\Omega-n\right)/\Gamma_{\rm T}$ and the circularization $t_{\rm circ}=-{\rm d}t/{\rm d}\ln e$ with the observed properties of a sample of planetary systems. Our treatment is thus a generalization of previous treatments of the equilibrium tide, where we have taken into account for the first time all the relative inclinations.

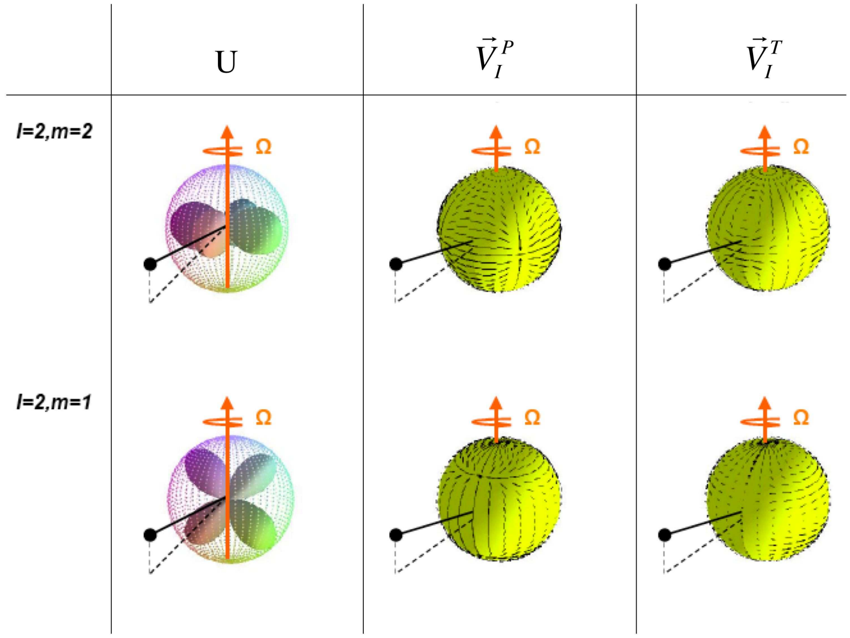

Fig. 3. The adiabatic equilibrium tide in an inclined binary component. The tidal potential has two angular modes $l = 2, m = 2$ and $l = 2, m = 1$, the $m = 1$ mode being due to the relative inclination between the orbital spin and that of the component. We show the angular function of each mode of the tidal potential, U, relative to the line component-companion, and the corresponding poloidal and toroidal parts of the adiabatic tide, $\vec{V}_{\rm I}^P$ and $\vec{V}_{\rm I}^T$, for an homogeneous sphere at $r/R_s = 0.95$ where R_s is the sphere radius. The dissipative tide $\vec{V}_{\rm II}$ has a much weaker amplitude than the adiabatic one, but the same spatial behavior than its toroidal components $\vec{V}_{\rm I}^T$.

5 Dynamical tide

In parallel, we have undertaken some improvements in the description of the dynamical tide, inside giant planets and stars. Our first contribution has been to generalize the previous treatment of the dynamical tide inside stably stratified regions by taking into account simultaneously, in the same way as it has been done for the equilibrium tide, the differential rotation and the relative inclinations in the system. The next step, in collaboration with the team of Michel Rieutord, will aim at a better description of the inertial waves inside convective regions, and at the excitation of the gravito-inertial waves by the tidal potential at the interface between convection and radiation zones.

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