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DEVELOPMENT OF ANISOTROPY IN MHD TURBULENCE

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Abstract. We perform numerical simulations to characterize the transition from strong to weak magnetohydrodynamic (MHD) turbulence for freely decaying conductive flows in presence of a uniform magnetic field at large scale. Due to reduction of energetic transfers along the background field direction, the flow anisotropy, as measured by Shebalin angles, increases with increasing intensity of the uniform field. Time evolution of global quantities of velocity and magnetic fluctuations shows charasteristic Alfvénic oscillations.

1 Introduction

In order to better understand the solar wind turbulence and its anisotropic properties due to the presence of a strong component of solar magnetic field (see Dasso et al. 2005), we perform tri-dimensionnal simulations of incompressible MHD flows as first approximation to astrophysical plasmas. The electrically conducting flow is thus described by the coupled Navier-Stokes and induction equations ;

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \nu \nabla^2 \mathbf{u} + (\nabla \times \mathbf{b}) \times \mathbf{B} \ , \tag{1.1}$$

$$\partial_t \mathbf{b} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{b} \ , \tag{1.2}$$

together with $\nabla \cdot \mathbf{v} = 0$, $\nabla \cdot \mathbf{b} = 0$, and with a unit mass density assumed. Here $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, where $\mathbf{B}_0$ is a uniform magnetic field, $\mathbf{u}$ and $\mathbf{b}$ stand for velocity and magnetic field fluctuations, P is the pressure, ν is the kinematic viscosity and η the magnetic diffusivity. The magnetic Prandtl number $P_M = \nu/\eta$ is hereafter chosen to be unity.

Turbulence occurs at Reynolds number $R_e = UL/\nu$ greater than a critical threshold (with U and L the characteristic velocity and length scale of the flow). Note that, at $P_M = 1$, the magnetic Reynolds number $R_M = UL/\eta$ equals to R_e. The kinetic and magnetic energy transfers create structures of smaller and smaller size up to dissipative and diffusion scales. The external magnetic field induces the development of Alfvén waves and flow anisotropy along the $\mathbf{B}_0$ direction. At $\mathbf{B}_0$ intensity strong enough, this can even lead to bi-dimensionnalization of the flow. Turbulent motions and waves propagation interplay with two different characteristic times : the eddy turnover time of structures of size l, $\tau_{nl} \sim l/u_l$ (with u_l the typical velocity at this scale), and the Alfvén time $\tau_A \sim l/V_A$ (with $V_A = B_0/\sqrt{4\pi}$ the Alfvén velocity). For B_0 intensities well above the *rms* level of kinetic and magnetic flutuations, the flow dynamics becomes dominated by the Alfvén waves dynamics (Galtier it et al. 2000), leading to weak turbulence ($\tau_{nl} \gg \tau_A$) as opposed to turbulence also known as strong ($\tau_{nl} \sim \tau_A$).

2 Numerical setup and initals conditions

For different intensities of the background magnetic field, $B_0 = 0, 1, 5$ et 15, we integrate numerically the MHD equations (1.1 - 1.2), in a 2π-periodic box using a pseudo-spectral method with 256^3 collocation points, and a second-order finite-difference scheme in time. The initial kinetic and magnetic fields correspond to spectra proportional to $k^2 exp(-k/2)^2$ for $k = [1, 8]$, it i.e. a flat modal spectrum up to $k = 2$, with equal kinetic and magnetic energies $E_v(t = 0) = E_b(t = 0) = 1/2$. At scale injection, the initial kinetic and magnetic Reynolds numbers are about 800 for flows at $\nu = \eta \sim 4 \times 10^{-3}$. The dynamics of the flow then freely decays.

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3 Temporal analysis

Figure 1.a shows the temporal evolution of the total enstrophy, $\Omega^T(t) =< [\nabla \times (\mathbf{v} + \mathbf{b})]^2 > (t)$ (where $< \cdot >$ means space averaging), which measures the full dissipation level of the flow. The earlier time of the dynamics, near the first inflection point, is almost inviscid corresponding to the small scale generation. For the highest values of B_0, this generation slows down, while the maximum of dissipation decreases and time at which it occurs increases. In other words, the dynamics slows down as B_0 increases. In physical space, this corresponds to elongation of structures along $\mathbf{B_0}$ (i.e. a lack of small scale generation in that direction), and in spectral space, it is linked to inhibition of kinetic and magnetic energy transfers along that direction. This confirm our first results obtained at lower resolution with 128^3 grid points (Bigot et al. 2005).

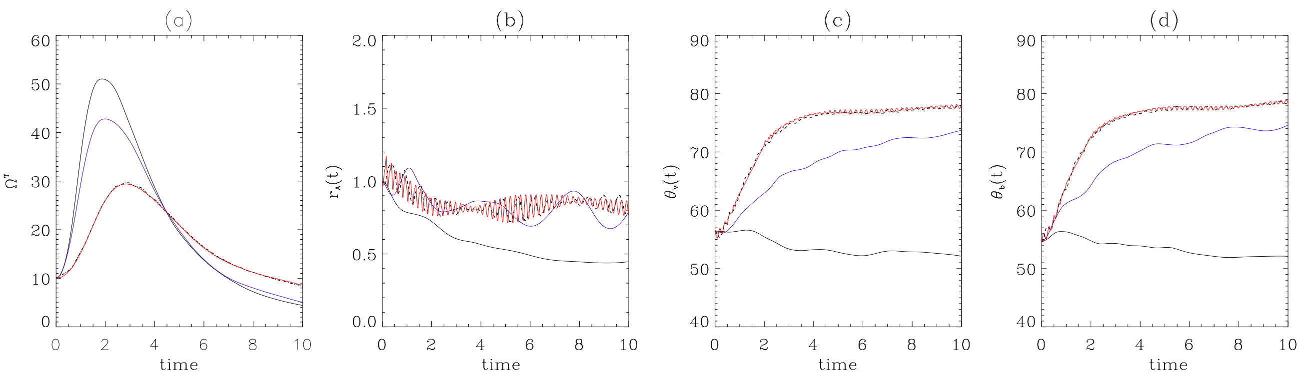

Fig. 1. Temporal evolutions : (a) Total enstrophy $\Omega^T(t)$ for $B_0 = 0$ (black), $B_0 = 1$ (blue), $B_0 = 5$ (dash black) and $B_0 = 15$ (red), (b) Alfvén ratio $r_A(t)$, and Shebalin angles: for velocity (c) and for magnetic field (d) (see text)

The Alfvén ratio $r_A(t) = E_v(t)/E_b(t)$ can be used to measure the prevalence of Alfvén waves (its departures from unity suggest the presence of non Alfvénic fluctuations). In presence of the external magnetic field, exchanges between magnetic and velocity fluctuations, due to Alfvén waves, produce oscillations as shown in Fig. 1.b. The oscillation periods are given by the Alfvén times: $\tau_A \sim 2$, 0.6 and 0.2 for $B_0 = 1$, 5 et 15, respectively, based on $L \sim \pi$, the characteristic lenghtscale of the flow.

To quantify the degree of anisotropy associated with a flow, we use the generalized Shebalin angles (see Oughton et al. 1994, and references therein), defined as $\tan^2 \theta_{\mathbf{q}} = \sum k_\perp^2 |\mathbf{q}(\mathbf{k}, t)|^2 / \sum k_z^2 |\mathbf{q}(\mathbf{k}, t)|^2$, with $k_\perp^2 = k_x^2 + k_y^2$, and $\mathbf{q}$ stands for $\mathbf{v}$ or $\mathbf{b}$ (see Fig. 1.c and 1.d). Note that here $\mathbf{B_0}$ is in the $\mathbf{z}$-direction. Initially, $\theta_{\mathbf{v}} \sim \theta_{\mathbf{b}} \sim 54, 74°$ corresponding to an isotropic $3D$ flow, the two angles then similarly evolves, their behavior depending only on the intensity of $\mathbf{B_0}$. For $B_0 = 0$ the energy transfer is similar in all directions and the temporal evolution of Shebalin angles remains almost constant, close to its initial value. For $B_0 = 5$ and $B_0 = 15$, the Shebalin angles quickly increase and stabilize around 78°. Thus, as expected, anisotropy develops along the $\mathbf{B_0}$-direction. However, the flow is not totaly confined in planes perpendicular to $\mathbf{B_0}$, which should be the case of a purely bi-dimensional flow with current and vortex lines perpendicular to flow plane. This $2D$ configuration compares to asymptotic value of 90° of Sheblin angles: spectra have all their energy in modes perpendicular to $\mathbf{B_0}$.

From all temporal results, one can see that flows with highest values of B_0 behave quite similarly while for $B_0 = 1$, the flow presents a transitional regime from the case with no background magnetic field.

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