

MAGNETIC FLUCTUATION SPECTRUM IN THE INNER SOLAR WIND

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Abstract. Waves and turbulence are ubiquitous in the inner solar wind. Whereas Alfvén waves and Kolmogorov-type energy spectra are found at low frequencies, whistler waves and steeper magnetic fluctuation power law spectra are detected at frequencies higher than a fraction of hertz (at 1 AU). This multi-scale turbulence behavior may be investigated in the framework of 3D Hall MHD. In that context, I have developed a wave turbulence analysis which shows that the steepening of magnetic spectra may be attributed to dispersive nonlinear processes rather than pure dissipation as often stated.

1 Introduction

From the very beginning of *in situ* observations it was realized that the interplanetary medium was not quiet but rather highly turbulent and permeated by fluctuations of plasma flow velocity and magnetic field on a wide range of scales, from 10^{-6} Hz up to several hundred hertz (Belcher & Davis 1971; Coroniti et al. 1982; Leamon et al. 1998). The detailed analyses revealed that these fluctuations are mainly characterized (at 1AU) by power law energy spectra around $f^{-1.7}$ at low frequency ($f < 1$ Hz), which are generally interpreted directly as wavenumber spectra by using the Taylor “frozen-in flow” hypothesis (Goldstein & Roberts 1999). This spectral index is somewhat closer to the Kolmogorov prediction for neutral fluids ($-5/3$) than the Iroshnikov–Kraichnan prediction for magnetohydrodynamic (MHD) ($-3/2$) (see *e.g.* Pouquet 1993). Both heuristic predictions are built, in particular, on the isotropic turbulence hypothesis which is questionable for the inner interplanetary medium since apparent signatures of anisotropy are found through, for example, the detection of Alfvén waves (Belcher & Davis 1971) or the variance analysis of the magnetic field components and magnitude (Barnes 1981). Note that from single-point spacecraft measurements it is clearly not possible to specify the exact three-dimensional (3D) nature of the interplanetary turbulent flow which still remains an open question.

For timescales shorter than few seconds ($f > 1$ Hz), the statistical properties of the solar wind change drastically with, in particular, a steepening of the magnetic fluctuation power law spectra over more than two decades (see *e.g.* Coroniti et al. 1982; Leamon et al. 1998) with a spectral index on average around -3 . This new inertial range – often called dissipation range – is characterized by a bias of the polarization suggesting that these fluctuations are likely to be right-hand polarized (Goldstein et al. 1994) with a proton cyclotron damping of Alfvén left circularly polarized fluctuations. This proposed scenario seems to be supported by Direct Numerical Simulations (DNS) of compressible $2\frac{1}{2}$ D Hall-MHD turbulence (Ghosh et al. 1996) where a steepening of the spectra is found – although on a narrow range of wavenumbers – and associated with the appearance of right circularly polarized fluctuations. It is likely that what has been conventionally thought of as a dissipation range is actually a second – dispersive – inertial range and that the steeper power law is due to nonlinear wave processes rather than pure dissipation (Krishan & Mahajan 2004).

2 Hall Magnetohydrodynamics Turbulence

Spacecraft measurements made in the interplanetary medium suggest a nonlinear dispersive mechanism that will be modeled by the following 3D incompressible Hall-MHD equations

$$\nabla \cdot \mathbf{V} = 0, \nabla \cdot \mathbf{B} = 0, \tag{2.1}$$

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$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\nabla P_* + \mathbf{B} \cdot \nabla \mathbf{B} + \nu \nabla^2 \mathbf{V}, \quad (2.2)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{V} - d_i \nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}] + \eta \nabla^2 \mathbf{B}, \quad (2.3)$$

where \mathbf{B} is normalized to a velocity, P_* is the total pressure, ν is the viscosity and η is the magnetic diffusivity. The Hall effect appears in the induction equation as an additional term proportional to the ion inertial length d_i ($d_i \sim 100$ km at 1 AU) which means that it is effective when the dynamical scale is small enough. In other words, for large scale phenomena this term is negligible and we recover the standard MHD equations. In the opposite limit, *e.g.* for very fast time scales, ions do not have time to follow electrons and they provide a static homogeneous background on which electrons move. Such a model where the dynamics is entirely governed by electrons is called the Electron MHD (EMHD) model.

Recently, a rigorous analysis of nonlinear transfers in the inner solar wind has been proposed in the context of Hall-MHD wave turbulence (Galtier 2006) where Alfvén, ion cyclotron and whistler/electron waves are taken into account. This approach reconciles somehow the picture, in one hand, of a solar wind made of propagating waves and, in other hand, a fully turbulent interplanetary medium. The main rigorous result derived is a steepening of the anisotropic magnetic fluctuation spectrum at scales smaller than d_i with anisotropies of different strength, large scale anisotropy being stronger than at small scales. The Hall MHD turbulence spectrum is characterized by two inertial ranges, which are exact solutions of the wave kinetic equations, separated by a knee as in the solar wind. The position of the knee corresponds to the scale where the Hall term becomes sub/dominant. The single anisotropic phenomenology proposed recovers the power law solutions found and makes the link continuously in wavenumbers between the two scaling laws. This prediction for the total energy spectrum is

$$E(k_{\perp}, k_{\parallel}) \sim k_{\perp}^{-2} k_{\parallel}^{-1/2} (1 + k_{\perp}^2 d_i^2)^{-1/4}. \quad (2.4)$$

We see how the large scale prediction is affected by the Hall effect through the term proportional to the ion skin depth d_i : only the scaling in the perpendicular (to the mean magnetic field) wavenumber k_{\perp} is modified. We remind that in such a strongly anisotropic situation the most relevant dependence is in k_{\perp} . Thus we see that the power law becomes steeper at small ($k_{\perp} d_i > 1$) scales as it is reported by *in situ* measurements in the solar wind. Therefore this steepening may be attributed to dispersive nonlinear processes rather than pure dissipation.

Single spacecraft measurements are clearly not sufficient for data interpretation since signatures of anisotropy are found in the solar wind. Efforts are currently made with Cluster spacecraft data from which it is possible to extract the 3D magnetic turbulent spectra of the magnetosheath thanks to a k-filtering technique (Sahraoui et al. 2006). Cluster may exit from the terrestrial magnetosphere to make solar wind measurements. The application of the k-filtering technique to the high frequency part of the solar wind magnetic fluctuations seems then possible. It may lead, for the first time, to a direct and rigorous comparison with theoretical predictions. Strong turbulence in Hall-MHD is then an important issue since it is probably the dominant regime at 1 AU. This point is currently investigated (Galtier & Buchlin 2006).

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