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ON THE IMPACT OF DISK GRAVITY ON PLANETARY MIGRATION

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Abstract. Linear theory as well as numerical simulations of the tidal interaction between a protoplanetary disk and a planet embedded in predict the planet undergoes an orbital decay that brings it to the vicinity of the central star. This is known as planetary migration. The migration rate is very sensitive to the gravitational potential felt by the planet and the disk. In this communication, we show that both the planet and the disk must orbit in a same gravitational potential. If the disk gravity is accounted for, inward migration of low-mass planets is accelerated.

1 Introduction

A planet embedded in a protoplanetary disk excites density waves at resonances. Each disk's particle on a circular orbit has two natural frequencies (Binney & Tremaine 1987). If the particle is kicked radially by a perturbing potential, it oscillates radially at its horizontal epicyclic frequency κ, which then corresponds to its radial natural frequency. Furthermore, if the perturbing potential pushes the particle azimuthally, but not radially, its orbital frequency does not change so that the azimuthal natural frequency of the particle is zero. Resonances occur when the perturbing frequency of the planet, seen in the matter frame, matches one of the natural frequencies of the disk's particles: $\pm\kappa$ (inner and outer Lindblad resonances) or 0 (corotation resonances).

Goldreich & Tremaine (1979) showed that the planet deposits angular momentum in the disk only at the position of the resonances. This exchange of angular momentum corresponds to a torque exerted by the planet on the disk. Newton's third law implies the disk exerts a torque on the planet. The deposit of angular momentum at outer (resp. inner) Lindblad resonances induces a negative (resp. positive) torque on the planet. The sum of these inner and outer Lindblad torques, called the differential Lindblad torque, is always negative in realistic protoplanetary discs (Ward 1997). The accumulation of angular momentum at corotation resonances yields another torque, called the corotation torque, that scales with the gradient of the inverse of the disk vortensity (Goldreich & Tremaine 1979; Ward 1991). The corotation torque is usually a positive quantity, but less than the absolute value of the differential Lindblad torque. The sum of the corotation and differential Lindblad torques is therefore negative and causes the planet semi-major axis to decrease. The timescale of this inward migration is small compared with the characteristic time of the gas accretion onto a protoplanetary solid core and the lifetime of the disk gas. It jeopardizes current theories of planetary formation since it seems very unlikely to build a gaseous planet before its protoplanetary core has reached the very vicinity of the central star. Hence the challenge to find physical ingredients that would lower the migration rate, whose spearhead is undoubtedly the use of numerical simulations of disk-planet tidal interaction.

In this communication we study the impact of the disk gravity on the migration rate by means of numerical simulations.

2 Dependence of the migration rate on the disk surface density in linear regime

All results of numerical simulations were obtained using the staggered-mesh dedicated polar hydrocode Fargo (Masset 2000). In this section we focus on the case of a low-mass planet and check the linear dependence of

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its migration rate on the disk surface density at the planet radius (Goldreich & Tremaine 1979; Tanaka et al. 2002).

2.1 Case of a non self-gravitating disk

We perform a double numerical experiment involving a $M_p = 5 \times 10^{-6}$ $M_\star$ planet mass embedded in a non self-gravitating disk, and measure its migration rate for various disk surface densities at the planet initial radius. The corresponding range of disk masses extend from 1 to 10 Minimum Mass Solar Nebula (MMSN).

On the one hand, we assume that the planet is held on a fixed circular orbit. In this case, refered to as case 1, both the planet and the disk feel the star's gravity but do not feel the disk's. Thus, the planet has a strictly Keplerian orbital velocity. The migration rate is infered from the torque acted on the planet. On the other hand, we assume that the planet can evolve freely in the disk. In this case, refered to as case 2, the planet feels the star and the disk gravities whereas the disk does not feel its own gravity. This corresponds to the standard scheme of all simulations dealing with disk-planet tidal interaction. The migration rate is straightforwardly computed from the variations of the planet semi-major axis. Figure 1 shows the migrations rates obtained in cases 1 (triangles) and 2 (circles).

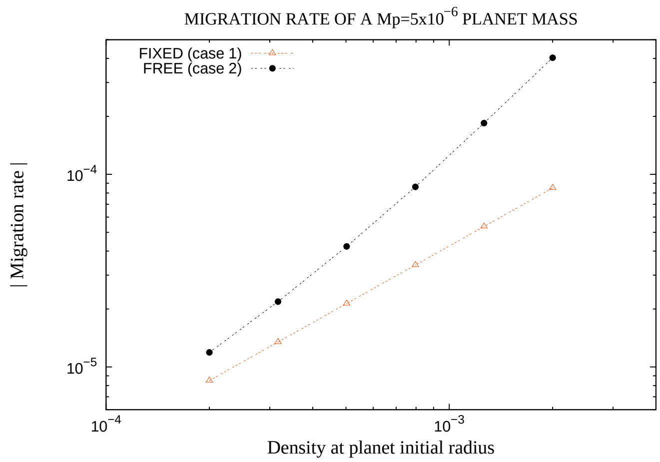

Fig. 1. Absolute value of the migration rate of a $M_p = 5 \times 10^{-6}$ $M_\star$ planet mass held on a fixed circular orbit (triangles, case 1) and freely migrating under the disk gravity (circles, case 2).

As expected, migration rates of case 1 scale with the disk surface density. Nonetheless, case 2 shows two surprising results. For a given surface density, the migration rate is far greater than expected from case 1. Moreover, the migration rate increases faster than linearly with the disk surface density.

These two results can be explained as follows. The planet orbital frequency is slightly higher in case 2 than in case 1, whereas the disk rotation profile remains the same in both cases. Thus, inner and outer Lindblad resonances are shifted inwards. While outer resonances get closer to the planet, which strengthens the outer Lindblad torque, inner resonances move away from the planet, which weakens the inner Lindblad torque. The differential Lindblad torque is then more negative, hence inward migration is faster. This shift of Lindblad resonances results from the fact that the planet and the disk do not feel the same gravitational potential. It is all the more important that the disk mass is large, which explains why drift rates of case 2 grow faster than linearly with the disk surface density. One should bear in mind that the torque acting on the planet is very sensitive to the position of Lindblad resonances: however slight the above shift may be, its impact on the migration rate can be dramatic. A natural solution to avoid this artificial resonance shift is to take the disk self-gravity into account.

2.2 Case of a self-gravitating disk

In this case, refered to as case 3, the planet and the disk orbit in the same gravitational potential, just like in case 1, except the disk mass is now accounted for. This is why it is meaningful to compare the drift rates obtained in cases 1 and 3. In addition to case 2, case 3 implies an outward shift of Lindblad resonances due to the increase of the disk orbital frequency. Thus, the sign of the shift of Lindblad resonances between cases 1 and 3 is not obvious since it deals with two opposite effects. Pierens & Huré (2005) showed analytically that both effects do not exactly cancel out: the net effect of the disk gravity, in linear regime, is to accelerate inward migration. However, their study involved axisymmetric fields so that they only took account of the axisymmetric component of the disk gravity.

We perform numerical simulations including a self-consistent treatment of the disk self-gravity. Radial and azimuthal self-gravitating accelerations can be read as convolution products, so be numerically computed with 2D Fourier transforms. Since considering a fully self-gravitating disk can be computationally expensive, we also consider the case of an axisymmetric disk potential, that only involves a 1D Fourier transform. Figure 2 compares the migration rates of case 1 (triangles) with those obtained in case 3 for an axisymmetric disk potential (asterisks) and for a fully self-gravitating disk (squares).

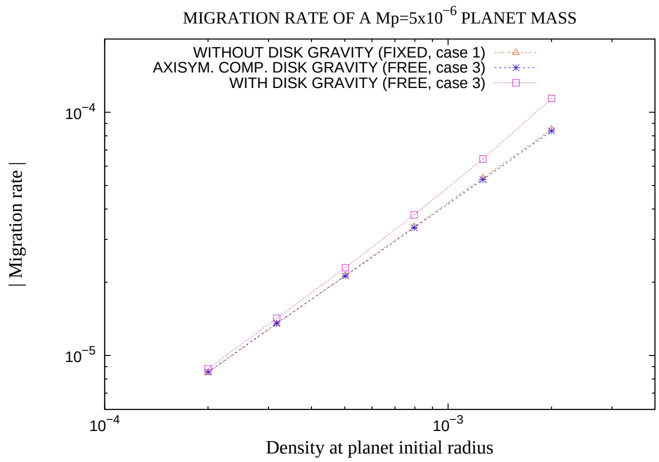

Fig. 2. Absolute value of the migration rate of a 5×10^{-6} $M_\star$ planet mass obtained in case 1 (triangles) and case 3, for an axisymmetric disk potential (asterisks) and a fully self-gravitating disk (squares).

Considering only the axisymmetric part of the disk potential reestablishes the proportional scaling of the migration rate with the disk surface density. In addition, whatever the disk surface density may be, it almost completely cancels out the artificial shift of Lindblad resonances underlined in case 2. This result should be compared to the analysis of Pierens & Huré (2005) who find a significant increase of the drift rate. Nevertheless, their analysis includes the fact that density waves propagate between Lindblad resonances in self-gravitating disks. An axisymmetric disk potential cannot take this effect into account and fails to reproduce this increase.

When a fully self-gravitating disk is considered, we do obtain a net increase of the migration rate from case 1. This increase scales with the disk surface density and reaches a value as significant as 30% of relative difference for a disk mass of 10 MMSN, which corresponds to a Toomre Q parameter of 5 at the planet radius. Thus, the qualitative trend of Pierens & Huré (2005) for the migration rate in linear regime is confirmed and generalized for a fully self-gravitating disk.

3 Dependence of the specific torque on the planet mass in linear regime

In this section, the disk surface density is fixed at the planet radius to correspond to a Toomre Q parameter of 5 at this radius. We aim at studying how previous results depend on the planet mass in linear regime. The planet is assumed to be held on a fixed circular orbit. This does not mean the planet cannot feel the disk gravity: the planet angular frequency is carefully initialized depending on whether we want the planet to feel the disk gravity or not. In the former case, the planet angular frequency brings in the disk gravity radial acceleration, while it is strictly Keplerian in the latter case. In figure 3 is depicted the specific torque acted on several planet masses for a non self-gravitating disk (triangles), an axisymmetric disk potential (asterisks) and a fully self-gravitating disk (squares).

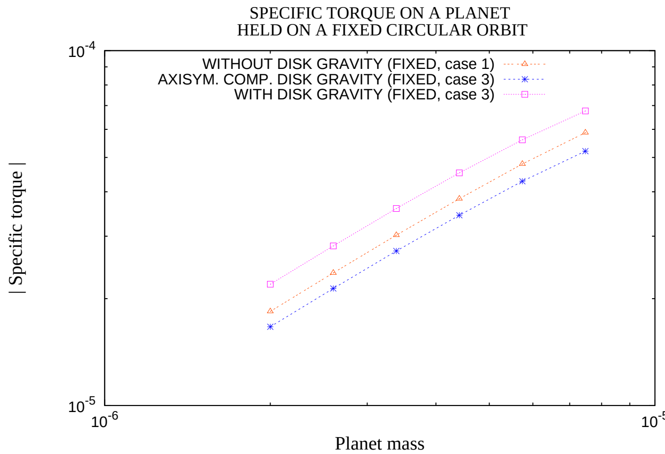

Fig. 3. Absolute value of the specific torque on a planet held on a fixed circular orbit for a non self-gravitating disk (triangles), an axisymmetric disk potential (asterisks) and a fully self-gravitating disk (squares).

Independently of the planet mass, taking account of the axisymmetric component of the disk self-gravity lowers the specific torque acted on the planet, while a fully self-gravitating disk increases it. The former trend results from the shift of Lindblad resonances that occurs when both the planet and the disk angular frequencies depend on the disk gravity. The latter result can be explained as follows. In a non self-gravitating disk, as well as in an axisymmetric disk potential, only the planet can deposit angular momentum at Lindblad resonances and then exert a torque on the disk. But in a fully self-gravitating disk, the circumplanetary disk mass around the planet also deposits angular momentum at Lindblad resonances. Thus, the torque acted on a planet by a fully self-gravitating disk is stronger since it involves the planet mass plus a function of the circumplanetary disk mass. Current work is in progress to determine precisely which amount of the disk mass participates to the torque exerted on the planet. Future work will generalize these results out of the linear regime.

4 Conclusions

The torque exerted on the planet, and then its migration rate, are very sensitive to the position of Lindblad resonances. Migration rates or torque computations infered from numerical simulations are reliable insofar as the planet and the disk orbit on the same gravitational potential. As far as the linear regime is concerned, inward migration is faster when the disk gravity is accounted for.

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