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DM DIMENSIONING FOR THE NEXT GENERATION AO SYSTEMS : STRATEGIES AND RULES

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Abstract.
Adaptive optics (AO) provides a real time correction of turbulence and allows to improve the angular resolution of the astronomical telescopes. The design of AO systems and of its key components (deformable mirror (DM), wave-front sensors...) is very important to provide the best performance. The definition rules are well known for 10m class telescopes. However the study of the next generation of telescopes, especially the Extremely Large Telescope (ELT), associated with the development of new technological solutions, particularly concerning DMs, require to refine and in some cases to reconsider these rules. Both turbulent parameters (seeing, outer scale ...), telescope effects, system design and performance requirements have to be considered in order to define the DM key parameters (stroke, inter-actuator stroke, influence function ...). We present a study based on the combination of analytical expressions and numerical simulation results. We do not give absolute rules of design but we present key aspects to keep in mind when designing the DM.

1 Introduction

The next generation of telescopes for astronomy is under study. The astronomers develop new high angular resolution instruments based on adaptive optics (AO) systems for 30m class telescopes (Roddier 1999, C.R Physique 2005). One of the key components of these systems are the deformable mirrors (DM) that allow to correct in real time the effects of the atmospheric turbulence.

This paper discusses the design rules of these DMs. The rules are well known for 10m class telescopes. However the study of the next generation of telescopes, especially the Extremely Large Telescope (ELT), associated with the development of new technological solutions, particularly concerning DMs, require to refine and in some cases to reconsider these rules. Both turbulent parameters (seeing, outer scale ...), telescope effects, system design and performance requirements have to be considered in order to define the DM key parameters (stroke, inter-actuator stroke, influence functions ...). We present a study based on the combination of analytical expressions and numerical simulation results. We do not give absolute rules of design but we present key aspects to keep in mind when designing the DM.

In a first part, we remind the principles and the equations of the atmospheric turbulence. Then we study the impact of the fluctuations of the turbulent phase in the pupil of the telescope. The last part presents the impact on the specification of a DM and particularly on the actuator command requirements.

2 Atmospheric turbulence

2.1 The theory of the atmospheric turbulence

The atmosphere is a turbulent environment. The turbulence is created by the kinetic energy produced by the displacement of the air masses. This energy creates whirlwind with different scales. Kolmogorov theory explains that the vortices with a diameter of about ten meters to hundred meters, called the outer scale of the turbulence (L_0), give their energy to smaller vortices with diameters down to a few millimetres, which

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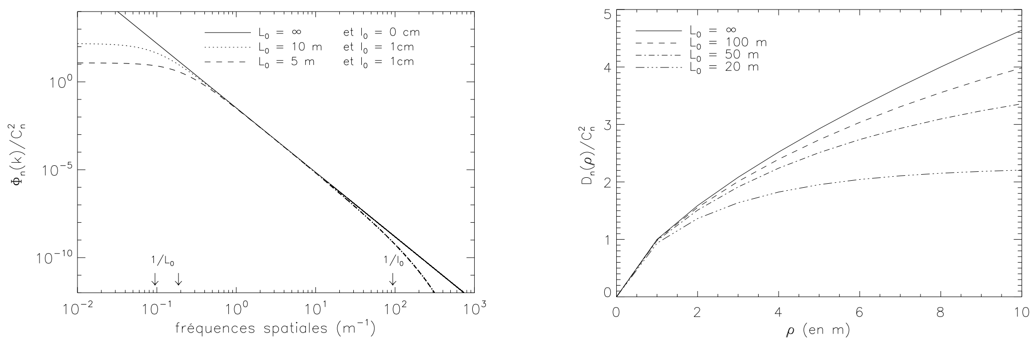

Fig. 1. Power spectral density of the indices fluctuations (left) and structure function of the indices fluctuations (right) for different values of the outer scale

corresponds to the inter scale of the turbulence (l_0). The domain between these two scales is the inertial domain. The theory of Kolmogorov allows to find the statistical equations of the turbulence in the inertial domain. In the atmosphere, mechanical turbulence is associated to refraction index fluctuations: this optical turbulence perturbs the propagation of the light. We can characterise the relative index fluctuations by its power spectral density (PSD) in the inertial domain given by:

$$W_{\Delta_n,h}(f) = 0.033(2\pi)^{-\frac{2}{3}}C_n^2(h)(f)^{-\frac{11}{3}} \quad (2.1)$$

where f is the modulus of the spatial frequency and $\frac{1}{L_0} > f > \frac{1}{l_0}$. $C_n^2(h)$ is the index structure constant related to the strength of the turbulence at the altitude h. Von Karman has extended the theory beyond the inertial domain and has given the following equation of the PSD:

$$W_{\Delta_n,h}(f) = 0.033(2\pi)^{-\frac{2}{3}}C_n^2(h)((\frac{1}{L_0}^2 + f^2)^{-\frac{11}{6}}exp(-(fl_0)^2) \quad (2.2)$$

These equations are valid for one layer at the altitude h. But the atmosphere is made of a lot of layers. The Fried parameter r_0 allows to quantify the global strength of the turbulence along the optical path. This parameter is very important in astronomy because it defines the equivalent diameter of a telescope when its angular resolution is limited by turbulence. r_0 is calculated thanks to the following equation:

$$r_0 = [0.42(\frac{2\pi}{\lambda})^2 \frac{1}{cos(\gamma)} \int_0^{\infty} C_n^2(h)dh]^{-\frac{3}{5}} \quad (2.3)$$

where γ is the zenithal angle and λ is the wavelength. Other quantities of interest can be deduced from the PSD and the Fried parameter. The structure function D_Φ of the turbulent phase corresponds to the variance of the phase between two points separated of the distance ρ:

$$D_\Phi(\rho) =< [\varphi(r) - \varphi(r+\rho)]^2 > \quad (2.4)$$

$$D_\Phi(\rho) = 6.88(\frac{\rho}{r_0})^{5/3} \ when\ L_0\ is\ infinite \quad (2.5)$$

The outer scale has an impact on the PSD and the structure function. It causes a saturation of the PSD for low frequencies (figure 1 left) and of the structure function for important spacings (figure 1 right).

2.2 The fluctuations of the turbulent phase

We study the variance of the phase (σ_φ) in the pupil for different values of the outer scale according to the position in the pupil. We note $\Phi(\vec{r})$ the turbulent phase defined on a infinite support and $\varphi(\vec{r})$ the phase in the telescope pupil S subtracted of its averaged piston. We have :

$$\varphi_{turb}(\vec{r}) = \Phi(\vec{r}) - \frac{1}{S}\int_S \Phi(\vec{\rho})d\vec{\rho} \quad (2.6)$$

The variance of the phase in the pupil is given with the equation:

$$\sigma_\varphi^2 = \langle \varphi_{turb}(\vec{r})^2 \rangle = \frac{1}{S}\int_S D_\Phi(\vec{r}-\vec{\rho})d\vec{\rho} - \frac{1}{2S}\int_S\int_S D_\Phi(\vec{\rho'}-\vec{\rho})d\vec{\rho}d\vec{\rho'} \tag{2.7}$$

The variance is not uniform in the pupil, it depends on the position of the point that we consider. Several authors have noticed this non stationarity of the phase (Heidbreder 1967, Wang 1978, Conan 1994).

Thanks to numerical simulation, we have studied the variance in the pupil for different values of the outer scale (See Figure 2). First, we can see that the variance is more important on the edge of the telescope pupil than in the centre. The influence of the outer scale is very important too. When L_0 decreases, the variance range decreases too. σ_φ is closely related to the displacement amplitude of the mirror surface. Generally, we consider that the peak-valley surface displacement is given by $6\sigma_\varphi$. Note that a measure in optical microns is twice bigger than a measure in mechanical microns (reflection on a mirror). So, the displacement of the DM can be linked to the phase variance range and reveals the movement at a given DM actuator location.

Note however that Figure 2 is given in normalised units for $D/r_0 = 1$ and that the absolute stroke of the DM increases with the telescope size. We only want to point out that for large diameters the parameter L_0/D is smaller and the stroke more uniform on the pupil.

Besides, we can show that when we remove the tip-tilt mode of the phase, the variance range across the pupil is also strongly reduced. In lots of telescopes, the correction of the low frequencies is made by a tip-tilt mirror and the DM corrects the higher order modes. The stroke constraints on the DM are then diminished a lot.

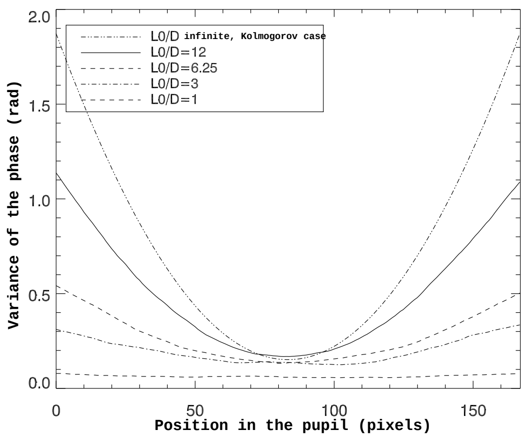

Fig. 2. The variance of the phase according to the position in the pupil for different L_0 for $D/r_0 = 1$

3 Dimensioning a deformable mirror

The design of a DM is related to the telescope diameter, to the site quality in terms of turbulence and of course to the technology of the DM itself (Séchaud 1999). We have seen that since the DM fits the turbulence, the variance of the phase is linked to the stroke expressed in optical surface displacement. However the mirror surface and the mirror command amplitudes are different notions. The command amplitudes depend on the DM technology. Afterwards, we do not consider a particular technology and the DM is considered to be linear. The correction phase is then given by the following matrix relation: $\varphi_{cor} = Nu$, where u are the actuator commands and N is the influence matrix of the DM. Each column of N corresponds to the mirror shape when we push a given actuator, so called influence function. Hereafter the influence functions are assumed to be identical Gaussian functions. They are normalised to their maximum in order to have φ_{cor} and u in the same units that is optical microns. We call coupling factor the value of the influence function at the actuator neighbour location.

To evaluate the ability of a DM to fit the turbulence, we simulate wavefronts which follow the theory of Von Karman. The simulation conditions are given in table 3. We do a random draw of the phase screens. We

Outer scale L_0	90 m
Fried diameter r_0	0.05 m at 500 nm
Diameter of the telescope D	8 m
Number of actuators	17x17 : 177 useful actuators
Coupling factor between two actuators	30% and 60%

Table 1. The parameters of the simulation

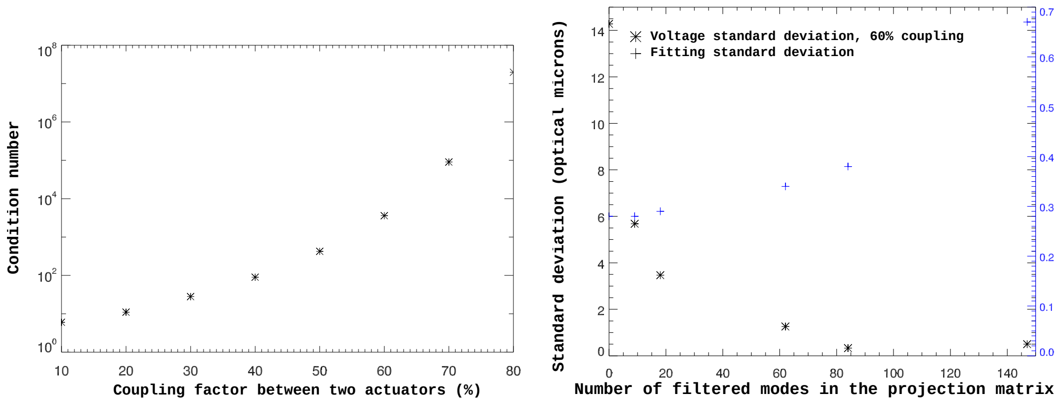

Fig. 3. Left :The condition number vs the coupling factor between two actuators. Right :Study of the influence of the number of filtered modes in the projection matrix P on the standard deviation of the actuator commands and the fitting of the DM. The coupling factor between two actuators is 60%

then subtract the piston and the tip-tilt mode of this phase. We consider two values for the coupling factor. We project each phase screen on the DM and we calculate its response in order to minimise the residual phase variance given by:

$$\sigma^2_{fitting} = \| \varphi_{res} \|^2 = \| \varphi_{turb} - \varphi_{cor} \|^2 = \| \varphi_{turb} - N \times u \|^2 \tag{3.1}$$

In order to obtain the corrected phase, we have to calculate the best commands u that minimise the residual variance, which corresponds to apply the generalised inverse of the influence matrix:

$$u = P \times \varphi_{turb} = (N^t N)^{-1} N^t \times \varphi_{turb} \tag{3.2}$$

Generally, the matrix $N^t N$ is ill conditioned. To stabilise the solution, we can truncate some modes to have a reasonable condition number. We have first studied the impact of the value of the coupling factor on the condition number. On the figure 3-left, the condition number increases with the coupling factor. So we see that for strong coupling factors like 60%, we have to truncate a large number of modes. For a coupling factor of 30%, we found that it is not necessary to truncate any mode. But what is the best criteria to truncate modes and how many modes should we truncate?

To evaluate the good number of truncated modes, we have to take into account two different effects. On one hand, we want the best fit of the turbulence, that means that the standard deviation of the residual phase ($\sigma_{fitting}$) must be low. And on the other hand, technological constraints generally impose to have low values of the actuator commands which means that the standard deviation of the actuator commands must be low too.

For 60% of coupling factor, we have studied the standard deviation of the actuator commands and the standard deviation of the residual phase according to the number of filtered modes. We can see on the figure 3-right that the standard deviation of the actuator commands reduces with the increase of the number of filtered modes. On the contrary, the standard deviation of the residual phase increases slowly because the DM has more difficulties to fit the turbulence. We can find a compromise between these two effects.

For a coupling factor of 60%, we choose to truncate twenty modes when we calculate the generalised inverse matrix. The table 3 gives the value of the standard deviation measured for different numbers of truncated

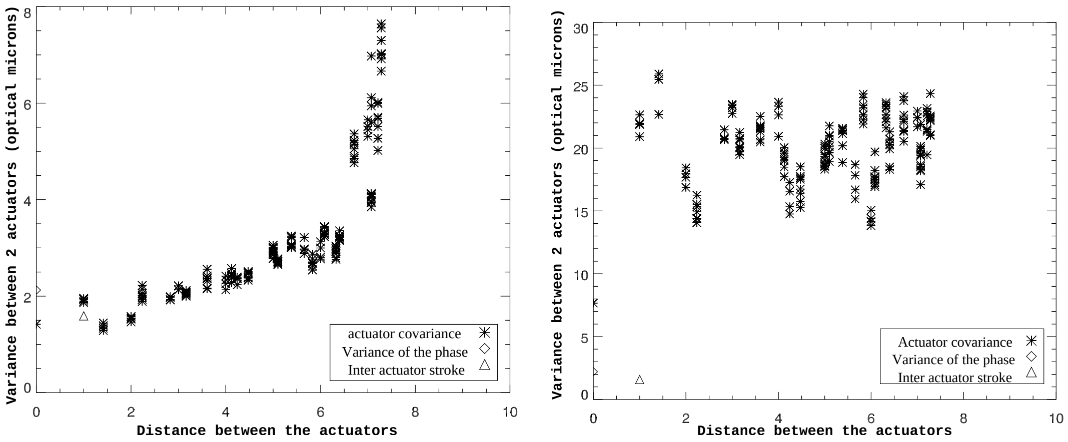

Fig. 4. Left :Variance of the difference between the actuator on the centre and his neighbours for 30% of coupling factor. Right : the same curve for 60% of coupling factor.

modes. We can see that we win a factor 4 on the standard deviation of the actuator commands when we truncate 20 modes but the standard deviation of the fitting is almost the same.

Number of truncated modes	0	20
Actuator command standard deviation σ_u	$14.3 optical\ \mu m$	$3.5 optical\ \mu m$
Fitting standard deviation $\sigma_{fitting}$	$0.28 optical\ \mu m$	$0.29 optical\ \mu m$

Table 2. Values of the standard deviation for different numbers of truncated modes

It is also interesting to compare the standard deviation of the centre actuator command with the standard deviation of the turbulence at the same location. In our simulation, we estimate $\sigma_{turb} = 2.1\mu m$ for the turbulent phase. For a coupling factor of 30%, we measure $\sigma_{u,30\%} = 1.4\mu m$ and for 60%, we measure $\sigma_{u,60\%} = 7.8\mu m$. It is interesting to note that for a low coupling the standard deviation of the actuator is lower than the standard deviation of the turbulence, while the reverse is observed for a large coupling. This is due to the fact that for low coupling one actuator is helped by its neighbours. On the contrary for an important coupling factor, the actuators are not helped by the neighbours, they are rather in competition.

In addition to the previous DM surface and command stroke considerations, one has also to analyse the differential stroke between actuators. We choose to study this between the centre actuator and all of its neighbours. We call u_o the central actuator command, u_i is the command of the neighbours, i corresponds to the distance between the actuators. We calculate the term $<\| u_o - u_i \|^2>$ for different i.

The results are presented in figure 4. For a coupling factor of 30% the variance between two actuators slowly increases with the actuator distance. For a coupling factor of 60%, the variance is much larger and rather independent of the actuator distance. We believe that this behaviour is related to the phenomenon previously mentioned: for a weak coupling factor, the actuators help each-others to fit the turbulence, while for a big coupling factor, the actuators have opposite movements even if they are neighbours.

4 Conclusion and Perspectives

The design and specification rules for DM is an important current topic. New AO strategies and ELT developments indeed lead to very demanding characteristics. In the meantime new DM technologies are under development. It is however difficult to provide simultaneously large optical strokes, large number of actuators, fast temporal response...

Without considering a particular technology we have shown general behaviours that have to be considered in the design process. We quantify the optical stroke as a function of both D/r_0 and D/L_0 parameters. We show that this stroke can strongly depends on the pupil location even if this effect is reduced for small D/L_0 and when subtracting low order modes that can be corrected by an auxiliary device.

We also demonstrate that optical stroke in terms of mirror surface displacement is generally quite different from the actuator command amplitudes. Assuming a linear DM model the command values strongly depend on the DM influence function and on their coupling factor. Of course one can not specify any kind of influence function since it depends a lot on the technology considered and associated constraints. Similarly for a given device there are different equivalent linear models. However one has to clearly identify the quantities of interest (voltages, currents...) and the related physical limitations.

Note that this work is mainly based on the statistical analysis of simulated turbulence occurrences. This simulation accounts for the influence functions of the DM and performs the proper projection on its subspace. It can also account for the DM temporal response. We believe that this numerical simulation is a powerful tool perfectly adapted to the dimensioning of the next generation of DM and AO systems.

The work presented in this paper will be applied to the design of the adaptive mirror of the E-ELT 40 m class telescope. It will be also applied to the specification of the DMs of the next generation of AO and wide field AO systems for the VLT and the E-ELT.

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