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DETECTING THE TIDAL DISSIPATION IN THE SATURNIAN SYSTEM

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Abstract.
We examine the problem of detecting the observational signature of tides in the Saturnian system. We show that, because of energy transfer encouraged by the mean-motion resonances, S-1 Mimas' secular acceleration should be detected by observing S-3 Tethys instead of Mimas itself. We have a similar conclusion for the Enceladus/Dione resonance. We also show that detecting secular accelerations of these satellites will give clues on Saturn's internal dissipation, but very few information on the dissipation inside the satellites.

1 Introduction

The main Saturnian satellites are S-1 to S-8 are respectively Mimas, Enceladus, Tethys, Dione, Rhea, Titan, Hyperion and Iapetus. Their orbital dynamics is overwhelmed by 3 orbital resonances:

- $2\lambda_1 - 4\lambda_3 + \Omega_1 + \Omega_3$ between Mimas and Tethys
- $\lambda_2 - 2\lambda_4 + \varpi_2$ between Enceladus and Dione
- $3\lambda_6 - 4\lambda_7 + \varpi_7$ between Titan and Hyperion

where λ_i are the mean longitudes, ϖ_i the longitudes of the pericentres, and Ω_i the longitudes of the ascending nodes.

Up to now, their motions have been modelized only with conservative models, the dissipations being too small to be evaluated properly. We here discuss about some clues that could improve the detectability of the tidal dissipation.

2 Dynamical consequences of tidal dissipation

The energy dissipations in the planet (here Saturn) and its satellites induce secular variations of their orbits, especially variations of the semimajor axes and damping of the eccentricities. In the case of a synchronous satellite i with small obliquity, small inclination, and an orbital period longer than the rotation period of its parent body, we have (Kaula 1964, Peale & Cassen 1978):

$$\frac{1}{n_i}\frac{dn_i}{dt} = -\frac{9}{2}\frac{k_2^p n_i m_i R_p^5}{Q_p a_i^4 M_{♄}}\left[1+\frac{51}{4}e_i^2\right] + \frac{63}{2}\frac{k_2^i n_i M_{♄} R_i^5}{Q_i m_i a_i^4}e_i^2 \tag{2.1}$$

and

$$\frac{de_i}{dt} = \frac{57}{8}\frac{k_2^p n_i m_i}{Q_p M_{♄}}\left(\frac{R_p}{a_i}\right)^5 e_i - \frac{21}{2}\frac{k_2^i n_i M_{♄}}{Q_i m_i}\left(\frac{R_i}{a_i}\right)^5 e_i \tag{2.2}$$

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	n_i	mean mean motion of satellite i
	k_2	Love number
	a	semimajor axis
with :	e	eccentricity
	m_i	mass of the satellite
	Q	dissipation function
	R	radius

These equations consist of two parts: one containing $\frac{k_2^p}{Q_p}$ gives the effects of the dissipation inside the parent planet, while the one containing $\frac{k_2^i}{Q_i}$ deals with the dissipation inside the satellite. We can see that these two contributions have contrary effects: the dissipation inside the planet tends to increase the semimajor axis a, while the dissipation inside the satellite decreases a and damps the eccentricity e. The two effects on the semimajor axis have quite the same magnitude, because the dissipation inside the satellite has no 0-degree effect in eccentricity and is compensated by the very small dissipation inside the planet ($Q_p >> Q_i$). The absence of 0-degree effect is due to the synchronous rotation of the satellite: in that case, the tidal bulge is always directed to the planet if the orbit is circular.

The effect that we hope to detect in the observations deals with (Eq.2.1), ie a secular acceleration of the satellite i. This could be seen in the observations by a shift in longitude between the observations and the positions predicted by dissipationless models.

3 Influence of the resonances

The mean-motion resonances existing in that system of satellites induce some transfers of orbital energy that induce change in the semimajor axes. So, a tidal acceleration of satellite i has dynamical consequences on satellite j if these two satellites are locked in a mean motion resonance.

More precisely, in the case of two satellites 1 and 2 locked in the resonance whose argument is $\phi = p\lambda_1 - (p+q)\lambda_2 + q_1\varpi_1 + q_2\varpi_2 + q_3\Omega_1 + q_4\Omega_2$, the dynamical equations of the mean mean motions n_1 and n_2 can be written as

$$\frac{dn_1}{dt} =< 3pn_1^2\alpha m_2 e_1^{|q_1|}e_2^{|q_2|}\gamma_1^{|q_3|}\gamma_2^{|q_4|}f(\alpha)\sin\phi > + \left(\frac{dn_1}{dt}\right)_M \tag{3.1}$$

and

$$\frac{dn_2}{dt} = - < 3(p+q)n_2^2 m_1 e_1^{|q_1|}e_2^{|q_2|}\gamma_1^{|q_3|}\gamma_2^{|q_4|}f(\alpha)\sin\phi > + \left(\frac{dn_2}{dt}\right)_M \tag{3.2}$$

with $\alpha = \frac{a_1}{a_2}$ ($a_2 > a_1$).

We have, at the first order of masses,

$$\begin{aligned} \ddot{\phi} &\approx p\dot{n}_1 - (p+q)\dot{n}_2 \\ &= (3p^2n_1^2\alpha m_2 + 3(p+q)^2n_2^2m_1)e_1^{|q_1|}e_2^{|q_2|}\gamma_1^{|q_3|}\gamma_2^{|q_4|}f(\alpha)\sin\phi \\ &+p\left(\frac{dn_1}{dt}\right)_M - (p+q)\left(\frac{dn_2}{dt}\right)_M \end{aligned} \tag{3.3}$$

From $<\ddot{\phi}>= 0$ we deduce

$$< e_1^{|q_1|}e_2^{|q_2|}\gamma_1^{|q_3|}\gamma_2^{|q_4|}f(\alpha)\sin\phi >= -\frac{p\left(\frac{dn_1}{dt}\right)_M - (p+q)\left(\frac{dn_2}{dt}\right)_M}{3p^2n_1^2\alpha m_2 + 3(p+q)^2n_2^2m_1} \tag{3.4}$$

and finally obtain :

$$\frac{\dot{n}_1}{n_1} = \frac{pn_1\alpha m_2}{p^2n_1^2\alpha m_2 + (p+q)^2n_2^2m_1}\left((p+q)\left(\frac{dn_2}{dt}\right)_M - p\left(\frac{dn_1}{dt}\right)_M\right) + \left(\frac{\dot{n}_1}{n_1}\right)_M \tag{3.5}$$

$$\frac{\dot{n}_2}{n_2} = \frac{(p+q)n_2m_1}{p^2n_1^2\alpha m_2 + (p+q)^2n_2^2m_1}\left(p\Big(\frac{dn_1}{dt}\Big)_M - (p+q)\Big(\frac{dn_2}{dt}\Big)_M\right) + \Big(\frac{\dot{n}_2}{n_2}\Big)_M \tag{3.6}$$

Tab. 1 and 2 give the numerical consequences of the resonances respectively in the cases Mimas-Tethys and Enceladus-Dione. These numerical values have been computed using the parameters indicated Tab.3.

Table 1. Consequence of the resonance on Mimas' and Tethys' secular accelerations. The two first columns are possible values for the secular accelerations of Mimas and Tethys due to tidal dissipation, while the two last columns are computed values.

$\left(\frac{\dot{n}_1}{n_1}\right)_M$	$\left(\frac{\dot{n}_3}{n_3}\right)_M$	$\frac{\dot{n}_1}{n_1}$	$\frac{\dot{n}_3}{n_3}$
-2×10^{-11}	-2×10^{-11}	-1.997×10^{-11}	-2×10^{-11}
-10^{-12}	-2×10^{-11}	-1.830×10^{-11}	-1.833×10^{-11}
10^{-12}	-2×10^{-11}	-1.813×10^{-11}	-1.816×10^{-11}

Table 2. Consequence of the resonance on Enceladus' and Dione's secular accelerations.

$\left(\frac{\dot{n}_2}{n_2}\right)_M$	$\left(\frac{\dot{n}_4}{n_4}\right)_M$	$\frac{\dot{n}_2}{n_2}$	$\frac{\dot{n}_4}{n_4}$
-2×10^{-11}	-10^{-11}	-1.137×10^{-11}	-1.135×10^{-11}
10^{-11}	-10^{-11}	-7.300×10^{-12}	-7.290×10^{-12}

Table 3. Physical and dynamical parameters of the main Saturnian satellites. The periods and eccentricity come from TASS1.7 theory (Vienne & Duriez 1995, Duriez & Vienne 1997), while the masses come from (Jacobson et al. 2006).

Satellites	radius (km)	period (d)	eccentricity	mass ($\times 10^{-7} M_{♄}$)
S-1 Mimas	198.6	0.94	0.015	0.66
S-2 Enceladus	249.4	1.37	0.0048	1.9
S-3 Tethys	529.8	1.89	$< 10^{-3}$	10.9
S-4 Dione	560	2.74	0.0022	19.3
S-5 Rhea	764	4.52	0.0010	42.1
S-6 Titan	2575	15.95	0.0289	2455.6
S-7 Hyperion	133	21.28	0.1	0.10
S-8 Iapetus	718	79.33	0.0294	33.0

We can see that, in the two cases, the 2 involved satellites have roughly the same secular acceleration, that are in fact respectively Tethys' and Dione's tidal secular accelerations, because of the large ratio of masses Tethys/Mimas and Dione/Enceladus. In the case Titan-Hyperion, the ratio of masses is so huge (about 9000) that Hyperion's tidal secular acceleration has actually no significative influence.

4 Detecting the tidal effects

It is now interesting to compare the predicted secular accelerations of the satellites with the accuracy of the astrometric observations. Tab.4 indicates the mean accuracies of these observations.

We dispose of 133 years of astrometric observations of the Saturnian satellites, since 1874. Tab. 5 gives estimations of the necessary timespan to detect secular accelerations of the Saturnian satellites:

We can see that detecting the secular accelarations of the Saturnian satellites is very difficult now. Moreover, the first secular acceleration that should be detected is Tethys', instead of Mimas', while Mimas is the closest to Saturn.

5 Conclusion

Up to now, we do not know the secular acceleration of the Saturnian satellites, and the first to be known should be Tethys'. We can note that, thanks to Tethys' very small eccentricity, its secular acceleration is only due

Table 4. Mean accuracy of the astrometric observations of the Saturnian satellites. The values for the classical observations come from (Vienne 2001), while the values for the mutual events have been estimated from (Noyelles et al. 2003).

satellite	classical observations		mutual events	
S-1 Mimas	130 mas	910 km	37 mas	259 km
S-2 Enceladus	100 mas	700 km	29 mas	203 km
S-3 Tethys	70 mas	490 km	23 mas	161 km
S-4 Dione	70 mas	490 km	23 mas	161 km
S-5 Rhea	70 mas	490 km	28 mas	196 km
S-6 Titan	70 mas	490 km	48 mas	336 km

Table 5. Evaluations of the timespans necessary to detect tidal secular accelerations. These timespans have been evaluated using the accuracy of the classical astrometric observations.

satellite	estimated $\left(\frac{\dot{n}}{n}\right)_M$	secular drift ($km.century^{-2}$)	estimated t (y)
S-1 Mimas	-2×10^{-11}	45	338
S-2 Enceladus	-10^{-11}	20	451
S-3 Tethys	-2×10^{-11}	36	300
S-4 Dione	-10^{-11}	16	451
S-5 Rhea	-10^{-13}	0.13	5400
S-6 Titan	5×10^{-13}	0.44	3900

to Saturn's internal dissipation. So, detecting Tethys's secular accelration means knowing Saturn's internal dissipation, that could be used in searching the internal dissipation of the other Saturnian satellites.

References

Duriez L. & Vienne A., 1997, A&A, 324, 366

Jacobson R.A., Antreasian P.G., Bordi J.J. et al., 2006, AJ, 132, 2520

Kaula W.M., 1964, Rev. Geophys. Space Phys., 2, 467

Noyelles B., Vienne A. & Descamps P., 2003, A&A, 401, 1159

Noyelles B., 2005, *Effets des marées sur la dynamique des satellites de Jupiter et de Saturne*, PhD thesis, Paris Observatory, Chap.9 (in French)

Peale S.J. & Cassen P., 1978, Icarus, 36, 245

Vienne A. & Duriez L., 1995, A&A, 297, 588

Vienne A., 2001, *Différents aspects de la dynamique des huit principaux satellites de Saturne*, HDR thesis, Université des Sciences et Technologies de Lille (in French)