DYNAMICAL EVOLUTION OF THE GLIESE 581 PLANETARY SYSTEM

H. Beust¹, X. Bonfils² and X. Delfosse¹

Abstract. Gl581 is an M-type dwarf harbouring a 3 Earth-sized planets system (Udry et al. 2007). The two outer planets lie potentially within the habitable zone of this star. We investigate the dynamical stability of this system using the symplectic N-body code SyMBA (Lee et al. 1998). The systems appears remarkably stable. The eccentricity of the small planet ranges secularly between 0 and 0.15. We also test different inclinations and eccentricities. In any case (apart from nearly pole-on configurations), we have stability. We can safely stress that this system, and thus the planetary climate, is stable. This makes the habitable planets potentially suitable for life development.

1 Introduction

The M dwarf Gliese 581 has been the subject of recent interest with the identification of its 3-planet system. One of the planets (Gl 581 b), a Neptune-mass object orbiting the star on a 5.4-day orbit (and eccentricity 0.014), was known for two years (Bonfils et al., 2005). Recently, Udry et al. (2007) reported the discovery of two additional super Earths (Gl 581 c and d), revolving around the star in 12.9 and 83 days with 0.16 and 0.12 eccentricities respectively (see details in Table 1 from Beust et al. 2007). Considering Gl 581's luminosity, Udry et al. (2007) inferred the equilibrium temperature of both planets and conclude they may lie within the habitable zone of the star.

Detailed further modeling by von Bloh et al. (2007) and Selsis et al. (2007) address the habitability of the planets. von Bloh et al. (2007) and Selsis et al. (2007) found that Gl 581 c's surface temperature is likely higher than the equilibrium temperature calculated by Udry et al. (2007). However, they do not rule out habitability for this planet. Conversely, both studies agree that the outermost planet (Gl 581 d) is a good candidate for habitability, although close to the outer edge of the habitable zone.

An important and unsettled issue about this system concerns its dynamical behaviour. It is first important to know whether the planetary system is dynamically stable and for which range of orbital inclinations. Beyond the basic stability, the secular evolution of the orbits may play an important role regarding planets' habitability. All climate calculations (von Bloh et al., 2007; Selsis et al., 2007) have been done with the present day determined orbits. The secular evolution of the orbits has the potential to affect the climate on the planets. The today determined eccentricities (0.16 and 0.12) are small enough to ensure climate stability. But one needs to know which maximum values they reach due to secular perturbations. In the present paper, we numerically investigate the secular evolution of the Gl 581 system.

2 The nominal case

We numerically integrate the Gl 581 system starting from the fitted solution and assuming $0.31 M_{\odot}$ for the mass of Gl 581. The integration is performed using the symplectic *N*-body code SyMBA (Duncan et al., 1998). The integration is carried out over 10^8 yr. Figure 1 shows the first 10^4 years of the integration. We see that the secular variations of the 3 planetary orbits are very regular. This solution is in fact very close to the one we can compute with a linear secular theory (Laplace – Lagrange), such as described by Bretagnon (1974, 1990).

¹ Laboratoire d'Astrophysique de Grenoble, UMR 5571 C.N.R.S., Université J. Fourier, B.P. 53, F-38041 Grenoble Cedex 9

 $^{^2}$ Centro de Astronomia e Astrofísica da Universidade de Lisboa, Tapada da Ajuda, 1349-018 Lisboa, Portugal



Fig. 1. The 10^4 first years of the integration of the nominal solution. The upper plots show the temporal evolution of the eccentricity of the three planets Gl581 b,c, and d, from left to right respectively; the lower plots are the same for the longitude of periastron ϖ

Table 1. Variation	ranges for some	orbital parameters f	o the 3 planets over	the 10^8 yr integration.
	0	*	*	

Planet	Semi-major axis (AU)	Eccentricity	Periastron (AU)	Apoastron (AU)
Gl 581 b	0.040609 - 0.046185	0.01 - 0.095	0.0368 - 0.0402	0.041 - 0.0445
Gl581~c	0.072885 - 0.073	0.07 - 0.16	0.0614 - 0.0678	0.078 - 00846
Gl581~d	0.2522 - 0.2528	0.1200 - 0.1246	0.2207 - 0.2227	0.2823 - 0.2843

In Table 1, we list the maximum evolution ranges for the orbital elements of the three planets. The semimajor axes are extremely stable, revealing a regular dynamics out of any mean-motion resonance configuration. The evolution ranges of the eccentricities are small, so that we may stress with a high level of confidence that the system is dynamically stable over 10^8 yr, and very probably over a much onger time scale.

Interestingly, the present day eccentricity of Gl 581 c corresponds roughly to its maximum values along its secular evolution; and the eccentricity of Gl 581 d as only little variations. Hence we expect the climate of both outer planets to be secularly stable.

3 Other solutions

The nominal solution corresponds to an inclination $i = 90^{\circ}$ (so the smallest possible planetary masses). Smaller inclinations and/or parameter's values slightly outside the best solution may lead to different dynamical behavior that are worth investigating.

In a first set of additional simulations, we assume various inclinations ranging from 0 (pole on) to 90° (edge on), but still holding the initial eccentricities to their nominal values. The planetary masses are augmented by a factor $1/\sin i$ with respect to the nominal case. In a second set of simulations we also assume different inclinations and moreover, we take the initial eccentricities at the upper limit of their error bars (we add 1σ to the initial eccentricities) For both sets of integrations, we plot the width of the evolution ranges obtained over the 10^5 yr integration for both the semi-major axes and the eccentricities of the three planets (Fig. 2).

As can be seen from Fig. 2, when the inclination decreases, the dynamical interactions increase accordingly. As expected, the semi-major axes and the eccentricities take a wider range of values for smaller inclinations. In the first set of integrations (nominal initial eccentricities), the system remains nonetheless stable down to $i = 10^{\circ}$. In the second set of simulations (1 σ augmented initial eccentricities), the dynamical interactions are



Fig. 2. Stability of the three planets system in various configurations. The maximum variation range for the semi-major axes (upper plots) and for the eccentricities (lower plots) is plotted as a function of the assumed viewing inclination of the system with respect to pole-on. Each cross corresponds to a single simulation. The left plots corresponds to simulations with the nominal eccentricities as initial conditions, and the right plots to simulations with 1σ augmented eccentricicites.

Fig. 3. Lyapunov exponents as a function of the inclination i, computed for all the simulations described in Fig. 2. Left plot : simulations with nominal eccentricities; Right plot : simulations with 1σ increased eccentricities.

slightly enhanced and the orbital elements take a wider range of values. The system is therefore found unstable below larger inclinations ($< 20^{\circ}$).

In any case, the instability appears very unlikely. If we assume that the rotation axis of the system is randomly distributed in the space, $i > 20^{\circ}$ occurs with a probability of 0.94. In conclusion, irrespective of its actual inclination, the Gl 581 planetary system is very probably stable.

A more sophisticated way for quantifying dynamical chaos is to compute Lyapunov exponents. For all simulations decribed above, we compute the Maximum Lyapunov exponents (MLE) for the three planets, following the technique by Benettin et al. (1978).

The result is shown on Fig. 3, where we have computed the MLEs for the three planets for all the simulations described in Fig. 2. We see that the MLEs slowly increase with decreasing inclinations, showing as expected that solutions at smaller inclinations are more chaotic, due to larger planetary masses. We nevertheless note that the variation is very small unless for $i < 20^{\circ}$. The system is not much more chaotic at $i = 20^{\circ}$ than at $i = 90^{\circ}$. This confirms that there is no real dynamical constraint on the inclination. We also note that solutions with 1σ increased eccentricities are not more chaotic. Hence the dynamics does not put any additional constraint on the planet eccentricities other than those derived from the radial velocity analysis.

4 Other planets

Our simulations were made with the three known planets orbiting Gl 581. However, the system may harbour additional, unknown planets. The presence of these planets may affect the stability of the whole system. There are upper limits to the presence of additional (mainly outer) planets. The maximum amplitude of the residuals

$\rm SF2A~2007$

in the 3-planet fits of Udry et al. (2007) is $\pm 2.1 \,\mathrm{m \, s^{-1}}$. Any additional planet should not generate a radial velocity with a larger amplitude, otherwize it would have been detected yet. This puts severe constraints on the mass m and distance d of the unseen planet. We derive $m/1 M_{\oplus} \leq 13.227 \times \sqrt{d/1 \,\mathrm{AU}}$. This constraint holds if the unseen planet generates full amplitude variations within the timespan of the radial velocity data, i.e., $\sim 1000 \,\mathrm{days}$ (Udry et al., 2007). This means that the orbital period of the unseen planet must not exceed \sim twice this time span to account for this constraint, i.e. an orbital distance $d \leq 5.5 \,\mathrm{AU}$. For more distant planets, the constraint is much weaker.

We thus performed new simulations with the nominal conditions, but to which we add an additional planet orbiting the star on a circular orbit at an arbitrary distance d, and with the maximum mass allowed by the above condition (see Beust et al. 2007 for details). In all cases, the whole system appears as stable as without any additional planet. The semi-major axis and eccentricity variation ranges for the three known planets are very similar in all cases. Therefore, we may stress that any additional outer planet that fits into the constraint of the radial velocity residuals does not affect the stability of the 3 planets system.

5 Discussion

We have computed the secular evolution of the Gl 581 planetary system in various possible configurations. The main conclusion is that the system is almost always stable. It is stable for inclinations as low as $\sim 20^{\circ}$ and even if the initial eccentricities are augmented by their 1- σ error bars.

The eccentricities of the two outer planets (both considered for habitability) reach values that are significantly above the Earth's value. Concerning Gl 581 c, the present day eccentricity is close to its maximum value. This planet is not expected to get much further away from its parent star and, to maintain a surface temperature cool enough to allow the presence of liquid water, a high water-cloud coverage ($\sim 75\%$) would be required. Regarding Gl 581 d, the nominal eccentricity is non negligible (~ 0.12) and also found to be very stable. It is significantly above the maximum value reached by the Earth throughout its secular evolution (~ 0.06 , see e.g. Laskar, 1988) and corresponds to a 24% variation of the radiation flux received from the star between apoastron and periastron. The anomalistic seasonal effects should therefore be important

Now, if the obliquity of the rotation axis of this planet is non-zero, this should combine with the obliquity seasonal effect and lead to climate differences between the hemispheres of this planet, much like Mars presently. The obliquity of Gl 581 d is of course unknown, but Selsis et al. (2007) and von Bloh et al. (2007) agree to claim that given the estimated age of the star (>2 Gyrs), the rotation of Gl 581 d should be already tidally locked with the orbital motion. In that case, we should expect the obliquity to have been set to zero by tidal effects. Selsis et al. (2007) also show that tidal locking is not in contradiction with the non-zero eccentricity of the orbit. Tides usually tend to both synchronize the rotation and circularize the orbit. The circularization time always larger than the synchronization time. For Gl 581 d, Selsis et al. (2007) estimate the synchronization time to 10 Gyrs, i.e. well above the present age of the system.

References

Benettin G., Galgani L., Giorgilli A., Strelcyn J.M., 1978, . R. Acad. Sci. Paris, Ser. A, 286, 431

Beust H., Bonfils X., Delfosse X., Udry S., A&A, to be submitted

von Bloh W., Bounama C., Cuntz M., Franck S., 2007, A&A, submitted, astro-ph/0705.3758

Bonfils X., Forveille T., Delfosse X., et al., 2005, A&A 443, L15

Bretagnon P., 1974, A&A 30, 141

Bretagnon P., 1990, A&A 231, 561

Duncan M.J., Levison H.F., Lee M.H., 1998, AJ 116, 2067

Laskar J., 1988, A&A 198, 341

Selsis F., Kasting J.F., Paillet J., Levrard B., Delfosse X., A&A, submitted

Udry S., Bonfils X., Delfosse X., et al., 2007, A&A, submitted, astro-ph/0704.3841