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Fig. 1. schematic view of the numerical nucleus.

AN IMPROVED NUMERICAL TREATMENT OF HEAT AND MASS TRANSFERS IN COMETARY NUCLEI MODELS.

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Abstract. Cometary nuclei models using the finite difference method may present intrinsic problems of mass and energy flux conservation when these bodies are close to perihelion along their orbit. In some circumstances, the calculated production rates of volatile species released by the nucleus can even be affected by errors of the same order of magnitude, as a result of the employed numerical method. In order to avoid these shortcomings, we present a one-dimensional cometary nucleus model that uses the finite volume method.

1 Introduction

In order to obtain accurate production rates of gas as well as a fair representation of the temporal evolution of the internal chemical structure in comet nuclei, the mass conservation must be maintained . The choice of the numerical method of integration used to solve the heat and mass transferts is then crucial. In the case of a long term study of a cometary nucleus, it may induce an error on the production rates and justify our research of a better balance between the volatiles that sublimate in the pores and those that escape of the nucleus. In this goal, we propose a numerical method that improves the treatment of mass conservation.

2 The finite volume method

We use the finite volume method that consists in dividing the domain of study in cells. The equations of conservation of mass and energy are integrated on each finite volume and the variables (Pressure and Temperature) are calculated in the center of each volume. The coefficient of heat conduction K and of gas diffusion G are calculated on the boundary of each volume of control. The result after integration is the exact conservation of flux of mass and energy through the cells.

To assess the improvements of our model to previous one, we compare it to the cometary nuclei models of Espinasse et al.(1991) and of Orosei et al.(1999). All codes solves the heat conduction and gas diffusion equations in 1D, within a spherically symmetric porous body. For all models, Ice can sublimate, gas can flow within the porous matrix and escape from the object. The resolution of the different equations differ by their numerical treatment : Espinasse and Orosei use a finite difference method to solve for equations of conservation, while we use a finite volume approach. Note that our model uses the same approach as Orosei (1999) to solve for gas and energy conservation equations.

3 Choice of the parameters

The original model of Espinasse et al . (1991) consists in a homogeneous mixture of H_2O and CO pur ices while the model of Orosei et al. (1999) incorporates a fraction of dust in addition to H_2O and CO pure ices. The

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Fig. 2. Temporal evolution of the relative error Q on the mass conservation of CO ice since the beginning of the computation for the comparaison of model 1 to hte one of Espinasse et al.(1991).

Fig. 3. The same as in Fig.1 but for the comparaison of the model 2 to the one of Orosei et al.(1999).

structure and the initial composition of the nucleus determine its thermal conductivity, as well as its physical and chemical evolution. The two models initially bear crystalline water ice with CO condensed in the pores. In order to compare our model with those of Espinasse et al. (1991) and Orosei et al. (1999), we define two different sets of input parameters. In the first case, we assume that our model has a composition similar to the one of Espinasse et al. (model 1). In the second case, our model has the composition of the one of Orosei et al. (model 2). Except for the presence of dust, all the models have identical physical (density, porosity, albedo, thermal conductivity ...) and dynamical properties (period, aphelion, perihelion, time step) that are summarized in table **??**.

Table 1. parameters for simulation

parameter	value
orbital semi-major axis	17.93 UA
orbital eccentricity	0.967
Radius of nucleus	1 km
Porosity	50 %
Initial temperature	30 K
ρ_{dust}	1000 $kg.m^{-3}$
Dust conductivity	4.2 $W.m^{-1}.K^{-1}$
CO/H_2O (in mol)	10 %

4 Comparison with others models and results

To evaluate the accurate conservation of integrals with time, we compare the relative error Q on the conservation for the different models:

$$Q = \frac{M^0 - M^t - M_{ejectas}}{M_{ejectas}} \tag{4.1}$$

where M^0 is the initial total mass of CO in the nucleus before the computation, M^t the current value of total mass of CO in the nucleus and $M_{ejectas}$ the total mass of CO ejected since the beginning of integration.

Figures **??** and **??** present the evolution of Q since the beginning of the computation for the two king of models. The result is clear : in both cases, the mass conservation is improved by several orders of magnitude with the finite volume method using calculated values in the center of the cells and coefficients of diffusion on the border.

5 Conclusion

A good conservation of mass will permit to best-differentiate the out-bursts that are physical phenomena from numerical artifact. We aim to modify the model in a near future to make it numerically stable in cases of large collisional input energy (Orosei et al. 2001) and to obtain a more refined treatment (including better knowledge of trapping) of the different phases of water ice, clathrate phase notably. With this model, we will show the thermodynamic evolution of a KBO that is induced by collisions with a range of frequencies and kinetic energies compatible with those envisioned in the early Edgeworth-Kuiper Belt.

References

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