SF2A 2007
J. Bouvier, A. Chalabaev, C. Charbonnel (eds)

VISHNIAC INSTABILITY: EXPERIMENTAL PREPARATION

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Abstract. In this article we are interested by the pressure driven thin shell overstability (PDTSO) discovered by Vishniac in 1983. We studied this hydrodynamic instability in three stages. Firstly, we found an analytical solution at the Vishniac perturbation equations and we established several criteria for the growth of instability. Secondly, we researched in astrophysical observation the contribution of Vishniac instability in supernova remnant. The counterpart of this instability in the rippling of the radiative shock front is not clearly identified in this object pattern. Thirdly, we present evaluation of fluid and radiative parameters quantities to establish the feasibility of a future french radiative shock experiment scheduled in 2008 on LIL (Ligne d'Intégration Laser, Bordeaux, France).

1 Introduction

In this work, we studied the Vishniac instability by several approaches. In first papers about this hydrodynamic instability (Vishniac 1983, Ryu & Vishniac 1987) the Vishniac's study is set in supernova remnant layout. The instability develops when the shock front becomes radiative and this condition is encountered in supernova remnant. From theoretical point of view, only a quite accurate situation can support the developpment of this instability. In a first time, we detail the initial conditions, the induced assumptions and the factors supporting the growth of instability. In astrophysics, this instability is regularly called to explain the fragmentation of the interstellar medium (Mac Low 1999, Raymond 2003), but it was never established that it was really at the origin of this process. From experimental point of view, some people attempted to realize Vishniac instability on shock fronts of adiabatic blast wave (Grun et al. 1991), but their measurements of the perturbation growth rate did not agree with the theory. Since few years thanks to a wide collaboration, we are specialized in performing astrophysical interest experiments driven by high-power lasers (Michaut et al. 2007, Koenig et al. 2006, Bouquet et al. 2004). Now we are able to propose to produce in laboratory a radiative shock presenting required conditions for the generation of the instability. Experimental requirements are drastic (small value range for the gas adiabatic index, high shock velocity. . .) and only a laser of LIL power class (Mégajoule laser prototype) will enable to carry out oscillation growth. Our experiment is scheduled in 2008 and we present here a feasibility study of Visniac instability conditions to prepare this experiment. To lead this work it is necessary to reconsider the initial equations bench by Vishniac, in order to take into account the strong radiation emitted by the shock front in an optically thick medium. This experiment should enable to advance knowledge on this instability and to find parameters allowing the instability astrophysical characterization.

2 Theory of Vishniac instability

Using Vishniac's work (Vishniac 1983, Ryu & Vishniac 1987), we introduce in this first part main concepts, equations and their analytical solutions to establish the Vishniac instability theory.

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2.1 Hydrodynamic equations

For an inviscid fluid, the three equations describing its dynamics are:

$$\text{Mass conservation :} \qquad \partial_t \rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0 \ , \tag{2.1}$$

$$\text{Momentum conservation :} \qquad \rho\,\partial_t\vec{v} + \rho\,(\vec{v}\cdot\vec{\nabla})\vec{v} + \vec{\nabla}\,P = 0 \ , \tag{2.2}$$

$$\text{Energy conservation :} \qquad \partial_t P + (\vec{v}\cdot\vec{\nabla})P + \gamma\,P\vec{\nabla}\cdot\vec{v} = 0 \ , \tag{2.3}$$

where ρ, $\vec{v}$ and P are respectively the fluid density, velocity and pressure, and where γ is the gas polytropic index including possible radiative effects.
On the shock front (*i.e.* radial position R_0, see in Sec. 2.3), fluid parameters ρ, u and P undergo discontinuity given by classical Rankine-Hugoniot conditions:

$$[\rho u_r] = 0 \ , \ [u_T] = 0 \ , \ [\rho u_r^2 + P] = 0 \ , \ \left[u\left\{\frac{1}{2}\rho u_r^2 + \frac{\gamma}{\gamma-1}P\right\}\right] = 0 \ , \tag{2.4}$$

where u is the fluid velocity relative to the shock front split in its radial u_r and tangential u_T components.

2.2 Vishniac instability conditions in astrophysical object

In astrophysics, the Vishniac instability can be involved to explain shock front morphology of supernova remnant or Wolf-Rayet star bubble... In supernova remnant, the observational confirmation of the Vishniac instability contribution is not well established because a lot of other instabilities can play a role for modeling the final gas structure. However in Wolf-Rayet stars as Eta Carinae, 2D simulations of Mac Low (Mac Low 1999) have shown the same cauliflower-like appearance. This result lets probe that the Vishniac instability seems to have a real astrophysical contribution on stellar evolution.
In theory, the instability is able to growth under particular conditions: when the remnant is in an advanced stage as the Sedov phase (non radiative) or the snowplow phase (radiative) and when the shock front ripples (see Fig. 1). This rippling occurs when the thin shell of accreted matter behind the shock front (post-shock matter) encounters ambient medium irregularities (pre-shock matter). The instability dynamics provides from the difference between pressures pushing on both sides of the thin shell and therefore hydrodynamic equations are considered only in the shell thickness. To calculate the discrepancy pressure, we introduce a dimensionless variable $\beta = P_i/\rho_e V_s^2$ which will used later.

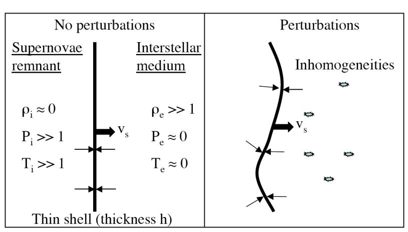

Fig. 1. Schematic principle of Vishniac instability on a thin shell where P_i is the thermal pressure (remnant hot gas) and $\rho_e V_s^2$ the ram pressure with ρ_e the interstellar medium density and V_s the shock velocity.

2.3 Vishniac instability criteria

To describe the thin shell dynamics, we introduce quantities averaged on thin layer thickness h: surface density $\overline{\rho}$, bulk radial velocity $\overline{v_r}$ and tangential velocity $\overline{\vec{v}_T}$. Then we impose a first order perturbation at these three variables and the resulting equation system is:

$$\frac{1}{\overline{\rho_0}}\left[\partial_t(\delta\overline{\rho}) + \frac{\delta R}{R_0}\partial_t(\overline{\rho_0})\right] = -\frac{2}{R_0}\left[\partial_t(\delta R) + \frac{V_s}{\overline{\rho_0}}\delta\overline{\rho}\right] + \frac{\rho_e}{\overline{\rho_0}}\left[\partial_t(\delta R) + V_s\frac{\delta R}{R_0}\right] - \vec{\nabla}_T\cdot(\delta\overline{\vec{v}_T}) \ , \tag{2.5}$$

$$\partial_t(\delta\overline{v_r}) = -\frac{1}{\overline{\rho_0}}\left[\delta\overline{\rho}\,\partial_t V_s + 2\rho_e V_s\,\delta\overline{v_r}\right] \ , \quad (2.6)$$

$$\partial_t(\delta\overline{\vec{v}_T}) = -\frac{1}{R_0\overline{\rho_0}}\left[\rho_e R_0 + \overline{\rho_0}\right] V_s\delta\overline{\vec{v}_T} - \frac{P_i}{\overline{\rho_0}}\vec{\nabla}_T\delta R \ . \quad (2.7)$$

In supernova remnants, the shock radius evolution can be described by a power law in time $R_0 \propto t^\alpha$ (homothetic solution due to the spherical geometry). In the same time, all perturbed parameters $\delta\overline{\rho}$, δR, $\delta\overline{v_r}$ and $\delta\overline{\vec{v}_T}$ takes the general separated variable form: $\delta\overline{x} \propto \overline{x_0}\ t^s\ Y_{lm}(\theta,\Phi)$ where x represents any variables mentioned previously. By introducing these solutions in Eq. (2.5) to Eq. (2.7), we obtain now that $\delta\overline{\rho}$, δR, $\delta\overline{v_r}$ and $\delta\overline{\vec{v}_T}$ explicitly depend on α, β and s. The instability growth rate s can be found by solving the new equation system. In previous works, authors give only numerical results of the growth rate $s(l)$, but we find an analytical solution for any couple (α,β), where l is the eigenvalue of spherical harmonic function Y_{lm}:

$$s = \frac{1}{2}\left[1 - 8\alpha \pm \sqrt{18\alpha^2 + 1 - 6\alpha \pm 2\alpha\sqrt{1 + 49\alpha^2 - 14\alpha - 12\beta\, l(l+1)\,\alpha(1-\alpha)}}\right] \ . \quad (2.8)$$

We replace α and β by their physical values and we find the minimal eigenvalue of the perturbation:

- for an adiabatic shock (*i.e.* Sedov-Taylor phase): $\alpha = 2/5$, $\beta = 1/2$, and $l_{min}(s=0) = 8.05$;
- for an isothermal shock (*i.e.* snowplow phase): $\alpha = 2/7$, $\beta = 1/6$, and $l_{min}(s=0) = 6.4$.

We need to have the maximal eigenvalue $l_{max}(s=0)$ for determine the polytropic index because in a first estimation $l_{max} = 3(\gamma+1/\gamma-1) = 3c$ with c the compression rate. According to calculations in (Ryu & Vishniac 1987) and using relations between unperturbed variables (given in bracket when needed), several required criteria are obtained allowing suitable layout for the Vishniac instability growth and determining experimental conditions. These criteria are respectively: polytropic index $\gamma < 1.1$ (ceiling value), compression rate $c > 11$, shell thickness $h \ll R_0$, perturbation wavelength $\lambda \sim R_0$ (with $\lambda = R_0/l$), shock velocity $V_s > 60\ km.s^{-1}$ ($V_s = \alpha R_0/t$). The Vishniac instability experiment is possible if all these criteria are verified. It is the subject of the next part.

3 New radiative shock experiment driven Vishniac instability

3.1 Hydrodynamic radiative conditions

In laboratory, it is necessary to take into account radiative effects in hydrodynamic equations because the shocked medium irradiates the initial gas which is enough optically thick to form a radiative precursor (Drake 2006). Under the diffusion approximation for radiation, three radiative terms are added in fluid equations: flux F_{rad}, pressure P_{rad} and energy density E_{rad}. Therefore Euler equations (Eq. (2.1) to Eq. (2.3)) become:

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0 \ , \quad (3.1)$$

$$\rho\frac{\partial\vec{v}}{\partial t} + \rho(\vec{v}\cdot\vec{\nabla})\vec{v} + \vec{\nabla}P_{tot} = 0 \ , \quad (3.2)$$

$$\frac{\partial}{\partial t}(\rho e + \rho\frac{u^2}{2} + E_{rad}) + \vec{\nabla}\cdot\left[(\rho e + \rho\frac{u^2}{2} + E_{rad})\vec{v} + P_{tot}\vec{v} + \vec{F}_{rad}\right] = 0 \ , \quad (3.3)$$

where e is the intern energy, and $P_{tot} = P_{th} + P_{rad}$ the sum of thermal pressure (including possible ionization of gas) and radiative pressure.

3.2 Evaluation of radiative parameters

We plan to perform radiative shock experiments in 2008 on LIL[1] and we expect to generate a Vishniac instability in the resulting fast ionized shock front. Firstly, from 2D simulations we obtain attempted values of density, thermal pressure and temperature are given in Table 1 for typical laboratory radiative shocks. Using these estimated values, we can determine if required criteria leading to Vishniac instability are fulfilled and what is the weight of radiative terms in Eq. (3.1) to Eq. (3.3). The gas polytropic index $\gamma(\rho,T)$ has been calculated

[1] http://www-lmj.cea.fr/html/cea.htm

taking into account the ionization degree and the excitation energy (Michaut et al. 2004). We found that the γ range for the foreseen LIL experiment is always less than the critical value 1.1 and it is around $\gamma = 1.03$ in the front shock and in the radiative precursor. This low value confirms that the gas is very ionized and the shock is very radiative. Secondly we consider this result to determine how radiative quantities take part in the gas

Table 1. Fluid parameters

	ρ $[kg.m^{-3}]$	P_{th} $[Pa]$	T $[eV]$
Shocked medium	3×10^2	2×10^{11}	60
Radiative precursor	12	6×10^4	50
Initial gas	0.5	10^4	3×10^{-2}

dynamics. We report the magnitude order of estimated radiative terms in Table 2. We conclude that radiative energy density E_{rad} and pressure P_{rad} are negligible in comparison with their thermal equivalents (E_{th}, P_{th}), but in opposite the radiative flux F_{rad} is very high compared to F_{th}. The shell could loss its energy quickly and could become thin as required for the instability growth. Thirdly we estimate the maximal perturbation

Table 2. Radiative and thermal parameters values

	E $[J.m^{-3}]$	P $[J.m^{-3}]$	F $[J.m^{-2}.s^{-1}]$
Radiation (rad)	2.10^8	6.10^7	9.10^{15}
Thermal (th)	2.10^{11}	2.10^{11}	2.10^{-4}

wavelength (with $l_{min}(s = 0)$) for an isothermal shock. In laboratory a shock radius is typically $R_0 \simeq 1$ mm, consequently $\lambda \simeq 100$ μm. Therefore we must observe the Vishniac instability using infrared diagnostic.
In conclusion, the instability growth are in principle possible because criteria and layout are well reached for a laboratory radiative shock planned on LIL. The witness of existence of the instability could be done by its wavelength measurement.

4 Conclusion

The Vishniac instability seems to be an interesting process for explain astrophysical object morphologies as supernova remnants and Wolf-Rayet bubbles. As we are specialized in laboratory experiment, we would like to study the radiative shock front behavior which will lead to a Vishniac instability growth and in same time we would like to prove its possible existence. In this goal we have estimated radiative quantities for the next LIL experiment with radiative precursor. We have show that *a priori* we will reach required conditions with this type of high-power laser.

5 Acknowledgments

Authors thank Tommaso Vinci from LULI (École Polytechnique) for MULTI 1D simulations used to establish experimental magnitude orders.

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