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CAN A DYNAMO OPERATE IN STELLAR RADIATION ZONES?

J.-P. Zahn[1], A. S. Brun[1 2] and S. Mathis[1 2]

Abstract. We examine the MHD instabilities arising in the radiation zone of a differentially rotating star, in which a poloidal field of fossil origin is sheared into a toroidal field. The numerical solutions built with the 3-dimensional ASH code are compared with the predictions drawn from an analytical study of the Pitts & Tayler instability. This instability is manifestly present in our simulations, with its conspicuous $m = 1$ dependence in azimuth. However, although the instability generated field reaches an energy comparable to that of the mean poloidal field, that field seems unaffected by the instability: it undergoes Ohmic decline, and is neither eroded nor regenerated by the instability. The instability is sustained as long as the differential rotation acting on the poloidal field is able to generate a toroidal field of sufficient strength but, up to a magnetic Reynolds number of 10^5, we observe no sign of dynamo action, of either mean field or fluctuation type, contrary to what was suggested by Spruit.

1 Introduction

There has been recently a surge of interest for stellar magnetism, due mainly to the discovery of magnetic fields in an increasing number of stars, and to their mapping through Zeeman imaging (cf. Donati et al. 2006). Theory benefits enormously from these new constraints, and quite naturally the main focus is on the generation of magnetic fields in turbulent convection zones, which can now been studied through high resolution numerical simulations (Brun et al. 2004, 2005; Dobler et al. 2006). But attention has been drawn also to the instabilities that may affect the magnetic field in stably stratified radiation zones. Spruit (1999) reviewed various types of instabilities that are likely to intervene in a magnetized radiation zone, and he concluded that the strongest among them were those which had been described by Tayler and his collaborators. They had shown for instance that a purely poloidal field would be unstable to non-axisymmetric perturbations, and that so would also a toroidal field (Tayler 1973; Wright 1973; Goossens et al. 1981). Later Pitts and Tayler (1985) proved that even in the presence of rotation a toroidal field would be unstable to such perturbations.

Spruit (1999, 2002) analyzed the latter instability in more detail, including Ohmic dissipation and radiative damping as was done before by Acheson (1978); he suggested that it could regenerate the toroidal field, and thus drive a genuine dynamo. Our interest in those instabilities was aroused when we observed them while verifying if a fossil field was able to prevent the spread of the solar tachocline, as had been proposed by Gough and McIntyre (1998): the results of these three-dimensional simulations are reported in Brun and Zahn (2006; hereafter referred to as BZ06). We then wanted to check whether these numerical results agreed with Spruit's analytical predictions; this incited us to re-examine his original derivation and, more importantly, to check whether the dynamo mechanism he suggested does actually operate. We report here the salient results of that study; for a complete account we refer the reader to Zahn, Brun & Mathis (2007).

2 Can the Pitts & Tayler instability sustain a dynamo?

The instability that affects the toroidal field, first described by Pitts and Tayler, could according to Spruit (2002) sustain a dynamo in stellar radiation zones, much as turbulent convection does in a convection zone.

[1] LUTH, Observatoire de Paris, CNRS, Université Paris-Diderot; Place Jules Janssen,92195 Meudon, France
[2] DSM/DAPNIA/Service d'Astrophysique, CEA Saclay, F-91191 Gif-sur-Yvette, France; AIM, UMR 7158, CEA - CNRS - Université Paris 7

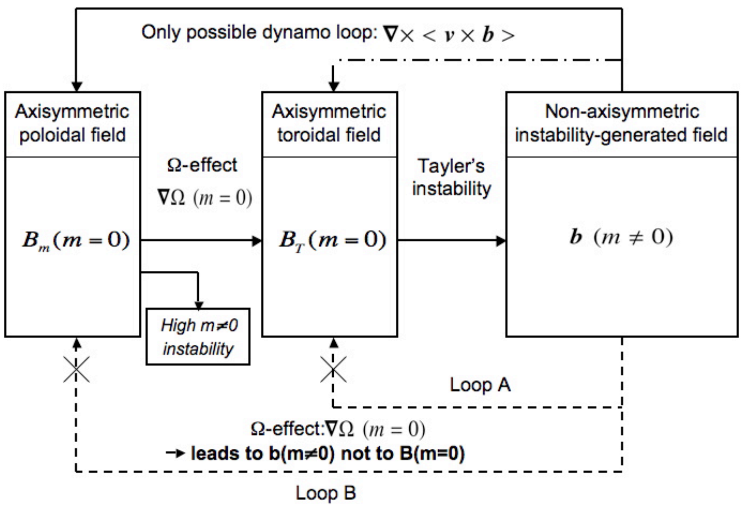

Fig. 1. How to close the dynamo loop involving the Pitts &Tayler instability. In dashed lines the loops proposed by Spruit (A) and Braithwaite (B). The only possible way to regenerate the mean toroidal and/or poloidal field is through the mean electromotive force $<\vec{v}\times\vec{b}>$ produced by the non-axisymmetric instability-generated field. But the dynamo must regenerate the poloidal field, in order to be fed from the differential rotation, and this leaves only the loop drawn in solid line. (Courtesy Astronomy & Astrophysics.)

We find his idea quite interesting, but on closer inspection we argue that this dynamo cannot operate as he describes it.

According to him, the instability-generated small scale field, which has zero average, is wound up by the differential rotation "into a new contribution to the azimuthal field. This again is unstable, thus closing the dynamo loop." But this shear induced azimuthal field has the same azimuthal wavenumber as the instability-generated field, i.e. $m\neq 0$ and predominantly $m=1$: it has no mean azimuthal component, and thus it cannot regenerate the mean toroidal field that is required to sustain the instability. For the same reason, the instability-generated field cannot regenerate the mean poloidal field, as was suggested by Braithwaite (2006). Therefore the Pitts & Tayler instability cannot be the cause of a dynamo, as it was described by Spruit and Braithwaite.

To close the dynamo loop, as is well known in mean-field theory (Parker 1955; Moffatt 1978), one has to invoke the so-called α-effect, which involves the non-zero mean electromotive force $<\vec{v}\times\vec{b}>$ that is produced here by the Pitts & Tayler instability. This can be read in the azimuthally averaged induction equation:

$$\frac{d\vec{B}}{dt}=\vec{e}_\varphi\left[\varpi\vec{B_m}\cdot\vec{\nabla}\Omega\right]+\vec{\nabla}\times<\vec{v}\times\vec{b}>-\vec{\nabla}\times(\eta\vec{\nabla}\times\vec{B}), \tag{2.1}$$

where $\vec{v}$ and $\vec{b}$ are the non-axi-symmetric parts of the velocity and of the magnetic field; the meridional advection has been absorbed in the Lagrangian time derivative. The first term on the RHS describes how the poloidal field $\vec{B_m}$ is wound up by the differential rotation to produce the mean toroidal field (the Ω-effect), the second how the mean electromotive force may (re)generate both the poloidal and the toroidal fields (the α-effect), and the last term represents the Ohmic diffusion, eventually enhanced by the turbulence (the β-effect). The only possible dynamo loop is depicted in Fig. 1 (solid lines); those proposed by Spruit and Braithwaite are shown in dashed lines. Moreover, for the dynamo to operate, the mean electromotive force must overcome the Ohmic dissipation.

Another type of dynamo has been observed in numerical simulations, with externally forced turbulence: it is the small-scale or fluctuation dynamo. There the magnetic field has no mean components, neither poloidal nor toroidal, but a quasi-stationary regime may be achieved when the magnetic Reynolds number Rm=VL/η exceeds some critical value (V and L are the characteristic velocity and length-scale of the turbulence). But

Parameter	Symbol	Sun	Case A	Case B
thermal diffusivity	κ	10^7	$8\,10^{12}$	$8\,10^{12}$
magnetic diffusivity	η	10^3	$8\,10^{10}$	$8\,10^9$
viscosity	ν	30	$8\,10^9$	$8\,10^9$
buoyancy frequency	N_t	$2.1\,10^{-3}$	$3\,10^{-4}$	$3\,10^{-4}$
rotation frequency	Ω	$3\,10^{-6}$	$3\,10^{-6}$	$3\,10^{-6}$

Table 1. Typical values of the relevant parameters in the upper radiation zone of the Sun, and values adopted for the numerical simulations (in cgs units).

it remains to be checked whether such a dynamo can be sustained by an imposed shear, such as a differential rotation.

3 3-D numerical simulations of the MHD instabilities

In Spruit's scenario, the toroidal field which undergoes the Pitts & Tayler instability is generated by winding up an existing poloidal field through a differential rotation $\Omega(z)$ that results from slowing down the star by a stellar wind. It is similar to the model we took to examine the possibility of confining the solar tachocline by a fossil field (BZ06). However, in our case the differential rotation is imposed in latitude by the adjacent convection zone, and the depth dependence of Ω near the polar axis is caused by thermal diffusion (cf. Spiegel & Zahn 1992). What distinguishes our model from that of Spruit, and several others (cf. Miesch et al. 2007; Arlt et al. 2007; Kitchatinov & Rüdiger 2007), is the presence of that large scale poloidal field which is allowed to evolve freely through advection by the meridional circulation, Ohmic diffusion, and eventually interaction with the instability-generated field.

Our simulations, together with the equations of the problem and the resolution methods, are described in full detail in BZ06. We used the global ASH code (Clune et al. 1999; Brun et al. 2004) to solve the relevant anelastic MHD equations (Eqs. 1 - 5 in BZ06) in a spherical shell representing the upper part of the solar radiation zone ($0.35 \leq r/R_\odot \leq 0.70$), with a resolution of $N_r \times N_\theta \times N_\varphi = 193 \times 128 \times 256$. For numerical reasons, we had to increase substantially the diffusivities of heat, magnetic field and momentum, as shown in Table 1, while respecting their hierarchy in the solar conditions. The characteristic evolution times are shortened accordingly, but contrary to BZ06 we made no effort here to rescale them by the Eddington-Sweet time in order to facilitate the comparison with the real Sun. In addition to the case A which is discussed in BZ06, we performed an additional series of simulations with a lower Ohmic diffusivity (by a factor of 10, case B), in order to reach a higher magnetic Reynolds number.

The temporal evolution of the magnetic fields and of the MHD instabilities is best followed in Fig. 2, where we display the energies of the poloidal, toroidal and non-axisymmetric components of the field. Initially a purely poloidal field of about 1 kG (when measured at the base of the computational domain) is buried in the radiation zone; it is unstable to non-axisymmetric perturbations of high azimuthal wavenumber ($m \approx 40$), as shown in BZ06 (cf. their Fig. 7), which is in agreement with the predictions of Markey and Tayler (1973). The poloidal field diffuses outward at a rate proportional to the Ohmic diffusivity and at some point (around 8,000 days in case A or 20,000 days in case B) it meets the differential rotation that has spread into the radiation zone, due to thermal diffusion. Their interaction induces there a toroidal field, whose strength is comparable to that of the poloidal field in case A; it becomes even larger in case B, where it keeps growing when we stop the simulation, at 60,000 days (corresponding to 1.3 Gyr when rescaled to the Ohmic diffusivity of the Sun). This toroidal field produces a strong non-axisymmetric MHD instability, with a dominant azimuthal wavenumber $m = 1$, which is clearly the signature of the Pitts & Tayler instability. In case A, this instability-generated field saturates at an energy comparable to that of the mean poloidal field, whereas in case B it is still increasing when we stop the simulation, much like the toroidal field.

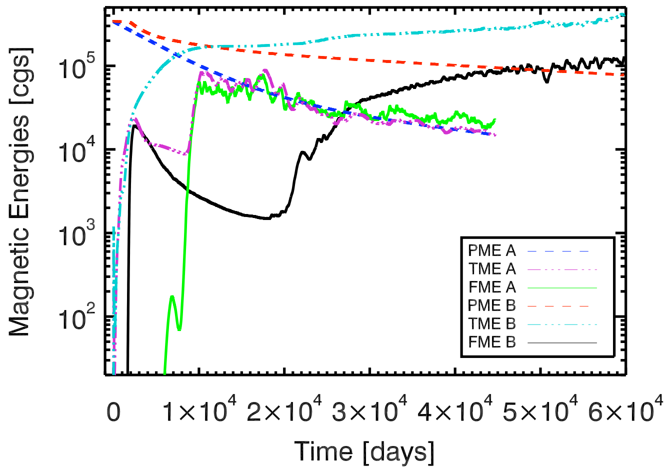

Fig. 2. Time evolution of the energies of the mean poloidal (PME), mean toroidal (TME) and non-axisymmetric (FME) components of the magnetic field. Cases A and B refer respectively to higher and lower magnetic diffusivity (cf. Table 1). Note the steady decline of the poloidal field, which is not affected by the irruption of the ($m = 1$) Pitts & Tayler instability (at t≈ 8,000 days in case A and ≈ 20,000 days in case B). (Courtesy Astronomy & Astrophysics.)

4 Looking for dynamo action

An important property of our numerical solutions is that the decline of the poloidal field is not affected by the instability-generated field. As can be seen in Fig. 2, this is true even once the (Pitts & Tayler) instability has reached its saturation level, where its energy is comparable with that of the mean poloidal field. This has two consequences. First it proves that the smallest resolved scales do not act on the mean poloidal field as a turbulent diffusivity: they seem to behave rather as gravito-Alfvén waves, with their kinetic energy balancing their potential energy (magnetic + buoyancy). Hence the saturation of the instability is not due to the mechanism suggested by Spruit (2002), which was inspired by thermal convection, namely that the magnetic eddy-diffusivity η_t adjusts such as to neutralize the Pitts & Tayler instability. In our model - and presumably also in stellar radiation zones - the regulation is apparently achieved through the action of the Lorentz torque on the differential rotation, which produces just the right amount of toroidal field that is required to sustain the instability, through small departures from Ferraro's law.

At saturation, the mean quantities are stationary on the instability time-scale Ω/ω_A^2, which translates into the following condition for the azimuthal component of the momentum equation:

$$4\pi\rho\frac{\partial}{\partial t}(\varpi^2\Omega) = \vec{B_m}\cdot\vec{\nabla}(\varpi B_\varphi) + <\vec{b_m}\cdot\vec{\nabla}(\varpi b_\varphi)> \approx 0. \tag{4.1}$$

Here $\vec{B}$ is the mean axisymmetric magnetic field with $\vec{B_m}$ being its meridional part, and likewise for $\vec{b}$, the non-axisymmetric field generated by the instability; $< \ \dots \ >$ designates the azimuthal average. Since the characteristic scales are comparable for the mean and fluctuating field, this equation tells us that the magnetic energy of the instability field must be - crudely speaking - of the same order as that of the axisymmetric mean field, as we observe in our simulations.

In our simulations we see no regeneration of the mean poloidal field: the α-effect plays a negligible role, at least up to the magnetic Reynolds number $Rm = R^2\Delta\Omega/\eta \sim 10^5$ (case B), for Prandtl number $P_m = \nu/\eta = 1$. Note that the β-effect, i.e. the turbulence-enhanced diffusivity, is also absent here; hence one should not expect much mixing of the stellar material. We thus conclude that in our simulations the Pitts & Tayler instability is unable to sustain a large-scale mean field dynamo, in the parameter domain that we have explored.

There is no sign either of a small-scale fluctuation dynamo, though one may argue that we inhibit this type of dynamo by imposing our large-scale fossil field. To check this point, we switched off the poloidal field at the latest stage of our low-η simulation (case B). The toroidal field decreases then rapidly, because it is no longer produced by the Ω-mechanism, and the instability-generated field accompanies its decline. Thus the fluctuating field does not maintain itself, although the magnetic Reynolds number, $\mathrm{Rm} = R^2 \Delta\Omega/\eta \sim 10^5$ (case B), should amply fulfill the necessary condition for a turbulent dynamo: at magnetic Prandtl number of order unity, as here, the critical magnetic Reynolds number should be of the order of 100, according to Ponty et al. (2006).

5 Why do our results differ from those of Braithwaite?

To our knowledge, the only simulation so far that claims to support a dynamo operating in stellar radiation zones is that by Braithwaite (2006): he showed that a sufficiently strong differential rotation can amplify a seed field to a level where it seems to be maintained, while undergoing cyclic reversals. According to him, his results confirm the analytical expectations of the role of the Pitts & Tayler instability, but to us it is not clear whether that specific instability plays any role in his simulation: for instance the author does not mention the $m = 1$ signature of the instability-generated field. Instead, he may have triggered a fluctuation dynamo.

Why do we reach different conclusions about the existence of such a dynamo? It is true that our set-ups differ somewhat, even when we suppress the large-scale poloidal field: in Braithwaite's case, differential rotation is enforced by a body force with strong relaxation, whereas in ours it spreads from the boundary of the computational domain, which is more realistic.

We differ also in the boundary conditions applied to the magnetic field, that are known to play a sensitive role in numerical dynamo. We connect our internal field to a potential field outside, as if the convection zone were a perfect conductor, whereas Braithwaite imposes the field to be normal to the boundary. Using a geometry similar to that of Braithwaite, Gellert et al. (2007) do not find dynamo action either, and they loose the Pitts &Tayler instability when they switch off the exterior field, much as we find when we switch off our poloidal field.

But the main difference probably resides in the way the equations are solved. Our code uses (enhanced) physical diffusivities; it is of pseudo-spectral type with a resolution of $128 \times 256 \times 192$, and this method is known to have exponential convergence and machine accuracy in evaluating derivatives. This allows us to reach a magnetic Reynolds number of 10^5, and when we fail to observe dynamo action this is certainly not due to a insufficient resolution. Braithwaite uses instead a 6th order finite difference scheme, with a resolution of $64 \times 64 \times 33$; the numerical diffusion is tuned to ensure stability for the chosen resolution, but it is not straightforward to infer from it the magnetic Reynolds number that characterizes the simulation (Braithwaite, private communication).

6 Conclusion

We have re-examined the non-axisymmetric instabilities affecting a toroidal magnetic field in a rotating star, which have first been described by Pitts and Tayler (1985) in the ideal, non-dissipative limit. The problem was generalized later by Spruit (1999) to include the diffusion of heat and of magnetic field. We have extended his analytic treatment to the case where the medium is stratified both in entropy and in chemical composition. Our exact solutions fully validate his approximate results; they are presented in Zahn et al. (2007).

Then we have compared these analytical results with numerical solutions built with the 3-dimensional ASH code; in our model the toroidal field is produced by shearing a fossil poloidal field through the inward propagating differential rotation imposed by the convection zone. Our solutions clearly display the Pitts & Tayler instability with its dominant $m = 1$ mode, but they do not conform to the quantitative predictions of the analytical model. In our simulations the instability occurs well below the threshold predicted by the analytical model, and it is much less sensitive to the stratification. These discrepancies are probably due to the approximations made in order to simplify that analytical model, such as neglecting the poloidal field, the differential rotation and the radial component of the buoyancy force.

Furthermore, it appears that the saturation of the instability cannot be ascribed to a turbulent diffusivity fulfilling the critical conditions, as in Spruit's scenario: it occurs when the energy of the instability-generated field reaches approximately that of the mean fields. The mean poloidal field steadily declines due to Ohmic dissipation, while it is wound up by the differential rotation to produce the toroidal field. Contrary to Spruit's expectation, which as we have shown in §2 is based on questionable grounds, we detect here no sign of a dynamo that could regenerate the mean fields; moreover, the small scale motions do not act as an eddy diffusivity on the mean poloidal field. Unlike the turbulent motions present in a convection zone, the instability-generated motions produce here no α and no β-effect. Neither do we observe a fluctuation dynamo, in spite of the relatively high magnetic Reynolds number, in contrast with the findings of Braithwaite (2006), who however considers a somewhat simpler model.

But the Pitts & Tayler instability persists as long as the toroidal field remains of sufficient strength, i.e. a few Gauss in the conditions prevailing below the solar convection zone, which puts a similar requirement on the poloidal field. We have shown in Brun & Zahn (2006) that such a poloidal field does not exist in the Sun, because it would imprint on the radiative interior the differential rotation of the convection zone, and that is ruled out by the helioseismic diagnostic. The Pitts & Tayler instability could well occur in other stars hosting a large-scale toroidal field, but we doubt that it may cause there any significant transport of matter and angular momentum, since in our simulations the motions associated with the instability behave rather as Alfvén waves than as turbulence. To settle that issue, observational tests, such as those expected from the COROT mission, will play an irreplaceable role.

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