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NONLINEAR MIRROR MODE STRUCTURES

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Abstract. We present a brief survey of recent direct numerical simulations and asymptotic or phenomenological models, aimed to understand the formation and time evolution of pressure-balanced coherent structures observed in the solar wind or in planetary magnetosheaths, and usually viewed as nonlinear mirror modes. Special attention is paid to the conditions for the generation of magnetic holes or humps, and to the dynamical role of kinetic and hydrodynamic effects.

1 Introduction

Magnetic structures in the form of pressure-balanced magnetic holes and humps with a size of a few ion Larmor radii, anticorrelated with density fluctuations, are often observed both in the solar wind and in planetary magnetosheaths close to the magnetopause, in regions characterized by anisotropic ion temperatures ($T_\perp > T_\|$) and a sufficiently high β (Kaufmann et al., 1970; Sperveslage et al., 2000; Winterhalter et al., 1994). Such conditions can in particular be met under the effect of the plasma compression in front of the magnetopause (Hellinger & Travnicek, 2005). These structures are quasi-static in the plasma reference frame and display a cigar-like shape, elongated in a direction making a small angle with the ambient field (Horbury et al. , 2004, and references therein). By permitting a distinction between spatial and temporal variations, the multispacecraft observations of the Cluster mission have provided an unambiguous detection of these structures that are usually viewed as resulting from the nonlinear saturation of the mirror instability, although alternative theories, in terms of slow mode solitons have also been presented (Stasiewicz, 2004b). It was in particular suggested that the mirror instability could play the role of a trigger generating high amplitude fluctuations that evolve such as to become typical solitary structures of isotropic plasmas (Baumgärtel et al., 2005). Nevertheless, in contrast with the linear mirror instability that has been extensively studied using particle in cell (PIC) simulations (Gary, 1992; McKean et al., 1992, 1994), phenomenology (McKean et al., 1993; Southwood & Kivelson, 1993) and, near threshold, the kinetic theory in the low-frequency limit (Vedenov & Sagdeev, 1959; Hall, 1979; Pokhotelov et al., 2002, 2004; Hellinger, 2007), the nonlinear saturation of this instability is still poorly understood, and the origin of the observed structures remains partly unsettled. Magnetic holes are in particular observed in regions where the plasma is linearly stable. Furthermore, in realistic situations, the mirror instability can be competing with the ion cyclotron anisotropic (ICA) instability, especially at relatively low β and directions making a moderate angle with the ambient field, although the presence of helium He^{++} can enhance the relative importance of the former effect (Gary, 1992; McKean et al., 1992, 1994).

In this paper, we briefly review recent results based on numerical simulations (PIC and Eulerian integrations of the Vlasov-Maxwell equations) and on asymptotic or phenomenological models (Passot et al., 2006; Passot & Sulem, 2008; Califano et al., 2007; Borgogno et al., 2007), with the aim to provide a qualitative interpretation of observational data usually viewed as nonlinear mirror modes.

2 Pressure-balanced magnetic structures

Inspection of mirror-like structures recorded by spacecraft missions early suggested that magnetic humps are preferentially present in regions of relatively low magnetic field, while magnetic holes are rather observed in

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regions of high field (Lucek et al., 1999). More precisely, the presence of the former or latter structures is strongly correlated with larger or smaller value of the parameter β (Joy et al., 2006). It in particular turns out that the presence of magnetic humps does not require the exceptionally large values of β suggested by previous observations (Erdös & Balogh, 1996). A more quantitative picture is obtained when characterizing the nature of the magnetic structures by the skewness of the magnetic fluctuations that appears to be directly related to the distance to threshold (Génot et al., 2006). Negative skewness (magnetic holes) is observed below or slightly above threshold, while positive skewness (magnetic humps) are measured in more unstable regimes (see also Soucek et al. (2007)). The phenomenon of bistability (associated with the existence of non trivial solutions in the linearly stable regime), together with the preference of magnetic humps at larger values of β and/or for larger distance from threshold is consistent with a nonlinear stability analysis based on an energy minimization argument, performed in the framework of ordinary anisotropic MHD with a specific equation of state derived from the stationary fluid hierarchy by assuming a bi-Maxwellian distribution function (Passot et al., 2006). It is noticeable that in contrast with bi-adiabatic equations of state or their generalizations (Hau &Sonnerup, 1993), the present closure where the parallel ion temperature is uniform and the perpendicular one a homographic function of the magnetic fluctuation (Passot & Sulem, 2006; Chust & Belmont, 2006), correctly reproduces the mirror instability threshold, an indication that such a fluid model retains essential ingredients associated with the mirror bifurcation. Nevertheless, lacking kinetic effects, this model is not suitable to accurately reproduce the time evolution of linear mirror modes. It could however be relevant to describe the very large magnetic holes (hundreds to thousands of ion Larmor radii) observed in the solar wind (Stevens & Kasper, 2007).

3 Nonlinear evolution

The first proposed description for the linear saturation of the mirror instability is based on the quasi-linear theory (Shapiro and Shevchenko, 1964) that assumes space homogeneity and a large number of excited modes. It is characterized by a diffusion process in velocity space, leading to a flattening of the space-integrated distribution function. This description is supported by the early time evolution of numerical simulations of the Vlasov-Maxwell equations initialized with a bi-Maxwellian distribution function and performed along the direction of maximum linear instability growth, in an extended periodic domain slightly above threshold (the growth rate is of order 10^{-3} in ion gyrofrequency units) (Califano et al., 2007). Nevertheless, after a while, coherent structures form, making the quasi-linear theory no longer valid. One in particular notices that the instantaneous growth rate (that significantly differs from that associated with a bi-Maxwellian distribution function) becomes negative while the amplitude of the structures (together with the skewness) continues to grow during thousands of ion gyroperiods under the effects of nonlinearities. Big magnetic humps are formed whose long-time evolution is characterized by a coarsening process that progressively reduces their number, leading eventually to a few large-amplitude peaks whose evolution becomes extremely slow. Note that a simpler dynamics develops in a small computational domain where geometrical constraints prevent the quasi-linear regime to develop, resulting in an early formation of coherent structures with a distribution function that does not appear significantly perturbed and in particular does not display evidence of particle trapping. The question however arises whether the same conclusions hold far from onset (Pantellini et al., 1995) or in deep magnetic holes.

In order to address the dynamics of nonlinear mirror modes, an asymptotic model based on a reductive perturbative expansion was developed near threshold, a regime where all the unstable modes are located at large scale (Kuznetsov et al., 2007). In this limit, kinetic effects are expected to act only at the linear level, which permits a simplified approach where the nonlinear contributions are estimated from the drift kinetic equation. A systematic perturbative analysis from the Vlasov-Maxwell system justifies this approach (Califano et al., 2007). This asymptotics leads to a pseudo-differential equation of gradient type for the parallel magnetic fluctuations, where, as announced, Landau damping and finite Larmor radius (FLR) effects arise at the linear level only, the nonlinearity originating from hydrodynamic effects. The Landau damping drives the system, while the FLR effects quench the instability at small scales. It turns out that the only asymptotically relevant nonlinearities tend to reinforce the linear instability, leading to a finite time singularity that can be viewed as the signature of a sub-critical bifurcation, not amenable to a perturbative calculation. To cope with this situation, a phenomenological model was constructed by supplementing the asymptotic equation with nonlinear FLR effects associated with the local variation of the ion Larmor radius in the coherent structures (Kuznetsov et al., 2007; Califano et al., 2007). In regions of weaker magnetic field, the Larmor radius is larger, leading to a more efficient stabilizing effect than within the linear theory. As a consequence, the mirror instability

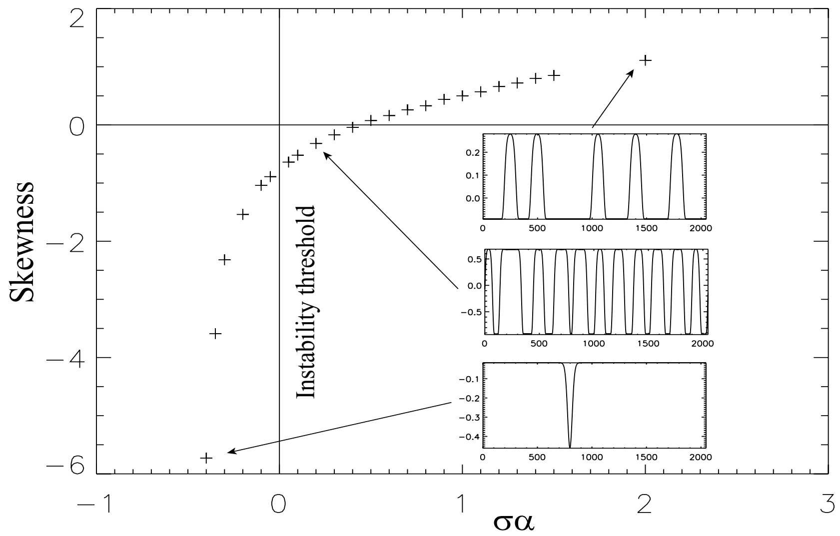

Fig. 1. Variation of the magnetic field skewness with $\sigma\alpha$ predicted by the model equation (3.1), where α combines the distance to threshold with the value of β, and $\sigma = \pm 1$ characterizes the positive or negative distance to threshold. The insets display typical quasi-stationary solution profiles.

is more easily quenched in magnetic minima than in magnetic maxima, making magnetic humps more likely to form during the saturation phase of the mirror instability. In contrast with the previous phenomenological descriptions based on the cooling of a population of trapped particles (Kivelson & Southwood, 1996; Pantellini, 1998) that mostly predict deep magnetic holes (except for very large β) and do not refer to the bistability phenomenon, the present model successfully reproduces spacecraft observations and numerical simulations. It indeed predicts the formation of magnetic humps above threshold and also the existence of subcritical magnetic holes (when the system is initialized by large-amplitude perturbations). In dimensionless units (depending on the distance to threshold), this phenomenological model is governed by the equation

$$\partial_\tau U = \widehat{K}_\xi \left[\sigma U - 3U^2 + \frac{1}{1+\alpha U}\partial_{\xi\xi} U - \frac{4\nu}{9(1+\alpha U)^2}\partial_{\xi\xi\xi\xi} U. \right], \tag{3.1}$$

Here ξ is the spatial coordinate in a direction quasi-transverse to the ambient field, and $\widehat{K}_\xi = -\mathcal{H}\partial_\xi$ (where $\mathcal{H}$ denotes the Hilbert transform) reduces in Fourier space to the multiplication by the wavenumber modulus $|k|$. The function U is related to the longitudinal magnetic perturbation b_z by $b_z/B_0 = \alpha U$ (here, B_0 refers to the magnitude of the ambient field). The parameter

$$\alpha = \frac{2\beta_\perp}{1+\beta_\perp}\left[\beta_\perp\Big(\frac{\beta_\perp}{\beta_\parallel} - 1\Big) - 1\right] \tag{3.2}$$

scales like the distance $\beta_\perp(\beta_\perp/\beta_\parallel - 1) - 1$ to threshold at moderate $\beta_\perp$, while it varies proportionally to $\beta_\perp$) when the latter is small. Furthermore, $\sigma = \pm 1$, depending on the positive or negative distance to threshold. The coefficient ν fixes the domain size (the value 10^{-2} was typically used in the simulations). An interesting quantity also used to analyze satellite data is the skewness of the magnetic perturbations, that is independent of the performed rescalings. This quantity is plotted versus $\sigma\alpha$ in Fig. 1. Above threshold ($\sigma = 1$), Eq. (3.1) is initialized with a small random noise, while in the subcritical regime ($\sigma = -1$) a much larger random initial perturbation is needed. The resulting graph is qualitatively very similar to that designed from Cluster data (Génot et al., 2006). The inserted graphs refer to the corresponding typical profiles of quasi-stationary solutions. The existence of stable subcritical magnetic holes is confirmed by direct numerical simulations of the Vlasov-Maxwell equations, where an initial large amplitude magnetic depression survives after a rapid adjustment. Such magnetic holes are also observed to persist in direct simulations above threshold (then developing a strong overshoot) (Califano et al., 2007).

It is interesting to note that magnetic holes can also be generated in a possibly more natural manner in an extended domain, at moderate β, in a regime far enough from the linear instability threshold. Numerical simulations performed in this regime show that the system does not relax to a quasi-steady state and that the early formed magnetic humps gradually transform into magnetic holes, an effect that can be related to a decrease of the β of the plasma that takes place as time elapses. No similar evolution is obtained in a small computational domain where no qualitative change is produced by similar increase of the distance to threshold.

4 Are mirror modes amenable to a fluid description?

Reproducing the linear mirror instability requires an approach retaining both Landau damping and (linear) FLR effects. It is possible to construct such a "FLR Landau fluid" (Sulem & Passot, 2008; Passot & Sulem, 2007) that generalizes the anisotropic Hall MHD by retaining these effects. It mostly consists in extending MHD Landau fluid models (Snyder et al. , 2997), by including transverse scales comparable to the ion Larmor radius, within a gyrokinetic scaling, suitable for quasi-transverse dynamics (Howes et al., 2006). The FLR Landau fluid is essentially obtained by closing the fluid hierarchy at the level of the fourth rank moment (in order to retain the nonlinear dynamics of the heat fluxes), in a way consistent with the low-frequency linear kinetic theory. It improves a simpler closure made at the level of the pressure tensor (Passot & Sulem, 2006). The FLR Landau fluid retains all the hydrodynamic nonlinearities together with a linear (or possibly semi-linear) description of the low-frequency kinetic effects. It thus contains all the ingredients entering the asymptotic equation derived by the reductive perturbative expansion, with nevertheless the property that the fluid model involves rich enough nonlinearities to arrest the singularity. In the linearly unstable regime, the system thus evolves towards the formation of sharp magnetic holes and also to transient humps at larger values of β (Passot & Sulem, 2008). During the saturation phase, mean ion parallel and perpendicular temperatures evolve in a way that reduces the distance to threshold. The structures that survive after their number has been reduced by coarsening effect, are not perfectly stationary, and their amplitude decreases on a very long time scale. The model that does not incorporate the variation of the ion Larmor radius, is unable to reproduce the formation of mirror humps at moderate β.

Since the structures tend to slowly relax to the uniform state, it is of interest to enforce the stationarity by maintaining constant the mean ion parallel and perpendicular pressures. This can be viewed as a simple way of imposing a forcing that in more realistic situations is obtained through boundary conditions, such as for example an inflow. The fluid model then correctly captures the phenomenon of bistability, associated with the existence of magnetic holes below threshold. In this case, it is interesting to note that it also reproduces the numerical observation that the magnetic component perpendicular to the plane defined by the ambient field and the direction of propagation is symmetric with respect to the center of the magnetic hole. This contrasts with the soliton models based on anisotropic Hall MHD with no Landau damping nor (linear) FLR (Stasiewicz, 2004a,b; Mjølhus, 2006). It is also interesting to notice that while in a small domain the system evolves towards a stationary regime, in an extended domain and for a regime far enough from threshold, a spatio-temporal chaotic dynamics develops where holes form and disappear in an unpredictable way (Borgogno et al., 2007).

5 Conclusion

A main conclusion of this study is that the saturation of the mirror instability preferentially leads to the formation of magnetic humps, although large amplitude magnetic holes are also stable solutions of the Vlasov-Maxwell equations, both below and above threshold, indicating the existence of bistable regimes. These observations are consistent with previous PIC simulations performed at lower resolutions (Baumgärtel et al., 2003). There is however no evidence that large-amplitude magnetic holes can be reached under the effect of the linear mirror instability. Furthermore, simulations in extended domains near threshold do indicate the development of an early quasi-linear regime that is arrested by the formation of coherent structures under the effect of hydrodynamic nonlinearities. It is unclear whether there exist conditions such that the system can stabilized in a quasi-linear regime before hydrodynamic nonlinearities become relevant. The influence of hydrodynamic nonlinearities was captured using a reductive perturbative expansion of the Vlasov-Maxwell equations. It turns out that these effects actually strenghten the linear instability leading to a finite-time singularity. This early breakdown of the asymptotics enables nonlinear kinetic effects to rapidly become relevant. As a consequence, their effect does not reduce to a smoothing of the singularity but actually drastically modifies the nature of the nonlinear structures. Nevertheless, a simple phenomenological correction of the asymptotic equation retaining the variation of the local ion Larmor radius associated with the structures turns out to be sufficient to reproduce main qualitative aspects of numerical and observational data. The question then arises whether there exists a regime of parameters for which a signature of the singularity predicted by the asymptotic equation could be visible.

The FLR Landau fluid model is able to accurately capture not only the linear dynamics of the mirror instability but also the finite amplitude solutions in the form of magnetic holes commonly observed in Vlasov simulations and in satellite data. This approach is relevant when the nonlinearities originate mainly from the hydrodynamic effects, and thus, further developments would consist in retaining, possibly in a phenomenological manner, the effects of nonlinear kinetic effects such as particle trapping and, possibly more important, nonlinear FLR effects. The latter effects are captured by the so-called gyrofluids (Brizard, 1992; Dorland & Hamett, 1993; Beer et al., 1996; Brizard & Hahm, 2007) obtained by closing the moment hierarchy derived from the gyrokinetic equation (Howes et al., 2006), but such a formalism has however not yet been developed in a regime where the equilibrium state is anisotropic, as needed for the mirror instability to occur.

Another interesting issue concerns the observation in the solar wind of magnetic holes bounded by discontinuities (Tsurutani et al., 2003) or extending up to thousands ion Larmor radii (instead on a few units) (Stevens & Kasper, 2007), whose origin is hardly understood, although they display important properties of nonlinear mirror modes. Finally, the question arises of the role played by mirror mode structures on the magnetopause boundary, and in particular whether they can trigger micro-reconnection events.

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