SF2A 2008
C. Charbonnel, F. Combes and R. Samadi (eds)

HIGH AREA-TO-MASS RATIOS GEOSTATIONARY SPACE DEBRIS : STABILITY AND SECONDARY RESONANCES (MEGNO AND FREQUENCY ANALYSIS)

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Abstract. Recently a new unexpected population of 10 cm size space debris near the geostationary orbit (GEO) has been discovered. These objects with high area-to mass ratios sometimes present highly eccentric orbits (Schildknecht et al., 2004, 2005). Recent numerical and analytical investigations (Anselmo & Pardini, 2005; Liou & Weaver, 2005) prove that this newly discovered population consists of near geosynchronous objects with high area-to-mass ratios. The large area-to-mass ratios space debris have a dynamical behavior dominated by the solar radiation pressure and the resonance with C_{22} spherical harmonic. In this paper we develop further the analysis done by Valk et al. (2008) by using a frequency analysis to study both the stability and the resonances of such particular orbits.

1 Introduction

As the GEO region is indisputably all-important for both commercial and scientific missions, ESA has recently initiated an optical search for fragments in the geostationary ring in order to improve the knowledge about the debris population and to understand their future evolution (Schildknecht et al. 2005). These observations have been performed, on behalf of ESA, by the Astronomical Institute of the University of Bern (AIUB) by using the 1 meter telescope located in Tenerife (Canary Islands). The recent observational discoveries in high altitude Earth's orbit (for the most part in geosynchronous orbits) stimulated the revisit of direct radiation pressure models. In particular, numerical investigations were recently performed in order to assess the time evolution of objects subject to such extreme situations (Anselmo & Pardini 2005; Liou & Weaver 2004). In this framework, short-term as well as long-term evolutions of geosynchronous space debris were studied in detail. These objects sometimes present highly eccentric orbits with eccentricities as high as 0.55 (Schildknecht et al. 2004, 2005). In addition, these authors and others, such as Chao (2006) and later Valk et al. (2007) presented some detailed analytical results concerning the short- and long-term evolution of high area-to-mass ratios geosynchronous space debris subjected to direct solar radiation pressure. More specifically, these latter authors mainly focused their attention on the long-term variation of both the eccentricity and the inclination vector. Recently, Valk et al. (2008) presented investigations of the long-term stability (intrinsic stability of such uncommon orbits) of high area-to-mass ratios space debris subjected to the direct solar radiation pressure, by means of the `MEGNO` criterion, moreover a relevant class of additional resonances (w.r.t the main resonance due to C_{22} spherical harmonic) appearing in the phase space were emphasized. In this paper, we improve such results based on the `MEGNO` indicator, by performing a preliminary frequency analysis of these additional resonances. The paper is organised as follows, first we summarize the Valk et al. (2008) paper. Secondly, we sketch the main ideas of the FMA (Frequency Map Analysis) theory and we lastly conclude by presenting our results based on the frequency analysis results.

2 Short review of the state of the art

In a recent paper (Valk et al. 2008) we studied the global dynamics of high area-to-mass ratios geosynchronous space debris, applying a recent technique developed by Cincotta et al. (2000), Mean Exponential Growth factor

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of Nearby Orbits (MEGNO), which provides an efficient tool to investigate both regular and chaotic components of the phase space. For the purpose of this study, we considered the modeling of a space debris subjected to the influence of the Earth gravity field (includes the central body attraction, the second degree and order harmonics J_2, C_{22} and S_{22}), as well as the combined attractions of the Sun and the Moon. The perturbing effects of the direct solar radiation pressure were also taken into account for a high area-to-mass ratio fixed to $A/m = 10$ m^2/kg. The GEO objects exhibit a resonance due to the C_{22} harmonic, thus the main dynamics of a large set of GEO object can be described by the double pendulum. In this paper we improve the main results of the Valk et al. (2008) reported in Figure 1. The interested reader could find there more details in Valk et al. (2008).

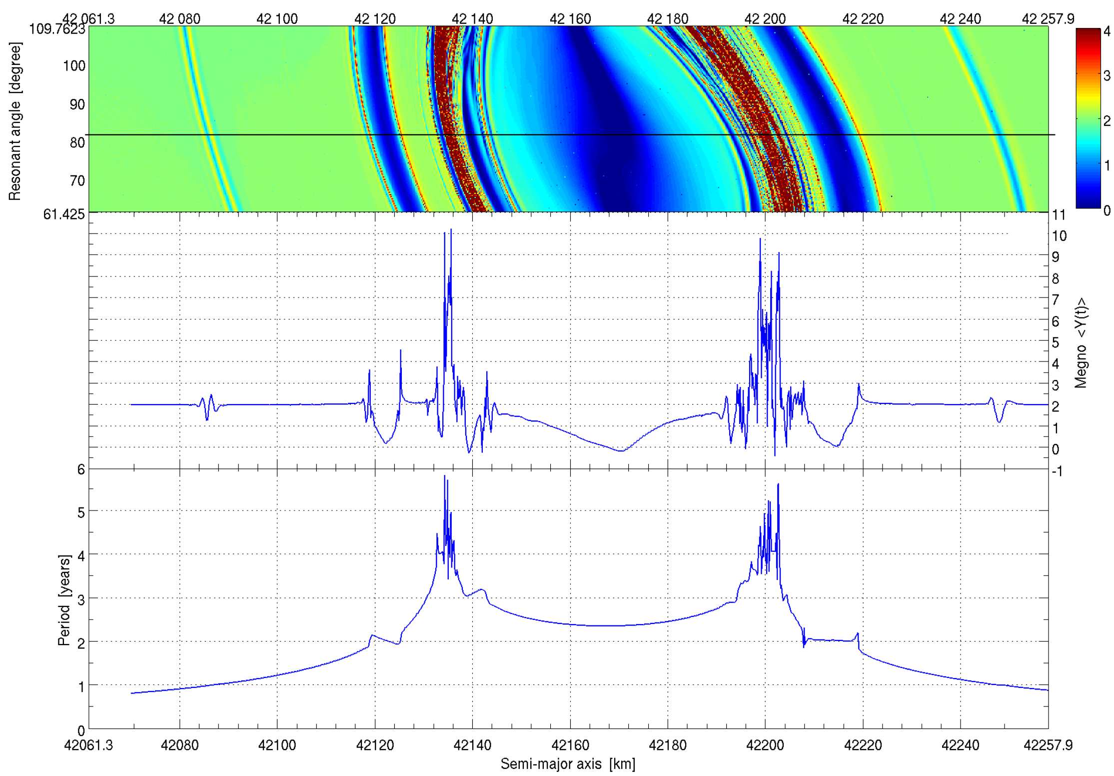

Fig. 1. Blow-up of the phase space around 1:1 resonance due to C_{22} [Graphics and caption from Valk et al. (2008)]. For a resonant angle section (horizontal black solid line), evolution of the MEGNO (middle panel) and the fundamental period of σ with respect to the initial semi-major axis a_0 (bottom panel).

3 Extended numerical analysis: Frequency Map Analysis

We now introduce a second original approach to study the stability of the high area-to-mass ratio space debris. We use a powerful frequency analysis FMA (Laskar 1990, 1993, 1995), previously introduce and used to study, for example, the chaotic motion of the solar system (Laskar 1990). To the best of our knowledge the present study provides the first application of FMA to the geodesy.

Let us briefly sketch the main idea of the FMA. Let $f(t)$ be a quasiperiodic regular function of the time t, with values in a complex domain, whose Fourier coefficient a_k have decreasing amplitudes with k. The FMA is a efficient numerical method allowing to obtain an approximation $f'(t)$ of the given signal with a given number (N) of harmonics, from the numerical knowledge of $f(t)$ over a finite time span $[-T, T]$ (Laskar 1990, 1995)

$$f(t) = \sum_{k=1}^{\infty} a_k e^{i\nu_k t} \qquad\qquad f'(t) = \sum_{k=1}^{N} a'_k e^{i\nu'_k t}$$

For our results, we use the FMA software (Laskar 1990). For the same resonant angle section (Fig. 1) but within a simpler model (only J_2, C_{22} and radiation pressure with circular sun, over a time span of 500 years) we calculate the first fundamental period of resonant angle (Fig. 2 bottom). To improve the direct analysis of the frequency curve ($\nu_1(a)$), we calculate (Laskar 1993) the numerical second derivative ($\delta\delta\nu_1(a) = \nu_1(a) - 2\nu_1(a-h) + \nu_1(a-2h)$) of the frequency of the resonant angle (Fig. 2 middle pannel). We can also calculate the diffusion of the main frequency with respect to the time (Laskar 1993). This is obtained by computing a first value $\nu_1^{(1)}$ of the main frequency of the resonant angle over a time span T_1 (for example to $-T$ untill 0) and a second value $\nu_1^{(2)}$ of this main frequency of the same signal but over a time span T_2 (for example to 0 untill T). Then we estimate the diffusion rate (Fig. 2 top) by computing a numerical indicator of diffusion of frequency $\log|1 - \nu_1^{(2)}/\nu_1^{(1)}|$.

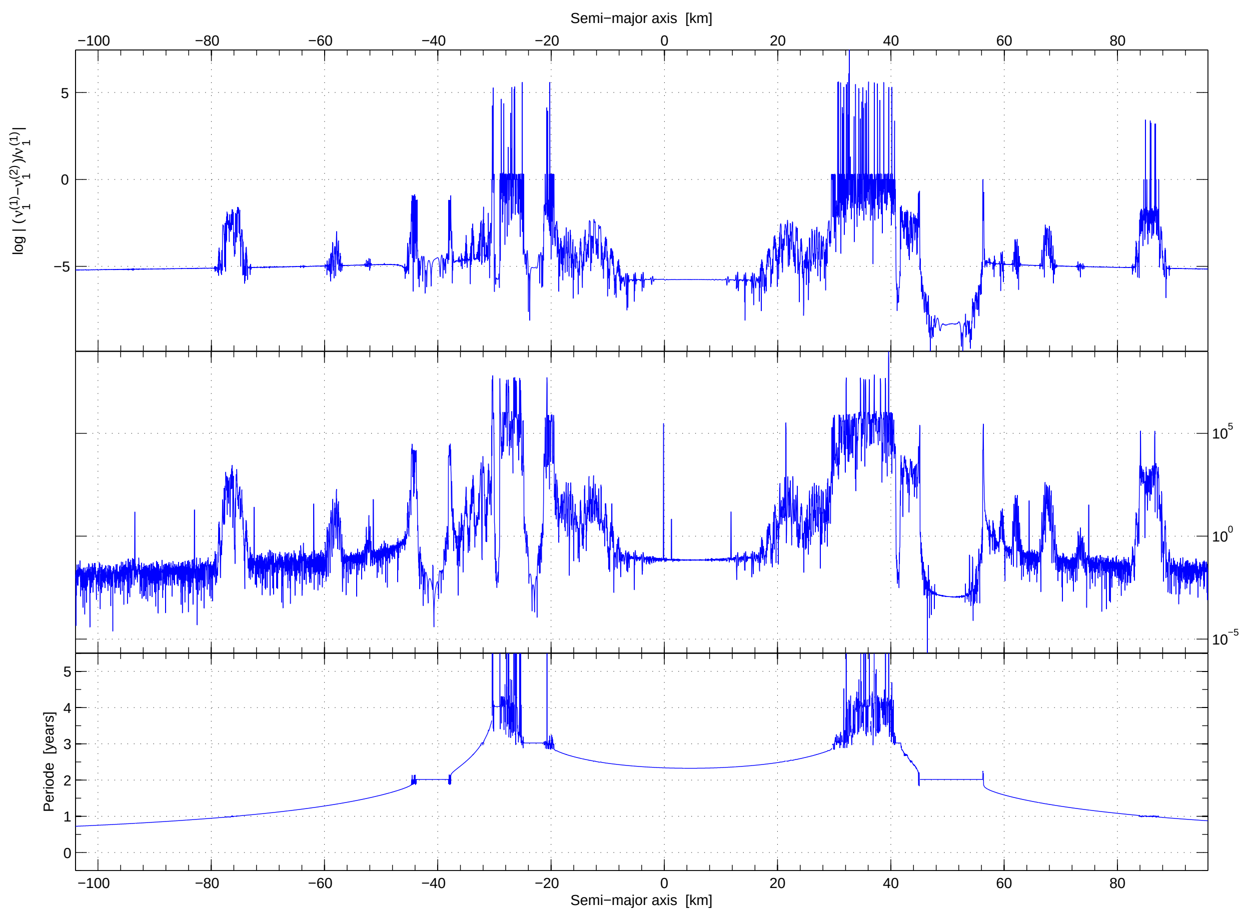

Fig. 2. The same resonant angle section in Fig. 1 but with a simpler model. Evolution of the fundamental period (bottom panel), second derivative (middle panel) and diffusion indicator (top panel), with respect to the initial semi-major axis a_0.

We can observe the same structure on both plots for derivative and for diffusion indicator. But the former exhibits less noise. One can easily recognize the main additional resonances when the period is equal to 2 years (between 42 and 58 km above the "eye" of the resonance due to C_{22} and between -36 and -44 km below the "eye" of the resonance). One can also observe an additional resonance when the period is equal to 1 year (between 84 and 86 km above the "eye" of the resonance due to C_{22} and near to -76 km below the "eye" of the resonance). Our analysis suggest that other additional resonances can be observed. A first one when the period is equal to 3 years (at $\pm 42km$ above the "eye" and $\pm -32km$ below the main resonance); a second resonance when the period is equal to 4 years (at $-30km$ below the main resonance) and a third one when the period is equal to 3/2 years and equal to 4/3 years. We can also observe a secondary resonance (inside the main "eye" of the resonance due to C_{22}) when the period of the resonant angle is equal to 3 years, clsoe to the separatrix (at $-20km$ to $-24km$ above the main resonance) and a weak evidence of secondary resonance (in the main "eye" of the resonance due to C_{22}) when the period of the resonant angle is equal to 2.5 years (at $-16km$ below and $24km$ above the main resonance).

4 Conclusions

In this paper, using numerical estimato, we brought to the fore a relevant class of additional resonances in the GEO for space debris with high area-to-mass ratio. Some of these resonances correspond to commensurabilities between the primary resonant angle and the ecliptic longitude of the Sun were also underlined in Valk et al. (2008). A set of these resonances have been discovered using the MEGNO indicator. Because of its better resolution, a larger set of resonances has been discovered by using the FMA (corresponding to periods equal to 4/3 and 3/2 years).

An analytical analysis of these additional resonances will be investigated in a forthcoming publication. Moreover a whole atlas, as in Valk et al. (2008), has been created using frequency analysis confirming the existence of a large number of additional resonances that wasn't discovered using the MEGNO indicator.

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