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PLANETARY PERTURBATIONS ON THE ROTATION OF MERCURY

Dufey, J.[1], Noyelles, B.[1, 2], Rambaux, N.[1,2] and Lemaître, A.[1]

Abstract. The space missions MESSENGER and BepiColombo require precise short-term studies of Mercury's rotation. In this scope, we perform analytically and numerically by Hamiltonian approach a synthetic 2-dimensional representation of its rotation, using complete ephemerides of the orbital motions of the planets of the Solar System. This representation allows us to derive the librations in longitude and latitude of the resonant arguments. We point out that the contributions of Venus and Jupiter cannot be neglected in the study of the librations in longitude. We also show that the librations in latitude are much smaller, with an amplitude of about 0.2 arcsec, whereas the librations in longitude have an amplitude of about 40 arcsec. Moreover, we mention the possibility of a resonance involving the free libration in longitude and the orbital motion of Jupiter. All these results are compared to those given by SONYR model, which integrates simultaneously the orbital motions of the planets and the rotation of Mercury, and therefore gives the full spin-orbit coupling motion of Mercury.

1 Introduction

Over the last decade, there has been a renewed interest in Mercury with the space missions BepiColombo and MESSENGER, along with radar measurements revealing a molten core (Margot et al. (2007)). To have a better understanding of the observations and the measurements, an analytical model as accurate as possible is necessary.
In the first section, we study the libration in longitude of Mercury. After stating our basic hypotheses, we use a Hamiltonian formalism to analyse the different planetary contributions on the libration in longitude. The results are compared with the numerical model SONYR and with a study of Peale et al. (2007).
In the second section, to analyse the libration in latitude, we use a more complete model of the rotation. Using a similar approach to the study of the libration in longitude, we analyse the planetary perturbations acting on the ecliptic obliquity and the angle representing the 1:1 commensurability between the rotational and orbital nodes.

2 Libration in longitude

In this section, we study the libration in longitude in the planar problem of the rotation of Mercury. In other words, we consider that the inclinations of all the planets are equal to zero. As a result, all the planets move in the same plane (the ecliptic J2000).
Before explaining this problem in more details, let us mention that we choose Mercury's equatorial radius as length unit, Mercury's mass as mass unit and the year of the Earth as time unit.

2.1 Basic hypotheses

Here are the hypotheses for this first model:

1. Mercury is a triaxial body. The development of the gravitational potential in spherical harmonics is limited at the second order. The values chosen for the coefficients are $J_2 = 6 \times 10^{-5}$ and $C_{22} = 10^{-5}$ (Anderson et al. 1987).

[1] University of Namur, Rempart de la Vierge 8, 5000 Namur, Belgium
[2] IMCCE, Observatoire de Paris - Universit Pierre et Marie Curie-Paris 6 - USTL, CNRS UMR 8028, Paris, France

2. Mercury is a two-layer body: a solid mantle and a spherical liquid core. There is no magnetic or viscous interaction between the mantle and the core. Consequently, in this short-term study, the rotation of Mercury is actually the rotation of the mantle.
3. The third axis of inertia and the direction of the angular momentum coincide.
4. Mercury is in a 3:2 spin-orbit resonance.
5. The orbit of Mercury is perturbated by the other planets. As ephemerides, we use a VSOP (Variations Séculaires des Orbites Planétaires) theory given by Simon in a private communication (Fienga and Simon (2004)).
6. All the planets move in the same plane. The orbital inclinations of the planets (with respect to the ecliptic J2000) are set to 0.

2.2 Hamiltonian formalism

Let us first define the angle σ_1, representing the 3:2 spin-orbit resonance: $\sigma_1 = g - \frac{3}{2}l_o - \varpi$, with g the spin angle, l_o the mean anomaly and ϖ the longitude of the perihelion. This angle actually represents the libration in longitude of the planet, in other words, it is the deviation from the exact 3:2 spin-orbit resonance.
With the basic hypotheses, the Hamiltonian of the problem consists of three parts (see papers of D'Hoedt and Lemaitre (2004) and Dufey et al. (2008) for a more detailed explanation of the Hamiltonian):

$$H = H_{2B} + T + V_G, \tag{2.1}$$

where H_{2B} is the Hamiltonian of the 2-body problem, $T = \frac{\Lambda_1^2}{2C_m}$ is the rotational kinetic energy, with Λ_1 the moment associated to the resonant angle σ_1, and $V_G = V_G(l_o, \varpi, e, a, \sigma_1, \Lambda_o, \Lambda_1)$ is the gravitational potential, with e the eccentricity, a the semi-major axis and Λ_o the moment associated to l_o.
It is possible to compute the fundamental period of σ_1:

$$T_{\sigma_1} = 11.21 \text{ years}, \tag{2.2}$$

with $C = 0.34$ and C_m/C=0.5.

2.3 Introduction of planetary perturbations

The planetary perturbations are introduced through the orbital elements a, e, l_o and ϖ, using the Poisson series of the planetary theory given by Simon.
Numerical tests allowed us to conclude that the Sun, Jupiter, Venus, the Earth and Saturn have the largest influence on the rotation. Consequently, we have five new variables in our Hamiltonian: $\lambda_M, \lambda_J, \lambda_V, \lambda_E, \lambda_S$, respectively the longitudes of Mercury, Jupiter, Venus, the barycenter Earth-Moon and Saturn. We only include the periodic contributions in the perturbations, not the Poisson terms. On a time scale of 100 years they are at least 100 times smaller than the periodic contributions.

2.4 The libration in longitude

There will be two steps in order to compute the libration in longitude (See Dufey et al. (2008) and references therein):

- First we use a Lie averaging process to average the Hamiltonian over all the angular variables (the short periodic terms). We then compute the generators of this transformation.
- We use the first-order generator (containing only short periodic terms) to compute the evolution of σ_1.

To confirm the results of the analytical method, we compare them with the results of the SONYR (Spin-Orbit N-bodY Relativstic) model, which is a dynamical model that integrates numerically the orbital motions in the Solar system and includes the coupled spin-orbit motion of the terrestrial planets and of the Moon (see Bois and Vokrouhlicky (1995) and Rambaux and Bois (2004) for references for SONYR).

Effect of	Period (years)	Amplitude	Relative amplitude
Sun (l_o)	0.2408	40.69	1
Jupiter (λ_J)	11.86	13.27	0.3261
Sun ($2l_o$)	0.1204	4.530	0.1114
Venus ($2l_o - 5\lambda_V$)	5.663	4.350	0.1069
Jupiter ($2\lambda_J$)	5.931	1.673	0.0411
Saturn ($2\lambda_S$)	14.73	1.233	0.0303
Earth ($l_o - 4\lambda_E$)	6.575	0.7038	0.0176

Table 1. The main planetary contributions on the resonant angle σ_1.

After this integration, a frequency analysis is used to find the contributions on the libration in longitude. The comparison of the methods is very satisfactory (see Dufey et al. (2008) for the detailed comparison). Table 1 states the result of our method.
The 88-day contribution is the largest, followed by Jupiter's contribution (11.86 years), which is around 33% of the 88-day forced librations. Then Venus' contribution (5.67 year-period) is around 10% and Saturn's (14.73 years) and the Earth's (6.58 years) are around 3% and 2%.
Compared to the results in the paper of Peale et al. (2007), we underline the contributions of the 11.86 year-period, and the contributions of Saturn and the Earth.
Let us mention that the fundamental period is proportional to $\sqrt{\frac{C_m}{B-A}}$ (A and B being the smallest moments of inertia). With the value given by Margot et al. (2007) ($\frac{B-A}{C_m} = 2.03 \times 10^{-4}$) the fundamental period would be $T_{\sigma_1} = 11.21$ years, which is much closer to Jupiter's orbital period. Consequently, the effect of Jupiter on the libration in longitude would be much larger.

3 Libration in latitude

The hypotheses for the 3-dimensional problem are

1. Mercury is a triaxial body.
2. It consists of a solid mantle and a spherical liquid core.
3. The third axis of inertia and the angular momentum coincide.
4. Mercury is in a 3:2 spin-orbit resonance and there is a 1:1 commensurabitlity between the orbital and rotational nodes, i.e. both nodes precess at the same rate.
5. The orbit of Mercury is perturbated by the other planets.
6. The orbital inclinations of the planets are different from zero.

We define a new resonant angle $\sigma_3 = -h + \omega_o$, where h is the argument of the rotational node and ω_o is the argument of the ascending (orbital) node. The angle σ_3 represents the 1:1 commensurability between the rotational and orbital nodes.
Due to the precession of the orbital node, the equilibrium of the ecliptic obliquity is now $K_{\text{eq}} = i_o + 1.2$ arcmin. We compute the evolution of σ_3 and of the ecliptic obliquity K using an approach similar to the previous section. Table 2 summarizes our results. The main observation here is that the amplitudes of the oscillations are very small, under 0.2 arcsec for σ_3 and under 1 mas for the obliquity K. Such amplitudes should be under the observational accuracy of the space misssions.
We also note that the great inequality ($2\lambda_J$-$5\lambda_S$) is the largest of the planetary contributions (aside from the orbital motion of Mercury around the Sun). Jupiter, Venus, Saturn and the Earth also have contributions, but much smaller in this case.

Effect of	Period (years)	Amplitude (as)
Sun (l_o)	0.2408	0.1524
Great ineq.($2\lambda_J - 5\lambda_S$)	883.3	0.0545
Sun ($2l_o$)	0.1204	0.0538
Sun ($3l_o$)	0.0803	0.0420
Jupiter ($2\lambda_J$)	5.931	0.0169
Venus ($2l_o - 5\lambda_V$)	5.663	0.0067
Jupiter ($1\lambda_J$)	11.86	0.0066
Saturn ($2\lambda_S$)	14.72	0.0052
Earth ($l_o - 4\lambda_E$)	6.575	0.0021

Effect of	Period (years)	Amplitude (mas)
Sun ($2l_o$)	0.1204	9.187
Sun ($3l_o$)	0.0803	5.607
Sun (l_o)	0.2408	4.415
Great ineq.($2\lambda_J - 5\lambda_S$)	883.3	4.045
Jupiter ($2\lambda_J$)	5.931	1.738
Jupiter ($1\lambda_J$)	11.86	1.340
Venus ($2l_o - 5\lambda_V$)	5.663	0.608
Saturn ($2\lambda_S$)	14.72	0.461
Earth ($l_o - 4\lambda_E$)	6.575	0.216

Table 2. Left table: main planetary contributions on the resonant angle σ_3. Right table: main planetary contributions on the ecliptic obliquity K.

4 Conclusion

Using a Hamiltonian approach, we are able to analyse the main planetary perturbations acting on the librations in longitude and latitude.

For the libration in longitude, in addition to the 40 arcsec coming from the 88-day orbital motion, we point out that Jupiter's signature (11.86 year-period) is about 13 arcsec, Venus' (5.67 years) is about 4 arcsec and Saturn's and the Earth's (14.72 and 6.58 years) are about 1 arcsec. We also observe that in the case of a close resonance between the orbital period of Jupiter and the fundamental period of the 3:2 resonant angle, Jupiter's contribution would be much larger.

Concerning the libration in latitude, we studied the ecliptic obliquity and the angle describing the 1:1 commensurability in nodes. The amplitude of the oscillations are really small (less than 0.2 arcsec for the resonant angle and less than 1 mas for the obliquity) and the largest planetary contribution (aside from the orbital motion of Mercury) is the great inequality between Jupiter and Saturn. We also detect the signatures of the other planets alone (Jupiter, Venus, Saturn and the Earth), but they are very faint.

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