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COMPARISON OF FAST LYAPUNOV CHAOS INDICATORS FOR CELESTIAL MECHANICS

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Abstract. For a long time, the estimation of the Lyapunov Characteristic Exponents (LCEs) had been used in Celestial Mechanics to caracterize the chaoticity of orbits. With the aim of gaining speed and accuracy in detecting this chaoticity, several indicators based on the theory of Lyapunov exponents have been developped. Here we present a comparison in terms of precision, CPU speed, and practicability of several of these indicators ; the FLI (Froeschlé *et al*, 1997) , MEGNO (Cincotta & Simó, 2000), and the GALI (Skokos *et al*, 2007). The GALI3 (using three tangent vectors) is the version of the GALI used here. While the FLI and MEGNO have been commonly used, the GALI has not yet been applied to Celestial Mechanics. However, this indicator has its own qualities and specificities. The final aim of the comparison of these indicators is the production of stability maps in the case of irregular satellites of giant planets, the examples and applications are shown in this sense.

1 Introduction

The three indicators presented here are variants of the Lyapunov Characteristic Exponent (LCE) which is defined by :

$$\sigma = \lim_{t\to\infty} (\frac{1}{t}) \ln \frac{\|\overrightarrow{w}(t)\|}{\|\overrightarrow{w}(0)\|} \tag{1.1}$$

with the use of a tangent vector obtained from the variational equation : $\dot{\overrightarrow{w}} = \frac{\partial \overrightarrow{F}}{\partial \overrightarrow{X}} \overrightarrow{w}$

2 Behaviour of the indicators in different situations

We compare the indicators with several criterions such as the contrast obtained for differentiating chaotic orbits, the integration and CPU times. The location in the orbital elements of the examples on Fig. 1 et Fig. 2 are chosen to be representative of different regimes undergone by a Jovian satellite in the restricted circular and planar three-body problem (Sun + Jupiter + satellite). The first example represent a part of the system dominated by chaotic and resonant orbits, while the second contains quasi-periodic, resonant and unstable orbits. In the figures, the final values over different integration times of the FLI and the MEGNO is shown, in addition with the inverse of the time τ_{GALI3} needed by the GALI3 to attain a particular treshold.

3 Discussions

We are making different objective tests to the methods, concerning mainly contrast between orbits and CPU time which are important for creating maps. Firstly, and as it is already thought, we found that the strong points of the FLI is its simplicity of use and its rapid computation. The FLI and MEGNO seems equivalent (Fig. 1 and 2), although it appears that the MEGNO can show structures and resonances faster in integration time than the FLI, but with more CPU time. Despite its slower speed, the MEGNO is perfectly adapted for the studies of resonant orbits thanks to its quasi-fixed value for stable orbits. Indeed, the value of the FLI and

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GALI3 for stable orbits depend (although faintly) on the location of the orbits (Fig. 2). Concerning the CPU time vs integration time, the MEGNO and GALI3 are slower by at least a factor 3 than the FLI, but in this case the GALI method is fundamentally different in its use : using in fact the time for indicator, it allows to deal with chaotic orbits very fast, making it more efficient for regions where chaos is largely extended. This fact can be for example interesting for the studies of orbits which escape rapidly from a giant planet's Hill radius. Conversely, the CPU time increase drastically for the case of stable orbits, although this can be overcome by adding more tangent vectors to the computation of the GALI, thus using a more convergent indicator. We must add that this indicator cannot be computed up to a certain threshold (i.e. integration time) corresponding to the numerical limit.

To continue the comparison, we are adding more tests as objective as possible, in particular comparing the detection of resonances by the different methods in function of time.

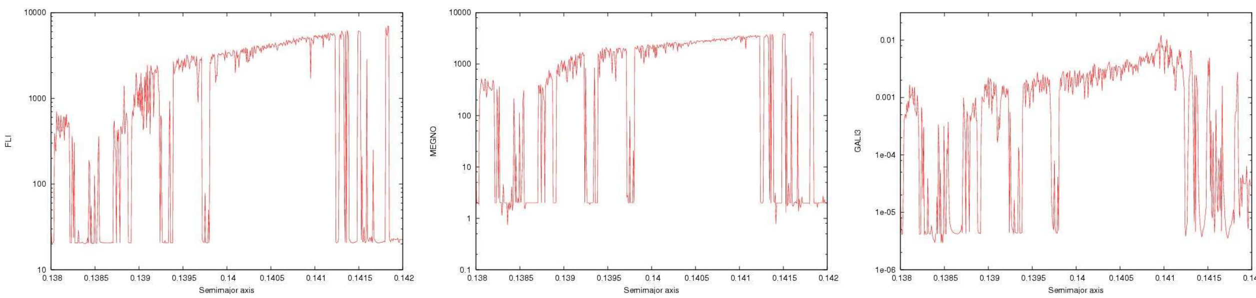

Fig. 1. Final values of the indicators for an integration time of 100,000 years (or treshold value of 10^{-16} for the GALI3), varying the semi-major axis in [0.138-0.142 UA] with e(0)=0.68 and f(0)=w(0)=0. FLI (**Left**), MEGNO (**Center**) and $\frac{1}{\tau_{GALI3}}$ (**Right**). The CPU time is respectively 3.5h, 10.5h and 7.25h.

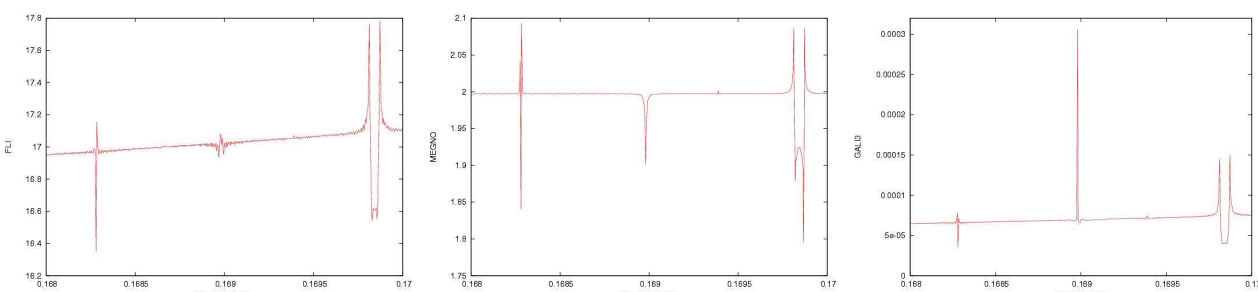

Fig. 2. Final values of the indicators for an integration time of 10,000 years (or treshold value of 10^{-11} for the GALI3), varying the semi-major axis in [0.168-0.17 UA] with e(0)=f(0)=w(0)=0. FLI (**Left**), MEGNO (**Center**) and $\frac{1}{\tau_{GALI3}}$ (**Right**). The CPU time is respectively 0.25h, 1h and 1.5h.

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