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EARTH'S INTERIOR WITH VLBI: PUSHING THE LIMITS

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Abstract. This paper reports on some recent Earth-related studies within the VLBI group of the SYRTE department at the Paris Observatory. Especially, we focus on the determination of some outer core properties through VLBI estimates of the Earth's nutation.

1 Earth's interior and Earth's rotation

The Earth's rotation is irregular, in response to various external and internal excitations. The atmosphere and the hydrosphere, by exchanging angular momentum with the lithosphere, induce variations in position of the rotation pole at seasonal, interannual and diurnal time scales, the latter being the result of the diurnal solar heating of the atmosphere and of the ocean tides. Effects of continental water, snow, and ice sheets, although significant at seasonal time scales, are totally negligible in the diurnal band. Besides, gravitational forces arise from celestial bodies. Combined with the internal structure and rheology of our planet, they determine the shapes of the mantle, and of the fluid-solid interfaces at the core-mantle boundary (CMB) and inner core boundary (ICB).

In presence of a mantle, a fluid outer core and a solid inner core, the dynamic of the system is described by the angular momentum conservation equations (see, e.g., Mathews et al. 1991)

$$\frac{d\vec{H}}{dt} + \vec{\omega} \times \vec{H} = \vec{\Gamma}, \quad \frac{d\vec{H}_{\rm f}}{dt} + \vec{\omega} \times \vec{H}_{\rm f} = \vec{\Gamma}_{\rm f}, \quad \frac{d\vec{H}_{\rm s}}{dt} + \vec{\omega} \times \vec{H}_{\rm s} = \vec{\Gamma}_{\rm s}, \tag{1.1}$$

where $\vec{H}$, $\vec{H}_{\rm f}$, and $\vec{H}_{\rm s}$ are the angular momentum of the Earth rotating at $\vec{\omega}$, the outer and inner cores, respectively, and $\vec{\Gamma}$, $\vec{\Gamma}_{\rm f}$, and $\vec{\Gamma}_{\rm s}$ are torques (including gravitational, interaction, and electromagnetic couplings). Such a 3-layer Earth admits normal rotational modes. The well-known Chandler wobble occurs with a period around 433 days in the terrestrial frame and is associated with the ellipticity of the Earth system (basically the mantle ellipticity plus an oceanic bulge contribution). Two normal modes associated with the free nutations of the outer and inner cores (free core nutation, or FCN, and free inner core nutation, FICN) affect the motion of the Earth's figure axis in space at diurnal frequencies. Finally, a fourth eigenmode is associated with the free motion of the inner core with respect to the mantle and is known as inner core wobble (ICW). The ratio of the nutational amplitude $\tilde{\eta}$ of the whole, 3-layer Earth, solution of (1.1), to its rigid counterpart is expressed in the frequency domain as

$$\frac{\tilde{\eta}(\sigma)}{\tilde{\eta}_{\rm R}(\sigma)} = \left[1 + \sigma' \left(\frac{\tilde{N}_{\rm CW}}{\sigma - \tilde{\sigma}_{\rm CW}} + \frac{\tilde{N}_{\rm FCN}}{\sigma - \tilde{\sigma}_{\rm FCN}} + \frac{\tilde{N}_{\rm FICN}}{\sigma - \tilde{\sigma}_{\rm FICN}} + \frac{\tilde{N}_{\rm ICW}}{\sigma - \tilde{\sigma}_{\rm ICW}} \right)\right], \tag{1.2}$$

where $\sigma' = \sigma + \Omega$, e is the Earth flattening, and the $\tilde{N}$ are functions of the frequency and of a limited number of geophysical parameters. The full expression of this transfer function can be found in Mathews et al. (1991; 2002) wherein (1.1) is solved in the same line as the 2-layer Earth of Sasao et al. (1980). One gets

$$\tilde{\sigma}'_{\rm FCN} = -\Omega \left(1 + \frac{A_{\rm f}}{A_{\rm m}}\right) \left(e_{\rm f} - \tilde{\beta} + \tilde{K}_{\rm CMB} + \frac{A_{\rm s}}{A_{\rm f}} \tilde{K}_{\rm ICB}\right), \quad \tilde{\sigma}'_{\rm FICN} = -\Omega \left(1 + \frac{A_{\rm s}}{A_{\rm m}}\right) \left(\alpha_2 e_{\rm s} + \tilde{\nu} - \tilde{K}_{\rm ICB}\right), \tag{1.3}$$

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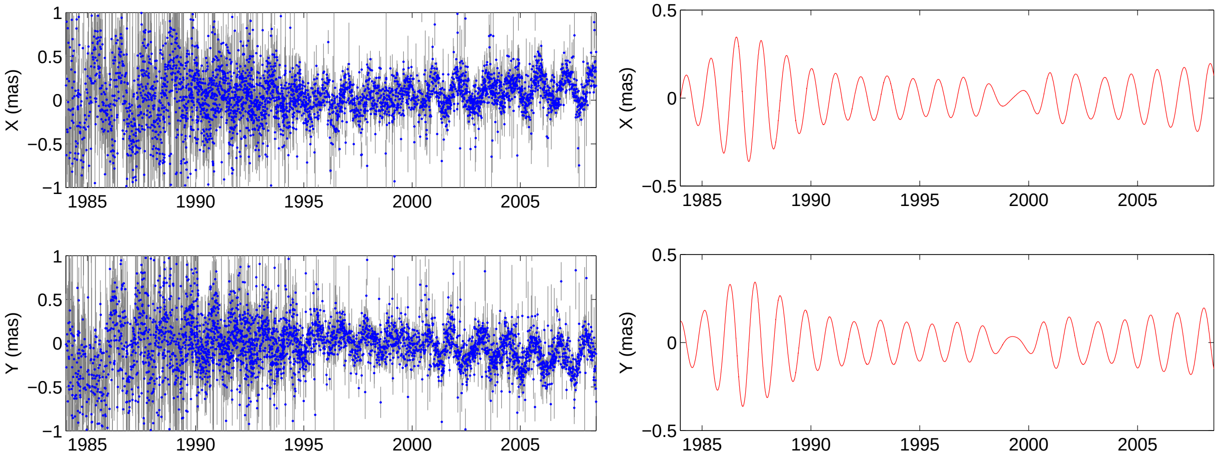

Fig. 1. Left: nutation offsets to Mathews et al. (2002)'s model as computed at the IVS OPAR VLBI analysis center. Right: FCN mode extracted by least-squares fit.

where e_f and e_s are the outer and inner core flattenings, $\tilde{\beta}$ and $\tilde{\nu}$ the compliances expressing the deformability of the CMB and the ICB under the centrifugal forces due to the outer and inner core wobbles, respectively. $\tilde{K}_\mathrm{CMB}$ and $\tilde{K}_\mathrm{ICB}$ are related to the electromagnetic torque at the relevant interface (see Buffett et al. 1992).

One can see from (1.2) that the normal modes amplify the response to an excitation occuring at frequencies close to the resonant frequencies. Thus, tidal forces that excite the Earth at diurnal frequencies would have their response enhanced by the FCN and the FICN modes. Comparison of observed nutations (which traduce the response of the real Earth to the tidal potential) against theoretical response of an hypothetical rigid Earth having the same moments of inertia would therefore lead to determine some Earth's interior properties. Especially, one can fit the resonant frequencies onto the observations and deduce constraints on geophysical parameters that appear in their respective theoretical expressions.

2 VLBI observations for geosciences

Very long baseline interferometry (VLBI) measures differential arrival times of radio signals emitted by extragalactic radio sources (e.g., quasars, BL Lac, AGN) on several antennas separated by thousands kilometers. On a typical 24-hr geodetic VLBI session, a network of 6–12 antennas observes about 80 sources. After correlation of the signals and some analyses, one gets the orientation of the network with respect to the polyhedra realized by the observed sources. Regular observations since ∼1984 (on the basis of one session every 3–4 days) had produced more than 5 million delays that are exploitable for precise geodesy. The improvement of the network geometry (from 4 antennas in the 1980's to 6–12 antennas nowadays) and receiver reliability makes the geodetic VLBI products reach an accuracy (repeatability) of about 6 mm (alternatively 0.2 milliarc second on the Earth orientation, or 20 ps). Geodetic VLBI observing program has long been set up by the NASA and the US Navy. Since 1998, the International VLBI Service for Geodesy and Astrometry (IVS) realizes most of that work through its coordinating center at NASA/GSFC and several analysis centers. It is important to figure out that VLBI is currently the only technique providing dense and accurate estimates of the Earth's nutation, and thereby constitutes a powerful technique for Earth's interior exploration along with local measurements of deformations or gravity variations and seismology. Note also that VLBI is the only technique that observes the free motion of the Earth associated with the FCN (see Section 5 of this paper).

Since 2007, the IVS analysis center located at the department SYRTE of the Paris Observatory (Gontier et al. 2006) and known as IVS OPAR, makes operational analysis of all geodetic VLBI sessions. We especially take care of reference frame effects to improve the reliability of Earth orientation parameters time series. The data sets used in this paper are made available at http://ivsopar.obspm.fr.

3 Estimates of the FCN resonant frequency and damping factor

Several studies have estimated geophysical parameters from VLBI data (Mathews et al. 2002; Vondrák et al. 2005; Lambert & Dehant 2007; Lambert et al. 2008; Koot et al. 2008). Among the interesting results that emerge from these papers, one can retain an estimate of $\tilde{\sigma}'_{\rm FCN}$ (Mathews et al.: 430.21 ± 0.6d; Lambert et al.: 430.31 ± 0.2d) that departs by about 4% from the theoretical value obtained using the seismology-based Earth model of Dziewonski & Anderson (1981) that assumes the CMB in hydrostatic equilibrium. The departure is attributed to an extra outer core flattening, that is confirmed by tomographic and gravimetric observations and explained by convection in the lower mantle (Defraigne et al. 1996).

The imaginary part of the FCN frequency is a damping factor $Q_{\rm FCN}$, such that ${\rm Im}\tilde{\sigma}'_{\rm FCN} = {\rm Re}\tilde{\sigma}'_{\rm FCN}/2Q_{\rm FCN}$, related to the viscous electromagnetic coupling at the CMB and the ICB. Though Mathews et al. yielded $Q_{\rm FCN} = 20\,000$ using VLBI data until 2002, an extra six years of data in Lambert et al. brings out a lower estimate $Q_{\rm FCN} = 19\,000$ where only the FCN frequency is estimated. Recently, one of the author (SBL) re-estimated the FCN frequency among other geophysical parameters and found a $Q_{\rm FCN}$ around 13 000, consistently with estimates from superconducting gravimeter data (Rosat et al. 2008). Such a result has been confirmed by Koot et al. These results bring possible new constraints on the value of the electromagnetic field at the CMB that will not be discussed here.

4 Celestial reference frame effects in geophysical results

Global analysis of a large number of sessions permit an estimate of both the Earth orientation parameters and the radio source coordinates. (Incidentally, on the terrestrial side, the positions and velocities of the observing sites are also estimated.) Time evolution of source coordinates due to various intrinsic phenomenon like plasma jets make the use of older coordinate determinations unreliable for precise geodetic analysis. However, estimating all sources' coordinates needs no-net rotation (NNR) constraints to ensure that the global set of quasars does not rotate with respect to the far universe. In practice, the NNR condition is applied to the coordinates of a certain number of 'defining' sources. This core ensemble has been verified to be non rotating by exhaustive tests. The current International Celestial Reference Frame (ICRF, Ma et al. 1998) proposes such a set of 212 defining sources, selected on the basis of VLBI observation until 1995. Since then, this set has become unstable and more reliable sets can be found (e.g., Feissel-Vernier 2003).

The sensitivity of VLBI products to celestial frame realization is still an open question. We give a brief review of various related studies in Lambert & Gontier (2008) where we also propose new sets of defining sources that provide celestial reference frames more reliable than the ICRF. In Lambert et al., we showed that the way of handling source coordinates during the analysis as well as the choice of the defining sources could produce substantial differences on Earth orientation parameter estimates. These differences result in uncertainties in estimates of the resonant frequencies that reach 0.2 day for the FCN and 300 days for the FICN. A spurious rotation of the celestial frame results in biases in the nutation angles, that reduces to zero when the system is perfectly non rotating. Effects on precession, rate and main periodic terms remain small, with no consequences on further geophysical analyses.

5 Outer core nutation excitation mechanisms

The signature of the FCN mode on the Earth's figure axis, observed from a space-fixed frame of reference, is a retrograde motion (opposite the Earth's rotation) that reaches an amplitude of $\sim$200 mas, variable in time, and with a variable phase. This signal clearly shows up in VLBI residuals. The apparent period oscillates between 430 and 460 days (Vondrák et al. 2005) and is most likely driven by diurnal atmospheric pressure variations (Gegout et al. 1998). Lambert (2006), under the assumption of a white noise excitation, showed that about a half of the observed amplitude could be explained using atmospheric general circulation models (GCMs), and that it is not possible to conclude whether the remaining half is due to another mechanism or to the poor quality of the diurnal atmospheric pressure data in the diurnal band.

Here, we propose to use a different approach based on a numerical resolution of the dynamical equations describing the Earth's nutation. We basically force the system (1.1) by the atmospheric data obtained from two distinct GCMs (US National Center for Environmental Prediction/National Center for Atmospheric Research reanalysis project, referred to hereunder as NCEP, and European Center for Medium range Weather Forecast,

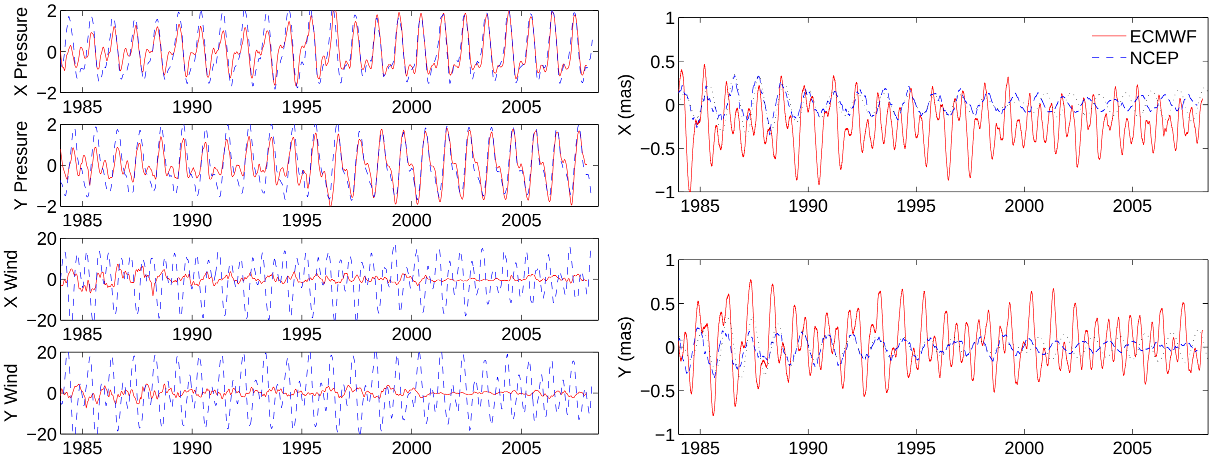

Fig. 2. Left: X and Y components of the modulation of the diurnal term in the atmospheric angular momentum from ECMWF (solid) and NCEP (dashed) in units of 10^{23} kg m^2/s. Right: predicted FCN mode.

referred to as ECMWF). This approach does not reduce the atmospheric signal to a white noise but takes any possible spectral signature into account. Moreover, we also consider wind data, in addition to surface pressure data, that were not studied in the work of 2006. Figure 2 displays the (mainly annual) modulation of the diurnal atmospheric angular momentum cycle. One can see significant discrepancies between NCEP and ECMWF, both in amplitude and phase, especially on the wind term. The right plot displays the predicted FCN mode, that must be compared to the VLBI reported in dotted line. It appears that the NCEP data (wind + pressure) fairly explains the VLBI-observed mode, including the decrase of the amplitude before 2000, but fails in explaning the amplitude at the very end. NCEP winds explain about 60% of the signal. However, ECMWF data lead to completely different results, far away from the observation. Although the pressure alone give a predicted mode that is very close the NCEP-predicted one, the wind part, which is smaller but noisier (with higher derivatives) produces a strong, unrealistic FCN mode. We thus conclude that the NCEP GCM is more reliable for study of diurnal atmospheric effects on the Earth's rotation. It is important to note that this does not disqualify the ECMWF data at all: one must understand that GCMs does not aim to produce diurnal data, since meteorological centers are mainly interested in 2- to 10-day forecasts.

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