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TITAN'S FORCED ROTATION - PART II: THE RESONANT WOBBLE

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Abstract. Our knowledge of the gravity field of Titan has been recently improved thanks to the fly-bys of Cassini spacecraft, that provided us first values of Titan's J_2 and C_{22}, unfortunately without any indication of the polar inertial momentum C. Anyway, these data allowed us to give last year a first 3-dimensional description of the rotation of Titan, seen as a rigid body. In particular, we pointed out an interesting phenomenon forcing the wobble (i.e. the angular separation between Titan pole axis and angular momentum), that we suspected to be nearly resonant. This year we present a study of this resonance, involving a free libration around the Cassini equilibrium and a proper mode given by the orbital ephemerides. The resonant argument has been clearly identified, and its behaviour has been investigated using the Second Fundamental Model of Resonance. We show that in case of capture, the wobble might be pumped to several degrees. Moreover, we propose an original formula to estimate the contribution of the wobble in the tidal internal dissipation of a synchronous satellite. A significant wobble might cause a wrong estimation of the rotation of Titan.

1 Introduction

Last year Noyelles et al. 2007 we presented a 3-degree of freedom theory of Titan's rotation, seen as a rigid body. Such a study is possible since the fly-bys of Cassini spacecrafts that provided us information on Titan's gravity field, especially J_2 and C_{22}. Unfortunately, a third useful parameter, i.e. the polar inertial momentum C, remains unknown. That is the reason why we study Titan's rotation for several realistic values of C.

Our study showed an interesting behavior of Titan's wobble (i.e. the angular separation between Titan's polar axis and its angular momentum), when C is close to $0.35MR^2$ (see Fig.1).

2 The resonant Hamiltonian

We start from the following Hamiltonian:

$$\begin{aligned}\mathcal{H} &= \frac{nP^2}{2}+\frac{n}{8}\left[4P-\xi_q^2-\eta_q^2\right]\left[\frac{\gamma_1+\gamma_2}{1-\gamma_1-\gamma_2}\xi_q^2+\frac{\gamma_1-\gamma_2}{1-\gamma_1+\gamma_2}\eta_q^2\right] \\ &\quad +n\left(\frac{d_0}{d}\right)^3\left(1+\delta_s\left(\frac{d_0}{d}\right)^2\right)\left[\delta_1(x^2+y^2)+\delta_2(x^2-y^2)\right] \qquad (2.1)\end{aligned}$$

in which the used variables are the modified Andoyer's variables. In this Hamiltonian the notations are as follows:

- P is the norm of Titan's angular momentum
- η_q and ξ_q locate the angular momentum in the body frame
- n is Titan's mean motion

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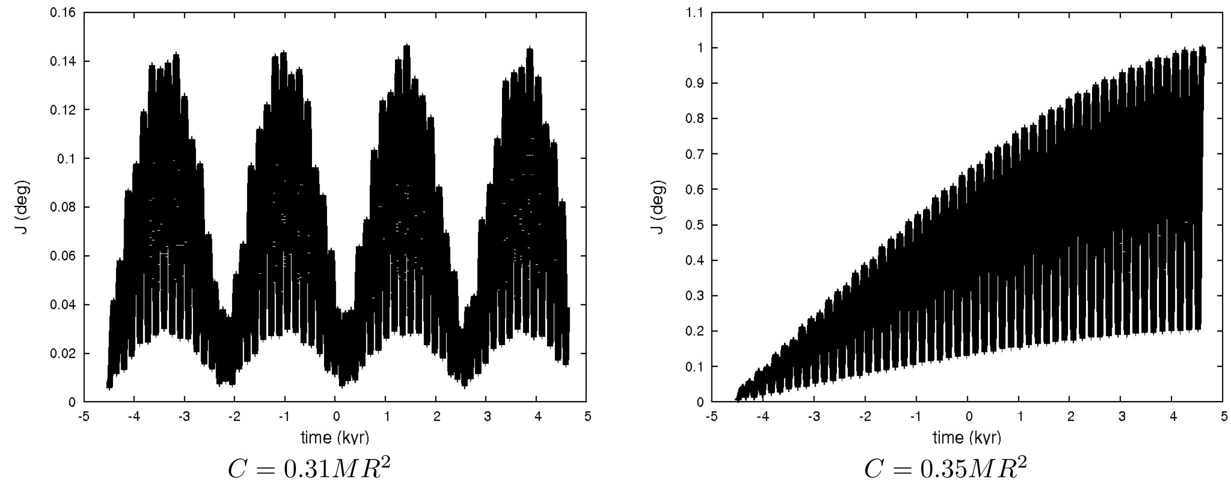

Fig. 1. Two different behaviors of the wobble J for 2 different values of the polar inertial momentum C. We note that for $C = 0.35MR^2$, J seems to have a quasi-resonant behavior, taking important values.

- (x, y, z) is the unit vector pointing to Saturn in Titan's reference frame
- γ_1, γ_2, δ_1 and δ_2 are associated to Titan's gravity field
- δ_s is associated to Saturn's oblateness coefficient J_2.

We now perform an analytical study requiring several canonical transformations, consisting in

1. Determining the equilibrium, and centering the Hamiltonian around it.
2. Untangling the proper modes Henrard & Lemaître 2005.
3. Doing a polar canonical transformation to express the small oscillations around the equilibrium in angle-action variables.

After these transformations, the Hamiltonian becomes

$$\begin{aligned} \mathcal{K} &= \mathcal{N} + \mathcal{P} \\ &= \omega_u U + \omega_v V + \omega_w W + \mathcal{P} \end{aligned} \tag{2.2}$$

in which $\mathcal{N}$ is the quadratic Hamiltonian depending only of the proper modes of the system (u, v, w, U, V, W), and $\mathcal{P}$ is the perturbation depending on the proper modes of the orbital motion.

Table 1. Synthetic representation of $\eta_q + \sqrt{-1}\xi_q = \sin J \exp(-\sqrt{-1}l)$, associated to the wobble, for $C = 0.31MR^2$. ϕ_6 and Φ_6 are proper modes of Titan's orbital dynamics. They are associated respectively to Titan's pericenter and ascending node.

N°	Amp. $\times 10^4$	Phase (°)	T (y)	Ident.	Cause
1	9.12391728	-51.69	306.33602	w	$\sqrt{W}$
2	6.01688587	51.69	-306.33605	$-w$	$\sqrt{W}$
3	5.73033451	158.48	351.70284	$\phi_6 - \Phi_6$	$e_6\gamma_6$
4	3.83212940	-158.48	-351.70284	$\Phi_6 - \phi_6$	$e_6\gamma_6$
5	0.63642954	-35.86	135.27368	$v - \Phi_6$	$\sqrt{V}\gamma_6$
6	0.38395548	35.86	-135.27368	$\Phi_6 - v$	$\sqrt{V}\gamma_6$

The Tab.1 gives the synthetic representation of the variable associated to the wobble, in the case of $C = 0.31MR^2$. We can see that the main forced contribution is $\phi_6 - \Phi_6$, with a period of 351.7 y. When C gets closer to $0.35MR^2$, the frequency of the proper mode w gets close to 350 y, so we can hint that the resonant argument is $w+\Phi_6-\phi_6$. This has been checked in numerical simulations.

Starting from the Hamiltonian 2.2 we perform this last canonical transformation:

$$\begin{array}{ll} u & U \\ v & V \\ \theta = w+\Phi_6-\phi_6 & \Theta = W \end{array}$$

giving this new Hamiltonian $\mathcal{T}$:

$$\mathcal{T} = \omega_u U + \omega_v V + \big(\omega_w + \dot{\Phi}_6 - \dot{\phi}_6\big)\Theta + \mathcal{T}_2. \tag{2.3}$$

We then average over every angle except the resonant one, considered as the only slow argument, and we obtain:

$$\mathcal{T} = \psi\Theta + \mu\Theta^2 + \epsilon\sqrt{2\Theta}\cos\theta. \tag{2.4}$$

This Hamiltonian is the classical *Second Fundamental Model of Resonance* Henrard & Lemaître 1983, describing the evolution of the system in deep resonance. Its equilibrium is the positive root of the cubic equation $x^3-3(\delta+1)x-2=0$ with $\delta=-1-sign(\psi\mu)\left|\frac{4}{27}\frac{\psi^3}{\mu\epsilon^2}\right|^{\frac{1}{3}}$. It corresponds to a forced value of the "free" amplitude of the wobble, forced by the resonance (see Tab.2).

Table 2. The wobble forced by the resonance.

$\frac{C}{MR^2}$	W_0 (forced)	$< J >$
0.34	(no real solution)	
0.35	0.342	80.368°
0.355	0.108	40.702°
0.3555	0.034	22.337°
0.355551	0.010	12.034°
0.35555146967191	0.009	11.413°
0.35555146967192	(no resonance)	

3 Internal dissipation

The differential gravitational attraction of Saturn on Titan raises a tidal bulge. The misalignement of the tidal bulge with the direction Titan-Saturn induces a loss of internal energy, that is usually expressed as:

$$\frac{dE}{dt} = \frac{21}{2}e^2\frac{k_2}{Q}f\frac{\mathcal{G}M_{♄}^2 nR^2}{a^6}. \tag{3.1}$$

In particular, we have $\frac{dE}{dt} \propto e^2$ because the misalignement of the tidal bulge is due to Titan's orbital eccentricity. If Titan's wobble J_0 is significant, it alters the orientation of the tidal bulge and thus it should be taken into account in the calculation of the internal dissipation. That is the reason why we propose this original formula :

$$\frac{dE}{dt} = \frac{3}{2}J_0^2\Big(\frac{n+w}{w}\Big)^2\frac{k_2}{Q}f\frac{\mathcal{G}M_{♄}^2 nR^2}{a^6} \tag{3.2}$$

and the total expression for the internal dissipation is now:

$$\frac{dE}{dt} = \left[\frac{21}{2}e^2 + \frac{3}{2}J_0^2\Big(\frac{n+w}{w}\Big)^2\right]\frac{k_2}{Q}f\frac{\mathcal{G}M_{♄}^2 nR^2}{a^6}. \tag{3.3}$$

A numerical application shows that the contribution of the wobble on the internal dissipation is predominant if $J_0 > 4.4°$, what is realistic. One can notice the term $n+w$ in the contribution of the wobble. It is the sum of two frequencies: the orbital frequency (that is also the spin rate) and the wobble frequency. It means that the frequency of the tidal excitation due to the wobble is not n (as it the case for the eccentricity) but $n + w \approx n$, because of a composition between the two motions (spin and wobble). If the wobble is significant, a measurement of the spin rate might be altered and a take a higher value than the actual spin rate.

4 Comparison with the observations

Recently, the Cassini RADAR Team measured a slightly super-synchronous rotation for Titan, the shift being about $+0.36°/y$ Stiles et al. 2008. An a priori neglected wobble could be an explanation, but at least another one exists: Tokano & Neubauer (2005) suggested that seasonal energy exchanges between Titan's atmosphere and its surface provoke a variation of the length of the day (i.e., the spin rate). Lorenz et al. (2008) interpret the measured super-synchronous rotation as the signature of an internal ocean, that decouples the rotation of Titan's crust from its mantle, and makes it highly sensitive to the seasonal energy exchanges. Unfortunately, one cannot discriminate a secular contribution to a seasonal one in Titan's measure of spin, because it has been computed from about 2-years irregularly spaced data. Their accuracy depends on the flybys of the spacecraft near Titan, and to the distance between Titan and the spacecraft. Moreover, the seasonal period is 29 years for Saturn and its satellites, far too large to be detected.

5 Conclusion

We have elaborated the first 3-degree of freedom theory of Titan's rotation, seen as a rigid body. This theory permitted us to enlight a likely resonance forcing the free wobble. This wobble might cause an alteration of the measure of the spin rate by Cassini spacecraft.

The next step is now to use a more realistic model for Titan, i.e. in modeling an internal ocean and also the atmosphere. Such a study has been initiated by Henrard (2008), in taking account of a liquid core in the rotation of Io. The goal is to obtain a convergence between the expected rotation and the observations.

This study has been published in Noyelles et al. (2008) and Noyelles (2008), in which the reader can find more explanations.

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