

ON THE GINI-STATISTIC AND THE SEARCH FOR VARIABILITY IN A HESS PKS 2155-304 TIME SERIES

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Abstract. The Gini statistic (V_n) considers individual photons and is standard normally distributed for exponentially distributed arrival times. The statistic can identify segments in a light curve within which the count rate changes and may therefore indicate the presence of variability. V_n is calculated for data from constant count rate and simple assumed intrinsic source simulations, windowing the photon lists on time-intervals between 10 s and 500 s. Plots of V_n vs. time show that peak values of the statistic identify sufficiently sharp changes in the slope of a light curve. Qualitatively, $V_n(t)$ identifies small flares that *are* 'visible' in the light curve and thus does not add information. Ultimately, both the binning and Gini (V_n) approaches are limited by the uncertainties of low counts and Poissonian statistics.

1 Introduction

Cherenkov telescope γ -ray observations produce photon lists which list the arrival times of the photons. The HESS PKS2155-304 flare of July 28, 2006 (Aharonian 2007), sparked great interest because of the very short time scales that are evident in the light curve, which could be identified due to the high count rate.

Poissonian statistical variations in the emission from a quiescent source can produce spurious short time-scale flare-like events. Very simply, the time scales that characterize a single flare are defined by points where the slope of the light curve changes, i.e. *change points*. Since the Gini-statistic discriminates between a constant count rate and any change in the count rate, it might be able to find variability (O. de Jager 2007 private comm.).

The Gini test is a powerful test for exponentiality against a range of alternatives (Gail & Gastwirth 1978). The statistic has a null-hypotheses, H_0 : The photon list is a realization of a constant count rate source. Let t_i be the photon arrival times, $x_i = t_{i+1} - t_i$ the time differences and $x_{(i)}$ the ordered statistics. Then V_n , the normalized Gini statistic, calculated as $G_n = \left[\sum_{i=1}^{n-1} i(n-i)(x_{(i+1)} - x_{(i)}) \right] / [(n-1) \sum_{i=1}^n x_i]$ and $V_n = (G_n - 1/2)[12(n-1)]^{1/2}$, is standard normally distributed for constant count rate data. However, the statistic does not distinguish between a constant count rate and a count rate that changes linearly with time. A large value may indicate *a change in the slope* of the light curve and may thus point out change points. If we calculate V_n within a time window T , $V_n \rightarrow$ for $n \rightarrow \infty$, if $x_{(i+1)} > x_{(i)}$ within the window, at least for some subset of $x_{(i)}$.

2 Calculation and analysis

A window of width T ($T = 20$ s . . . 500 s) is allowed to slide in steps of 10 s from the beginning to the end of a photon list, as described in de Villiers (2007). V_n can be plotted versus time for each window. Figure 1 shows an assumed source and the associated V_n values for different window sizes. Two or three peaks in V_n may be associated with a single flare. Narrow (small T) windows insufficiently sample around a possible change point to produce larger V_n values. On the other hand, wide windows (large T) may sample multiple change points and so resolution is diminished.

Figure 2(left) shows the light curve of the HESS July 28 flare (run 3), 30 s bins. The count rate changes substantially at a number of points, which may indicate time scales of varying source components, but the

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