SF2A 2008
C. Charbonnel, F. Combes and R. Samadi (eds)

COSMOLOGY AND ASTROPHYSICS WITH EXACT INHOMOGENEOUS MODELS

Célérier, M.-N.[1]

Abstract. It is commonly stated that we have entered the era of precision cosmology. The successes of the Concordance model are built on using a homogeneous background metric combined with first order perturbation theory. However, as we map out the universe around us — its mass distribution and flow patterns — in ever greater detail, the non-linear behaviour of cosmic structures becomes increasingly apparent. The homogeneity assumption — so essential in developing the basics of cosmology — must now be considered just a zeroth order approximation, and similarly linear perturbation theory a first order approximation. Since inhomogeneous solutions of Einstein's field equations provide models of both small and large structures that are fully non-linear, they seem to be best appropriate to properly interpret observations. We give here some instructive examples pertaining to inhomogeneous astrophysics and cosmology, from structure formation to the reproduction of cosmological data.

Structure formation is generally studied either in N-body simulations or in the linear perturbation formalism. However, the linear approach is no more valid once the density contrast becomes too large and it does not take into account the influence of the structure shape. The limitations of the N-body simulations are: the use of Newtonian mechanics, the assumption of a uniform Hubble expansion and the finite number of particles. Thus investigations based on exact solutions are mandatory.

Exact spherically symmetric inhomogeneous models, such as Lemaître–Tolman–Bondi (LTB) and Lemaître (otherwise known as Misner-Sharp) were used to describe: the formation of a galaxy with a central blackhole (Krasiński & Hellaby, 2004a), the formation and evolution of galaxy clusters and voids (Krasiński & Hellaby, 2002; Krasiński & Hellaby, 2004b) and the influence of radiation on void formation (Bolejko, 2006a). These investigations demonstrate that velocity perturbations are more efficient in generating structures than density perturbations are and that until several million years after last scattering radiation cannot be neglected in models of structure formation.

To estimate how two neighboring structures influence each other evolution, the evolution of a double structure in quasi-spherical Szekeres (QSS) models (inhomogeneous without symmetries) was compared with that of single structures in other models. In the studied QSS models, the growth of the density contrast as shown in Fig. 1 appears to be 5 times faster than in LTB models and *8 times faster* than in the linear approach (Bolejko, 2006b).

A QSS model for a triple structure was also studied by Bolejko (2007). An overdense region at the origin is followed by a small void which spreads to a given r coordinate. At a larger distance from the origin, the void is huge and its larger side is adjacent to an overdense region. Where the void is large, it evolves much faster than the underdense region closer to the "centered" cluster. The exterior overdense region close to the void along a large area evolves much faster than the more compact supercluster at the centre. This suggests that, in the Universe, small voids surrounded by large high densities evolve much more slowly than large isolated voids.

In cosmology, the last decade has witnessed a phenomenal increase of the available data. Analyzed in the framework of FLRW homogeneous solutions, they have yielded the Concordance model where more than 95% of the Universe content is of unknown nature. Hence the need of studying if these observations could not be given a more natural explanation in the framework of exact inhomogeneous models, even if other promizing

---

[1] LUTH, Observatoire de Paris, CNRS, Université Paris VII, 92195 Meudon, France

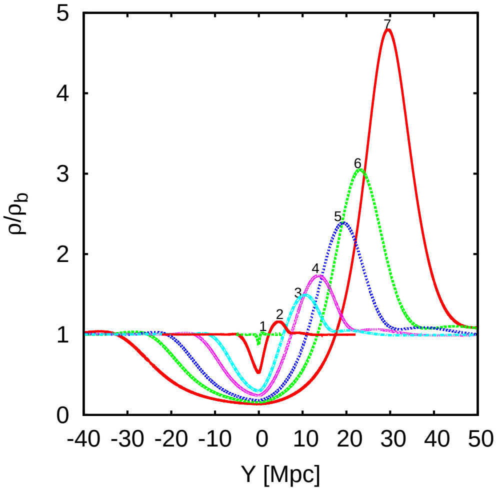

Fig. 1. Evolution of the density profile from 100 Myr after BB (1) to the present time (7).

averaging methods are currently put forward.

The horizon problem develops sooner or later in any cosmological model exhibiting a space-like singularity. Inflation adds a slice of de Sitter space-time to an otherwise space-like singularity FLRW model. It thus only postpones the occurence of the horizon problem to a future region of the observer's worldline. A delayed Big-Bang provides a permanent solution to the problem within of a particular class of LTB models exhibiting a non space-like singularity (Célérier & Schneider, 1998; Célérier, 2000; Célérier & Szekeres, 2002).

The luminosity distance-redshift relation obtained from the SN Ia data analyzed in the framework of an a priori homogeneous Universe yields an apparent accelerated expansion. However, recent studies show that these data can be reproduced by fitting the parameter functions of inhomogeneous solutions – LTB or LTB Swiss-cheese – without any "dark energy" component (see, e.g., Célérier, 2007, for a review).

This very short summary shows that there is still much to be learned about the effects of exact non-linear inhomogeneities in Geneneral Relativity. The Universe, as we observe it, is very inhomogeneous. There are groups and clusters of galaxies, huge cosmic voids and large elongated filaments and walls. In cosmology, however, the homogeneous and isotropic models of the FLRW class are used almost exclusively and structure formation is described by an approximate perturbation theory. This works well as long as the perturbations remain small, but cannot be applied once they become large and evolution becomes non-linear. The phenomena of fully relativistic inhomogeneous evolution do occur and cannot be ignored. This is where the methods of inhomogeneous cosmology must come into play.

References

Krasiński, A., & Hellaby, C. 2004a, Phys. Rev., D69, 043502
Krasiński, A., & Hellaby, C. 2002, Phys. Rev., D65, 023501
Krasiński, A., & Hellaby, C. 2004b, Phys. Rev., D69, 023502
Bolejko, K. 2006a, MNRAS, 370, 924
Bolejko, K. 2006b, Phys. Rev., D73, 123508
Bolejko, K. 2007, Phys. Rev., D75, 043508
Célérier, M.-N., & Schneider, J. 1998, Phys. Lett., A249, 37
Célérier, M.-N. 2000, A& A, 362, 840
Célérier, M.-N., & Szekeres, P. 2002, Phys. Rev., D65, 123516
Célérier, M.-N. 2007, New Adv. Phys., 1, 29