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THE SZ EFFECT : BIASES INTRODUCED ON COSMOLOGICAL PARAMETER ESTIMATION USING PLANCK

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Abstract. We examine the biases induced on cosmological parameters when the presence of secondary anisotropies is not taken into account in Cosmic Microwave Background analyses. We develop an exact analytical expression applicable when any additive signal (systematics, foregrounds) is neglected in the analysis. We then apply it, in the context of the upcoming *Planck* experiment, to the case of the thermal Sunyaev-Zel'dovich residual signal that remains after cluster extraction. We show that analyses neglecting the presence of this signal introduce important biases on the values of the cosmological parameters n_s and τ, at least 6.5 and 2.9 times larger than the expected precision on these parameters. The $\Omega_{\rm b}$ parameter is also biased to a lesser extent.

1 Introduction

The future Cosmic Microwave Background (CMB) experiments, such as *Planck*, will measure with unequalled sensitivities the amplitude of the CMB temperature and polarisation anisotropies up to angular scales of a few arcminutes in multiple frequency bands. These measurements will allow us to constrain cosmological parameters with a relative precision of the order of one percent. In this context, additional contributions to the signal (galaxies, point sources, secondaries arising from the interaction of CMB photons with matter after decoupling, etc.) can no more be neglected and a precise quantification of the biases on the parameters is needed.

The secondary anisotropy we focus on is that which dominates at small scales (e.g. Aghanim et al. 2008): the thermal Sunyaev-Zel'dovich (SZ) effect (Sunyaev & Zel'dovich 1972). The SZ effect is made of two contributions: the inverse Compton scattering of the CMB photons off the hot electrons of the intra-clusters gas, called the thermal SZ, and a Doppler shift caused by the motion of the clusters with respect to the CMB reference frame, called the kinetic SZ. The thermal SZ effect thus decreases the CMB intensity in the Rayleigh Jeans part of the black body spectrum and increases it in the Wien part. Using this characteristic spectral signature and the upcoming multifrequency surveys, one will be able to detect and extract galaxy clusters. Nevertheless we will be left, in the temperature anisotropy maps, with some level of residual SZ contribution from undetected clusters that generate some power excess at small scales.

We present a method we developed to calculate the biases on parameters induced by any signal that adds to or subtracts from the primary signal. We then apply this method to the case of the thermal SZ residuals in the context of the future *Planck* mission and discuss our results.

2 Biases induced by an additive contribution

Several studies have estimated the expected precision on the cosmological parameters that one can reach given the characteristics of an instrument (sky coverage, beam, sensitivity...) This information must be completed by an estimation of the sensitivity of the measurements to any secondary signal added to the primary. We present a general method to calculate the bias induced, on parameter estimation, by any additive signal (instrumental systematics, astrophysical contaminants).

As a reference used to estimate cosmological parameters, we consider a signal $C_\ell^{\rm D}$ that is the sum of a primary signal ($C_\ell^{\rm CMB}(\hat{\theta})$) that depends on the cosmological parameters $\hat{\theta}$ and of an additional signal ($C_\ell^{\rm add}$)

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that may, or may not, contain cosmological information. When this additional signal is taken into account in the parameter estimation one obtain the "true" cosmological parameters ${}_1\hat{\theta}$. On the contrary, when one fits the total signal with the primary only, one obtains a biased parameter set ${}_2\hat{\theta}$. We derived (Taburet et al. 2008) an analytical expression of the biases on the parameters : $\hat{b} ={}_2 \hat{\theta} -{}_1 \hat{\theta}$. Considering that the associated errors to each $C_\ell^{\rm D}$ data point have a Gaussian distribution, we showed that the biases on the parameters are $\hat{b} = \mathbf{G}^{-1}\hat{V}$, where

$$\begin{aligned} G_{ij} \quad = \quad & \sum_{\ell,X,Y} \mathrm{cov}_\ell^{-1}(C_\ell^X C_\ell^Y) \left[\left.\frac{\partial C_\ell^{X\mathrm{mod}}}{\partial\theta_i}\right|_{\hat{\theta}={}_1\hat{\theta}} \left.\frac{\partial C_\ell^{Y\mathrm{mod}}}{\partial\theta_j}\right|_{\hat{\theta}={}_1\hat{\theta}} + \left.\frac{\partial C_\ell^{X\mathrm{mod}}}{\partial\theta_j}\right|_{\hat{\theta}={}_1\hat{\theta}} \left.\frac{\partial C_\ell^{Y\mathrm{mod}}}{\partial\theta_i}\right|_{\hat{\theta}={}_1\hat{\theta}} \right. \\ & \left. - C_\ell^{X\mathrm{add}} \left.\frac{\partial^2 C_\ell^{Y\mathrm{mod}}}{\partial\theta_i\partial\theta_j}\right|_{\hat{\theta}={}_1\hat{\theta}} - C_\ell^{Y\mathrm{add}} \left.\frac{\partial^2 C_\ell^{X\mathrm{mod}}}{\partial\theta_i\partial\theta_j}\right|_{\hat{\theta}={}_1\hat{\theta}} \right] \end{aligned} \tag{2.1}$$

and

$$V_i = \sum_{\ell,X,Y} \mathrm{cov}_\ell^{-1}(C_\ell^X C_\ell^Y) \left[C_\ell^{Y\mathrm{add}} \left.\frac{\partial C_\ell^{X\mathrm{mod}}}{\partial\theta_i}\right|_{\hat{\theta}={}_1\hat{\theta}} + C_\ell^{X\mathrm{add}} \left.\frac{\partial C_\ell^{Y\mathrm{mod}}}{\partial\theta_i}\right|_{\hat{\theta}={}_1\hat{\theta}} \right]. \tag{2.2}$$

with $C_\ell^{X\mathrm{mod}} = C_\ell^{X\mathrm{CMB}}$ and $X, Y =$ TT, EE, TE. The expression of the coefficients of the covariance matrix, $\mathrm{cov}_\ell(C_\ell^X C_\ell^Y)$, can be found in Zaldarriaga & Seljak (1997), and the numerical values that we used can be found in the *The Scientific Programme of Planck* (also known as the *Planck Blue Book*, The Planck Colaboration 2006).

Our method presents multiple advantages : it is applicable to calculate the biases induced by any kind of additive signal. In contrast to previous studies (e.g. Zahn et al. 2005; Amara & Réfrégier 2008) the formula we derived is applicable even when the additive signal is dominant over the primary.

The biases on the investigated parameters become relevant only if they are larger than the expected confidence intervals. The latter can be computed through a Fisher matrix analysis. The 68.3% confidence interval on one parameter (the others being known) is given by

$$\delta\theta_i = \sqrt{F_{ij}^{-1}}, \tag{2.3}$$

where the Fisher matrix coefficients are

$$F_{ij} = \sum_\ell \sum_{X,Y} \mathrm{cov}_\ell^{-1}(C_\ell^X C_\ell^Y) \frac{\partial C_\ell^X}{\partial\theta_i} \frac{\partial C_\ell^Y}{\partial\theta_j}. \tag{2.4}$$

3 The thermal SZ residuals

In the context of the large upcoming multifrequency surveys (*Planck*, SPT...) and using the typical signature of the thermal SZ effect, one will be able to detect galaxy clusters through their SZ effect. It will allow us to build up SZ cluster catalogues and also to remove some of the thermal SZ signal from the CMB maps so as to retrieve the best primary CMB signal. We will however be left with some SZ residual signal in the temperature maps. It is thus important to quantify the biases on the cosmological parameters if the thermal SZ residuals are neglected in the *Planck* data analysis. Our approach is first to estimate which galaxy clusters will remain undetected, then to calculate the associated SZ residual signal and use it in equations (2.1) and (2.2) to get an estimate of the biases.

First we build up the theoretical selection function that determine the minimal mass $M_{\rm lim}(z)$ for a cluster at redshift z to be detected by the *Planck* instrument through its SZ contribution. We consider that a galaxy cluster is detected if its beam-convolved Compton parameter emerges from the confusion noise and if its integrated signal is above the instrumental limit, σ_Y, simultaneously in the 3 channels where the SZ signal is the strongest (100, 143 and 353 GHz).

Then we calculate the SZ residual angular power spectrum due to the undetected clusters. For each multipole ℓ, the contribution comes from each galaxy cluster (poisson contribution) and from the correlation between clusters. Following Komatsu & Kitayama (1999) who have shown that the latter is negligible compared to the former for $\ell > 300$, we consider only the poissonian contribution (Komatsu & Seljak 2002):

$$C_\ell = f_\nu^2 \int_0^{z_{\rm rec}} dz \frac{dV}{dz} \int_{M_{\rm Min}}^{M_{\rm Max}} dM \frac{dn(M,z)}{dM} \left|\tilde{y}(M,z)\right|^2 \tag{3.1}$$

where f_ν represents the frequency dependency of the SZ effect. The comoving volume V and the mass function $n(M,z)$ depend on the cosmology, and $\tilde{y}(M,z)$ depends on both the cosmology and the intra-cluster gas distribution (that we describe by an isothermal β-model). $M_{\rm Max} = M_{\rm lim}(z)$ is the highest mass of undetected clusters at redshift z.

Figure 1 shows that the detected clusters (difference between the dashed and the dot-dashed curves) contribute mainly at large scales, while the residual contributes at small scales where the primary CMB signal vanishes. For multipoles higher than 2000, the SZ contribution after extracting clusters detected at $3\sigma_Y$ dominates over the primary CMB signal.

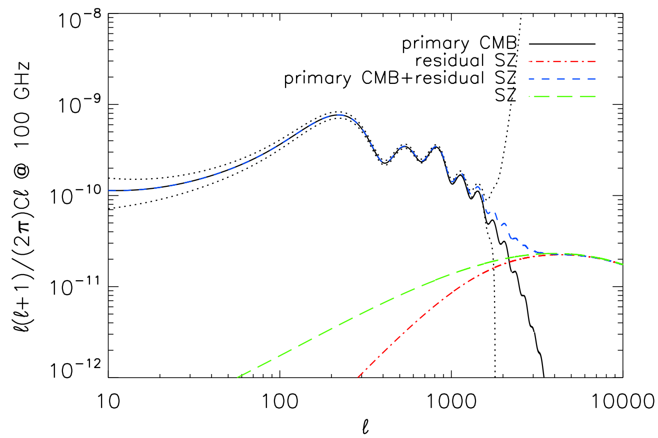

Fig. 1. At 100 GHz, primary CMB (solid line), SZ angular power spectrum of the whole cluster population (long-dashed green line), residual SZ spectrum after the extraction of clusters above 3 times the instrumental noise simultaneously at 100, 143 and 353 GHz (dot-dashed red line), primordial CMB + residual SZ spectrum (dashed blue line). The black dotted lines represent the $1\sigma_Y$ noise of the 100 GHz *Planck* channel.

4 Results and discussion

We apply the method outlined in section 2 to the *Planck* mission in order to estimate the biases introduced on the Ω_Λ, $\Omega_{\rm b}$, H_0, $n_{\rm s}$, σ_8 and τ parameters when the additive signal constituted by the thermal SZ residuals is neglected in the data analysis. The black solid lines on figure 2 represent the 68.3% confidence ellipses on these 6 parameters using the TT, TE and EE power spectra when a coherent analysis that takes into account the primary CMB and the SZ residuals dependency on the cosmological parameters is carried out (reference model). The distance between the centre of these and the centre of the shifted ellipses (dotted red and dashed green) represents the biases on the parameters when the thermal SZ residuals (respectively after a 1 and a 5 σ_Y cluster detection) are not taken into account in the analysis of the CMB signal.

First, and unsurprisingly, the biases induced on Ω_Λ and H_0 are negligible. At most, for a large residual contribution, they reach roughly 0.08 and 0.6 in units of the 1σ errors on the parameters. On the contrary, as expected, the parameters σ_8, and to a higher extent $n_{\rm s}$, $\Omega_{\rm b}$ and τ, which are the most sensitive to a power excess at large scales, and are also degenerate with each other, are significantly affected. An excess of power at high ℓ and a slight shift of the fourth and fifth CMB peaks towards smaller scales, due to SZ residuals, can be respectively accounted for by over-estimating the values of σ_8 and $\Omega_{\rm b}$. Since the amplitude of the CMB power spectrum strongly depends on σ_8, a relatively small variation of σ_8 is enough to fit the power excess. As a result, the bias on this parameter is rather small. For $\Omega_{\rm b}$ the biases are 1.4 and 2.4 in units of the error on Ω_b, for the 1 and $5\sigma_Y$ cases respectively. Higher $\Omega_{\rm b}$ and σ_8 values add more power at all multipoles, while the SZ residuals contribute significantly only at $\ell > 1000$. The resulting excess at large scales is thus accounted for by increasing the spectral index n_s, redistributing power from large to small scales ($\ell > 1200$). It is also compensated by a decrease of the optical depth τ, that reduces the CMB power. As a consequence, fitting data containing primary CMB and SZ secondary residual with a pure primary CMB spectrum induces quite an

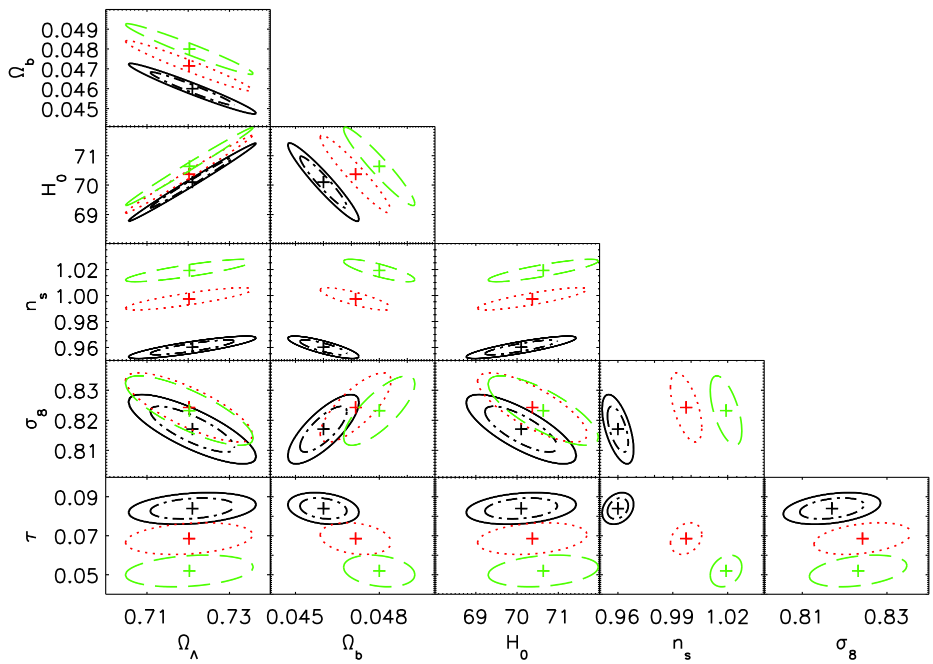

Fig. 2. 68.3% joint confidence regions for Ω_Λ, Ω_b, H_0, n, σ_8 and τ (solid line) obtained with the expected *Planck* TT, TE and EE spectra computed for the reference cosmological model. The dotted (red) and long dashed (green) shifted ellipses represent the 68.3% joint confidence regions around the biased parameters when respectively the $1\sigma_Y$ and $5\sigma_Y$ residual SZ contributions are not taken into account.

important bias on n_s and τ.

We show that a neglected thermal SZ residual signal in future high sensitivity high resolution CMB experiments introduces strong biases on the values of n_s, σ_8 and Ω_b. The high values of the biases emphasise the need of a coherent analysis (as was already pointed out in Douspis et al. 2006) of the CMB signal that includes full cosmological dependency of the SZ signal. One should also consider the importance of taking into account other additional signals contributing to the power excess at small scales, such as extragalactic point sources that also contribute to biasing the parameters when neglected.

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