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THE ROSSBY WAVE INSTABILITY AND PLANET FORMATION: 3D NUMERICAL SIMULATIONS

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Abstract. Models of planet formation do not explain yet the growth of planetesimals as in certain ranges of grain size collisions are too slow compared to estimated planet formation time. The Rossby wave instability (RWI) may solve this problem by the formation of Rossby vortices in the accretion disc, speeding up the accumulation of grains in their centre (Varniere & Tagger, 2006). Up to now, only two dimensions numerical studies of the RWI have been done. In this proceeding we present the results of three dimensions numerical simulations of the non-linear evolution of the RWI in a non magnetized disc and its vertical structure.

1 Introduction

The planet formation model based on gravitational collapse is a too slow mechanism as far as the collisions between grains with a size higher than one centimeter or meter, will destroy them. The other difficulty of this model is that the grains should fall on to the central star in a few thousand years. In order to speed up the planet formation, it has been proposed (Barge & Sommeira, 1995) that vortices in the protoplanetary disc would speed up the process by trapping the dust in their center. However vortices in discs are sheared away by differential rotation (Tagger, 2001) and should not be able to survive on the planet formation timescale. This difficulty could be overcome by the Rossby-Wave Instability (Lovelace et al., 1999) which under certain conditions can create long-lived, high-amplitude vortices (Varniere & Tagger, 2006).

The analytical study of this instability (Lovelace et al., 1999) was done in the case of a two dimensional disc and predicted the RWI linear evolution. This work has been confirmed by 2D numerical simulations of the linear evolution (Li et al., 2000) and have been extended to the non linear stage (Li et al., 2001). Other simulations of the RWI have been performed in the context of planet formation with a dead zone lying in the protoplanetary disc (Varniere & Tagger, 2006, Lyra, et al., 2008). Rossby wave instability may have already been seen in three dimensional numerical simulations (Machida & Matsumoto, 2003) but was not identified and therefore not studied in details. The point of the present simulations is to study the development and the vertical structure of this instability when it appears in a 3D accretion disc.

After a brief discussion about the Rossby wave instability, its characteristics and trigger mechanism,we show the numerical setup and initial conditions of the simulations. The results are presented in paragraph 4 and 5, and in the last part conclusions are given.

2 The Rossby wave instability

The Rossby wave instability was introduced by Lovelace et al. (1999) as a solution for the angular momentum transport in a un-magnetized disc. It is a hydrodynamical instability that occurs when the radial profile of the specific vorticity shows a local extremum:

$$\partial_r \mathcal{L} = 0 \tag{2.1}$$

$$\mathcal{L} = \frac{(\vec{\nabla} \times \vec{v})_z}{\Sigma} \tag{2.2}$$

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where r and z are the coordinates of the cylindrical basis (r, θ, z), $\vec{v}$ is the velocity of the fluid, and Σ is the surface density of the disc.

This condition may be fulfilled within a protoplanetary disc at the edge of the dead zone (defined as the radial interval where the ionization is too low to couple the gas with the magnetic field). Varniere & Tagger (2006) found that the dead zone naturaly creates the conditons to trigger the RWI: MHD instabilities such as the magneto-rotational instability (Balbus & Hawley, 1991) cause accretion outside the dead zone that creates a minimum of the density at its inner edge and a density maximum at its outer edge. These density extrema lead to extrema of the specific vorticity $\mathcal{L}$.

The RWI is a non axisymetric instability characterised by the formation of a Rossby vortex at the position of the extremum of $\mathcal{L}$ and density waves outside the vortex zone. Rossby vortices are usually discussed in meteorology and planetary atmosphere in general (Jupiter great red spot). In an accretion disc, a vortex would be sheared due to differential rotation, unless being sustained by some process; the RWI is one of the processes able to do so. Differential rotation is also responsible of the coiling of the compressional waves into spirals, as in spiral galaxies.

3 Setup

The 3D simulations of the RWI were done by the mean of the hydrodynamical module of the Versatile Advection Code (VAC,Toth, 1996) solving the adiabatic equations. The resolution in cylindrical coordinates $(r/r_0, \theta, z/r_0)$ is $300 \times 256 \times 32$ or 64 cells on the computational domain $[0.8, 10.] \times [0., 2\pi] \times [0., 0.8]$ with r_0 the radius of the inner edge of the disc.

The initial density profile is a usual profile with a surface density varying as $(r/r_0)^{-1/2}$ where we add an overdensity (a bump) at a radius of $3r_0$, to satisfy the instability condition.

$$\rho(r,0) = 10^{-2} + r^{-1/2}\Big(1 + th\big(\frac{r/r_0 - 1.5}{0.1}\big)\Big)\Big(1. + e^{-\frac{(r/r_0-3.)^2}{10^{-2}}}\Big) \tag{3.1}$$

The radial velocity and vertical density profile were chosen in order for the disc to be in equilibrium. The axisymmetry of the disc is broken due to small perturbations of the radial velocity that are of the order of 10^{-4} the keplerian velocity.

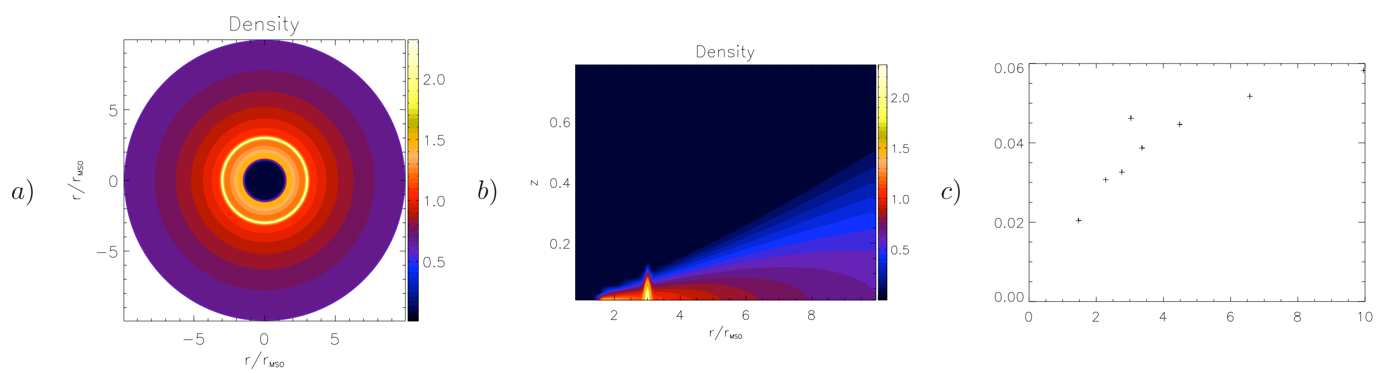


Fig. 1. Initial density profile of the disc, cut in the midplane (a) and vertical cut (b). c) Evolution of the aspect ratio h/r of the disc with the radius r.

These initial conditions have been chosen such that the Rayleigh stability criterion is ensured everywhere, despite the strong radial pressure gradient near the overdensity. The equilibrium of the disc have been checked with analogous simulations without the velocity perturbations.

4 RWI identification

The first aim of these 3D simulations was to show that the RWI can develop near the overdensity, at the edge of the dead zone of a proto-planetary disc. The instability can be identified thanks to its two components, vortices and density waves that appear clearly in the simulations. Those two components can be seen on the density profile on fig 2. In the following plots, we display the physical quantities once have been subtracted the axisymmetric component of each quantity.

4.1 *Acoustic waves*

The density waves are clearly identified when plotting the compressible part of the velocity ($\vec{\nabla}.\vec{v}$). An interesting point is that the bump zone is known to be a forbidden zone for the density waves and it is clear on fig 2 b) that those waves are only seen outside this zone.

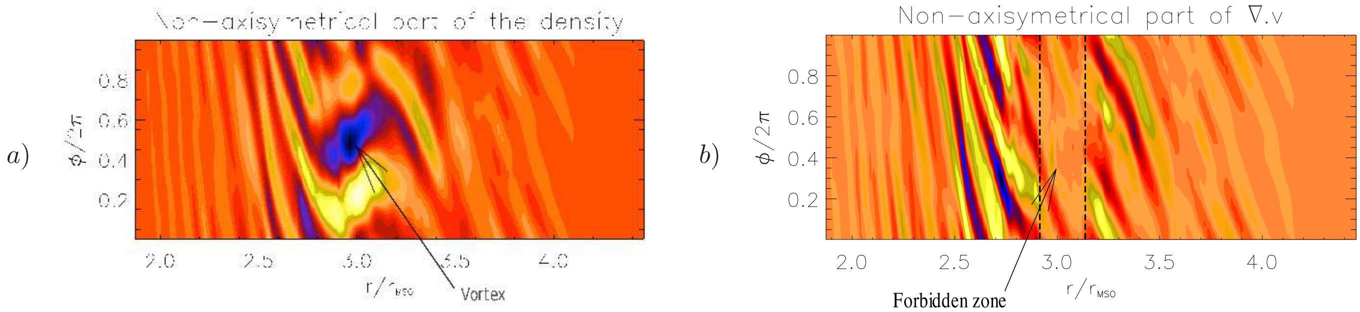


Fig. 2. The two plots are representations of the disc in a cartesian grid, where the radius of the disc is in asbscissa and the θ angle is in ordinate ($\phi = \theta/2\pi$). The plots are cut of the disc between $r/r_0 = 1.8$ and 4.2 . a) Non-axisymetrical part of the density of the disc, the two parts of the instability can be seen: the vortex and the acoustic waves. b) The plot of the non-axisymetrical part of $\vec{\nabla}.\vec{v}$ shows the compressible waves that are localised outside the bump zone.

4.2 *Vortices*

The second aim of the simulations was to know if this instability can help for the planet formation, that is the structure of the vortices in the vertical direction. In the planet formation model, it is usually supposed that the dust grains are concentrated in the mid-plane of the disc due to a lower thermal velocity for a given gas temperature; that is why it is important to know if the vortex occurs in the vertical structure of the disc and where it is located.

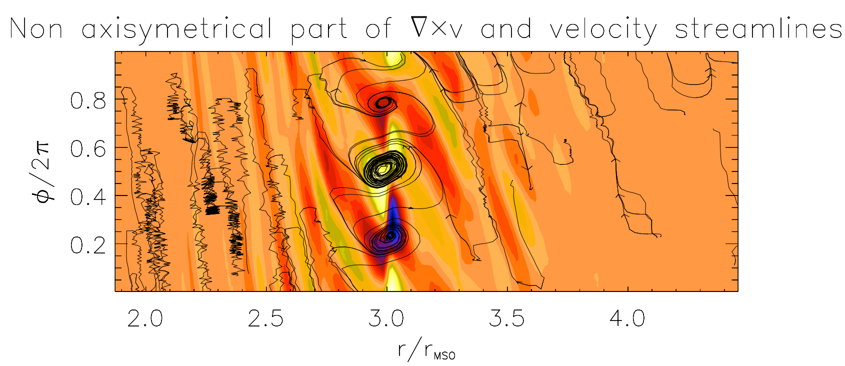


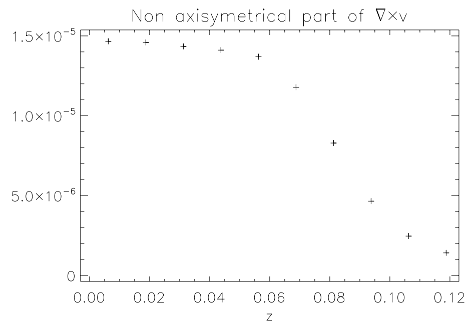
Fig. 3. The same projection as before is used. The color plot is the nonaxisymetric part of the vorticity $(\vec{\nabla} \times \vec{v})_z$ in the disc, it is concentrated around the position of the bump. The black lines are the streamlines of the non-axisymetric part of the velocity showing the vortices.

The vortices can be observed by plotting the vorticity of the stream that is the vertical projection of $\vec{\nabla} \times \vec{v}$ as can be seen on fig 3. The exact location of the vortices is confirmed by the velocity stream lines. The radius where we placed the extremum of $\mathcal{L}$ is clearly the radius of the center of the vortices and is a key parameter for the instability observed here.

5 Vertical structure of the vortices

Figure 4 shows the maximum value of the vorticity in function of z. This comparison with the density profile shows that when the disc has a vertical extension of ten cells, the vortex has a vertical extension over several cells before its amplitude diminishes with the density. The vortex is then more vertically extended than the density. This vertical extension of the vortex is coherent with the instability linear theory and with the planet formation mechanism because it will favour the agglomeration of dust.

a)



b)

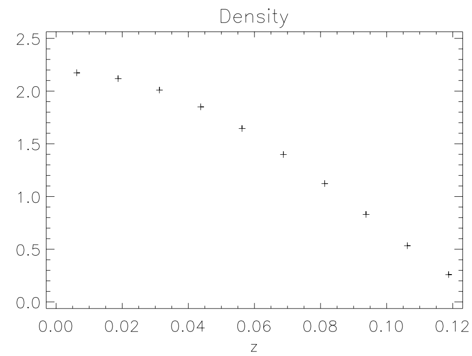


Fig. 4. Vorticity (a) and density (b) in function of the height z. This figure shows a vertical extension of the vortices as far as the vorticity decreases slower than the density

6 Conclusion

We have presented the first fully 3D simulations of the RWI. The radial and vertical structures of the instability are in agreement with the linear theory. The vortices exhibit finite vertical extension and survive during several disc rotations. These simulations confirm the physics of the instability, and its ability to help planet formation by creating strong long-lived vortices in a differentially rotating disc.

However only a few keplerian orbits of the vortices have been simulated here, future studies will include longer simulations to see the long time evolution of the vortices. A better vertical resolution is also needed to have a better idea of the vortices structure, and for such a simulation an adaptive mesh refinement (AMR) grid will be used. A two fluid simulations including the solid grains with a more realistic disc including the magnetised and dead zone would also be an interesting development of this work. In an other context we aim to study the RWI instability in order to understand the microquasar high frequency quasi-periodic oscillations.

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