SF2A 2009
M. Heydari-Malayeri, C. Reylé and R. Samadi (eds)

GRAVITY TESTS WITH INPOP PLANETARY EPHEMERIDES.

Fienga, A.[1], Laskar, J.[2], Kuchynka, P., Manche, H., Gastineau, M. and Leponcin-Lafitte, C.[3]

Abstract.
We present here several gravity tests made with the latest INPOP08 planetary ephemerides. We first propose two methods to estimate the PPN parameter β and its correlated value, the Sun J_2, and we discuss the correlation between the Sun J_2 and the mass of the asteroid ring. We estimate a possible advance in the planet perihelia. We also show that no constant acceleration larger than 1/4 of the Pioneer anomaly is compatible with the observed motion of the planets in our Solar System.

1 Introduction

Thanks to the high precision achieved with the observations deduced from spacecraft tracking, it becomes possible to estimate relativistic parameters γ and β of the Parametrized Post-Newtonian formalism of General Relativity (Will, 1993). Nevertheless, if γ plays a role in the equations of motion, it is worth noting that light propagation is only sensitive to that parameter. PPN γ can then be estimated with high accuracy by light deflection measurements by VLBI (Shapiro et al. 2004; Lambert & Le Poncin-Lafitte, 2009), by time delay during an interplanetary roundtrip, and by Doppler tracking data of a space mission (see for instance the Cassini experiment, Bertotti et al. 2003). This is also why, in the following, we assume $\gamma = 1$ in order to test only the sensitivity of PPN β on the perihelion's advance of planets. However, the Sun oblateness J_2 plays also a key role in this phenomena. Indeed, the usual expression of the advance of perihelion is given by (Will 2006)

$$\Delta\omega = \frac{2\varpi(2\gamma - \beta + 2)GM_{\rm sun}}{a(1-e^2)c^2} + \frac{3\varpi J_2 R_{\rm sun}^2}{a^2(1-e^2)^2} \tag{1.1}$$

where G and c are the newtonian gravitational constant and the speed of light in vacuum, respectively. J_2, $M_{\rm sun}$ and $R_{\rm sun}$ are the Sun oblateness, mass and equatorial radius, respectively, while a and e are the semi-major axis and the eccentricity of the precessing planet. The PPN β is, thus, correlated with the Sun oblateness J_2 through this linear relation. But, the β coefficient varies as $1/a$, while the J_2 coefficient is proportional to $1/a^2$. Using data from different planets will, thus, allow to decorrelate these two parameters. MEX and VEX tracking data have actually led to an important improvement of Mars and Venus orbits in INPOP08 (Fienga et al. 2009). Thanks to the information brought by the combination of very accurate tracking data of spacecraft orbiting different planets, the planetary ephemerides become thus an interesting tool for gravity testing. In the following, we give some examples of such tests.

2 Determination of PPN β and the Sun oblateness J_2

The advance of the perihelion induced by general relativity and the Sun J_2 has an impact very similar to the advance induced by the main-belt asteroids on the inner planet orbits. In INPOP08, a ring was added to average the perturbations induced by the main-belt asteroids which cannot be fitted individually by tracking observations. This ring has its physical characteristics (mass and distance to the Sun) estimated independently

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[1] Observatoire de Besançon- CNRS UMR6213, 41bis Av. de l'Observatoire, 25000 Besançon
[2] Astronomie et Systèmes Dynamiques,IMCCE-CNRS UMR8028,77 Av. Denfert-Rochereau, 75014 Paris, France
[3] SYRTE-CNRS UMR8630, Observatoire de Paris, 77 Av. Denfert-Rochereau, France.

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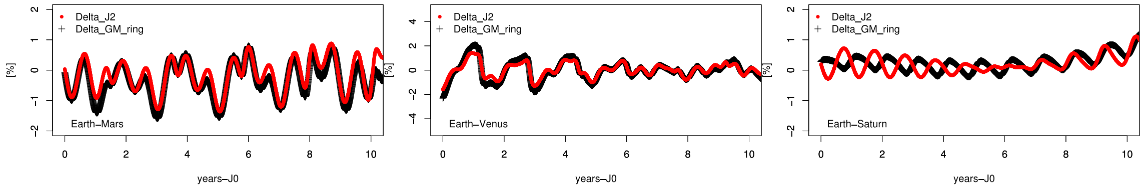

Fig. 1. Residuals obtained by comparisons between Mercury direct range, MGS/MO, MEX, VEX and Cassini range tracking data and ephemerides perturbed by a small change in the Sun J_2 (12%) and by a small change in the mass of the asteroid ring (17%).

Table 1. β intervals in which the residuals stay below the 5% limit. The values of β given here are estimated for $\gamma = 1$.

Data	β min	β max	Data	β min	β max	Data	β min	β max
MGS/MO+MEX	0.99995	1.0002	Jupiter VLBI	0.9996	1.0002	Viking	0.9995	1.0002
VEX	0.99990	1.0002	Saturn Cassini range	0.9998	1.0005	Mercury	0.9985	1.005

from the fit by considering the albedos and physical properties of 24635 asteroids (for more details see Kuchynka et al. 2008). As illustrated in figure 1, there is a correlation between the effect on the geocentric distance of the modeling of the ring as done in INPOP08, in one hand, and the effect of the Sun oblatness in the other hand. Indeed, on these plots, one may see how a small change in the value of the Sun J_2 (12%) induces, after the refit of the planet initial conditions a periodic effect very similar in amplitude and frequency as a change in the mass of the asteroid ring (17%). This effect is obvious on Mercury, Mars and Venus distances to the Earth, but not for Saturn. The Saturn-Earth distances are indeed not affected in the same way. We can also conclude that, when new accurate observations of outer planets will be obtained, they will be very useful to decorralate asteroid effects on planet orbits by combination with inner planet data. Finally, it stresses the crucial importance of having a model of the asteroid perturbations as a fixed ring, characterized independantly from the fit of planetary ephemerides. By fixing the ring, we limit then an overestimation of the value of the Sun J_2 merging in this value some effects induced by the asteroids.

In Fienga et al. (2009b), estimations of J_2 and β were done by least squares adjustements over different sets of data. The obtained results stress the correlations between J_2 and β. For more details, see Fienga et al. (2009b). An alternate strategy to study the sensitivity of the planetary ephemerides to J_2 and PPN β is to estimate how does an ephemeris built using different values for J_2 and PPN β and fitted on the same set of observations as INPOP08 differ from INPOP08. To estimate the sensitivity of the most accurate sets of data (Mercury direct range, VEX, MEX, MGS/MO, Cassini and Jupiter Galileo) used in the INPOP08 adjustment to the variations of values of J_2 and PPN β, we have estimated and plotted the S/N ratio defined as:

$$S/N = \frac{\sigma_{i,j} - \sigma_{0,0}}{\sigma_{0,0}}$$

where $\sigma_{i,j}$ is the 1-sigma dispersion of the postfit residuals of an ephemeris based on INPOP08 but with values of J_2 and PPN β different from the ones used in INPOP08 (which are $\beta = 1.0$ and $J_2 = 1.82 \times 10^{-7}$) and fitted to all the INPOP08 data sets, and $\sigma_{0,0}$ is the 1-sigma dispersion of the postfit INPOP08 residuals. Results presented as the S/N percentage, are plotted in figure 2 for MEX/MGS and VEX. For other plots see (Fienga et al. 2009b). As one can see in figure 2, the impact of the PPN β is not symmetric with respect to $\beta = 1$. In figure 2, one notices also the direct correlation between the S/N obtained with MGS/MO and MEX data and the one obtained for VEX. In table 1, we have gathered minimum and maximum values of PPN β defining the sensitivity interval of the different data sets. The sensitivity interval is the interval of PPN β for which the S/N remains below 5%. By considering figure 2 and table 1 it appears that MGS/MO and MEX data provide the most narrow interval of sensitivity with $0.99995 < \beta < 1.0002$. This interval is in agreement with the latest determinations done by Williams et al. (2009), Fienga et al. (2008) and Pitjeva (2006).

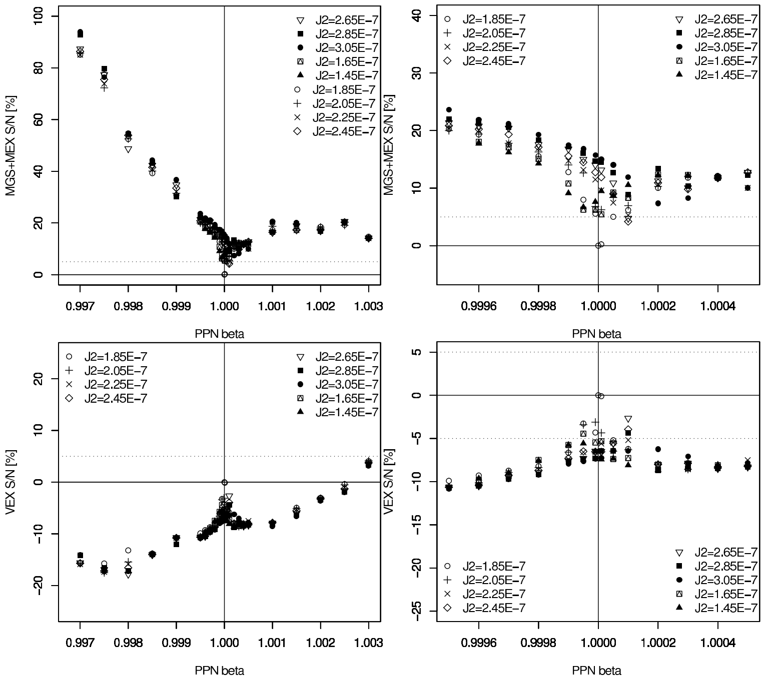

Fig. 2. Residuals obtained by comparisons between observations and ephemerides estimated with different values of PPN β (values given on x-axis of each subframes) and different values of the Sun J_2.

3 Secular advances of planetary perihelia

We are interested here in evaluating if the observations used to fit INPOP08 would be sensitive to supplementary precessions of the planet orbits. To estimate the sensitivity of the modern tracking data, we first fix J_2=1.8 × 10^{-7}, $\beta = 1$ and $\gamma = 1$. By fixing the value of the Sun J_2, we then isolated the impact of the secular advance of the perihelion, $\dot{\varpi}_{\rm sup}$, for one given value of J_2. For each different value of $\dot{\varpi}_{\rm sup}$, initial conditions of planets are fit to the INPOP08 observations and we compare the postfit residuals to the INPOP08 ones. The behaviour of the obtained S/N (as defined in section 2) is symmetrical to a minimum value, this minimal value being centered around $\dot{\varpi}_{\rm sup} = 0$ or not. This symmetry explains why we give an interval of $\dot{\varpi}_{\rm sup}$ for which the minimum of S/N is obtained. The best constraint on the Earth orbit is given by the Jupiter VLBI data set which gives the narrowest interval of $\dot{\varpi}_{\rm sup}$. For Saturn, an offset in the minimum of the S/N is obtained for the Cassini tracking data set (-10 ± 8) and the VEX data set (200 ± 160). These estimations lead to determinations of a supplementary precession of the Saturn orbit that are only marginally statistically significant. By comparisons, (Pitjeva 2009) the value is very close to the one we obtain by considering only the S/N induced on the Cassini observations. This result shows how important the description of the method used for evaluating such quantities. The investigation about a statistically significant advance in the Saturn perihelion has to be continued in using more Cassini and VEX data. Indeed, a prolongation of the interval of time covered by these two data sets will improve the accuracy of the estimations. For more details see (Fienga et al. 2009b).

4 Does the Pioneer anomaly impact the ephemerides ?

We investigate the question of the Pioneer anomaly by using the INPOP08 planetary ephemerides as a test bed for some hypothesis describing the pioneer anomalies. A classic description of the pioneer anomalies (PA) is the appearance of a constant acceleration of about $8.75 \times 10^{-10}\,\mathrm{m\,s^{-2}}$, Sun-oriented after 20 AU (Anderson et al. 2002). We, thus, add this constant acceleration in the equations of motions of Uranus, Neptune and Pluto. We have then fit the modified ephemerides to observations usually used to built INPOP08. Residuals obtained after the fit are plotted in Figure 3. As it appears clearly in the residuals of Uranus right ascension, a constant acceleration of $8 \times 10^{-10}\,\mathrm{m\,s^{-2}}$ added to the classical Einstein-Hoffmann equations of motion can not be missed,

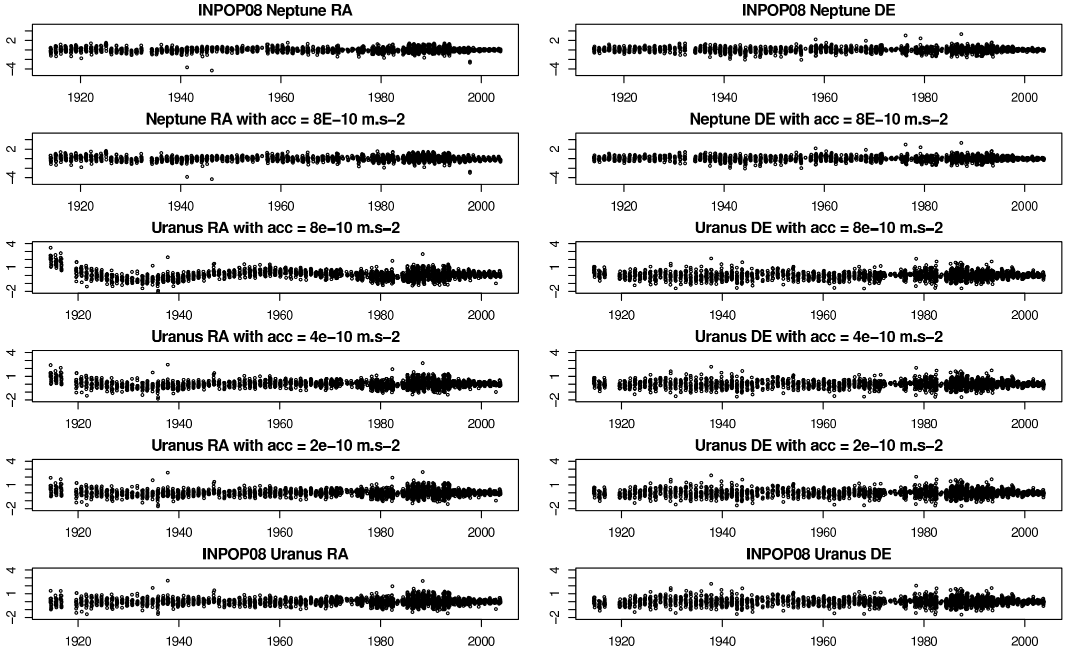

Fig. 3. Residuals in right ascension and declination of Neptune and Uranus obtained with INPOP08 (solution of reference) and fitted ephemerides including PA of different magnitudes: from 8 to 2 $\times 10^{-10}\mathrm{m\,s^{-2}}$. The x-axis are years and y-axis is in arcseconds.

even after the fit of the Uranus initial conditions. A systematic effect remains especially after 1930. This effect cannot be absorbed by the fit or by the noise of the old Uranus observations. By changing the value of the acceleration, one can see that the acceleration must be at least 4 times smaller than the one commonly adopted to be absorbed by the residuals. For Neptune and Pluto, the situation is different. For these planets, the effect of a constant acceleration is absorbed by the fit, as one can see on figure 3 with the postfit and prefit residuals of Neptune.

5 Conclusions

Concerning the determination of the PPN parameter β, an estimation of the sensitivity of planetary ephemerides to this parameter is done following two methods. Our results show that a global fit is needed in order to decorrelate parameters such as PPN β, the Sun J_2 and the asteroid pertubations. We have tested possible detection of an anomalous advance of perihelia of planets. More investigations are needed for the analysis of the perihelion rate of Saturn and more observations of Cassini and VEX data are necessary. Finally, the results obtained here for the Pioneer Anomaly conclude that no constant acceleration larger than 1/4 the PA can affect the planets of our solar system. If it was so, it would have been detected sooner. In the frame of the equivalence principle, this means that no constant acceleration larger than 1/4 the PA can be realistic.

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