SF2A 2009
M. Heydari-Malayeri, C. Reylé and R. Samadi (eds)

SELF-SIMILAR EXPANSION OF POLYTROPIC GAS: APPLICATION TO THE SUPERNOVAE PHOTOSPHERE DYNAMICS

Busschaert, C.[1], Falize, E.[2] and Drouin, M.[2]

Abstract. In this paper we analyze the self-similar expansion of polytropic gas in order to predict the evolution of the supernovae photosphere. We consider a specific solution that we obtained thanks to similarity considerations and which permits to extract an explicit expression of the photosphere dynamics. The latter is compared to silicium line dynamics of 25 SN Ia.

1 Introduction

Similarity analysis and self-similar solutions play an important role in many fields of physics and astrophysics. They give basic information about physical studied systems and are a crucial complement to numerical simulations. In most cases, these specific solutions describe the asymptotic dynamics of systems when a part of boundary and/or initial conditions are lost (Barenblatt & Zeldovich 1972) but in many situations they present important attractor properties. Different methods (dimension analysis (Sedov 1959), Lie group symmetries or Burgan-Feix-Munier transformation (Burgan et al. 1978, Falize et al. 2008) exist in order to obtain them. These methods are based on the invariance properties of equations or on the quasi-invariance principle (Burgan-Feix-Munier transformation),which allows to construct general solutions including those obtained by the standard invariance of equations.

In this paper we consider the dynamics of collisional plasma where the two-temperature effects, viscosity, radiative transfert and thermal conductivity are negligible. As a consequence, the polytropic gas dynamics allows to quantify the hydrodynamic response of matter when submited to high-density energy (Zeldovich & Raizer 1967). Thus, this regime allows to describe detonation, cylindrical and spherical implosion, foil explosion by laser, meteorit explosion, blast wave dynamics or first phase of supernova remnants. Different initial and/or boundary conditions entail various applications. Several analysis have been realized and different self-similar solutions were obtained (see for example Coggeshall 1991, Falize 2008, Falize et al. 2008, Sedov 1959, Simonsen & Meyer-ter-Vehn 1997) in order to describe the following physical systems. In this paper we consider the photosphere dynamics of type Ia supernovae in the vicinity of maximal luminosity. The main motivation of this paper is to use the standard self-similar solution on 25 type Ia supernovae (SN Ia) and find out the exponant dependance of density spatial, which appears as a free parameter depending on the past history of the supernova and particularly the explosion mechanism.

Firstly, we recall the main self-similar solutions and the link between them. Secondly, we apply these considerations to the supernovae photosphere dynamics. Finally we conclude on the main results of this study.

2 Self-similar polytropic gas dynamics

The dynamics of polytropic gas is given by the mass, momentum and energy conservation laws which are given respectively by (Sedov 1959):

$$\frac{\partial\rho}{\partial t}+\frac{1}{r^N}\frac{\partial}{\partial r}\left[r^N\rho v\right]=0,\quad \left[\frac{\partial}{\partial t}+v\frac{\partial}{\partial r}\right]v=-\frac{1}{\rho}\frac{\partial P}{\partial r},\quad \left[\frac{\partial}{\partial t}+v\frac{\partial}{\partial r}\right]P-\gamma\frac{P}{\rho}\left[\frac{\partial}{\partial t}+v\frac{\partial}{\partial r}\right]\rho=0, \tag{2.1}$$

---

[1] UFR Science - Magistère de Physique Fondamentale - Bat 470 - 91405 Orsay Cedex, France
[2] CEA-DAM-DIF F-91297 Arpajon Cedex, France

where N, ρ, P, v and γ are respectively the dimensionality (N=0, 1, 2 in plane, cylindrical and spherical geometry, respectively) of the problem, the density, the pressure, the velocity and the polytropic index. When, initially or asymptotically, kinetic energy is more important than internal energy, the pressure gradient becomes negligible and the evolution of the different physical quantities is given by the following general solutions:

$$\rho(r,t) = \frac{1}{r^{N+1}} f\left(\frac{r}{t}\right), \quad v = \frac{r}{t}, \tag{2.2}$$

The generality of the function f is very interesting since it is possible to introduce the structure of initial and/or boundary conditions in the form of solutions. For example we can introduce the rarefaction wave solution described by Falize et al. (2008):

$$f\left(\frac{r}{t}\right) = \rho_0 \left(\frac{r}{t}\right)^{N+1} \times \left(1 - \left[\frac{\gamma-1}{2c_s}\sqrt{N+1}\frac{r}{t}\right]^2\right)^{2/(\gamma-1)}, \tag{2.3}$$

but we can also introduce the Stanyukovich (Stanyukovich 1960) or Coggeshall (Coggeshall 1991) solutions. Among those, a very interesting one is a consequence of the similarity properties of polytropic gas. This is the case introduced by Chevalier (Chevalier 1982), particularly valuable within the context of our study, where the function f is given by:

$$f\left(\frac{r}{t}\right) = \left(\frac{r}{t}\right)^{-\alpha}. \tag{2.4}$$

With such a form of f we can deduce the evolution of the photosphere since it is defined by:

$$\frac{2}{3} = \int_{R_{ph}}^{\infty} \kappa(\rho,T)\rho.dr, \tag{2.5}$$

where κ and R_{ph} are respectively the opacity and the position of photosphere. Furthermore, we suppose that opacity is given by a power law, *i.e.* $\kappa(\rho,T) = \kappa_0 \rho^m T^n$. Consequently, we can explicitly derive the expression of the photosphere position which is given by:

$$R_{ph}(t) \propto t^{-\alpha[1+m+n(\gamma-1)]/[1-(N+1+\alpha)(1+m+n(\gamma-1))]}. \tag{2.6}$$

When the opacity is determined by Thomson scattering we find the classic result (Branch et al. 1988, Pearce et al. 1988) in spherical case:

$$R_{ph}(t) \propto R_0 \left[\frac{t}{t_0}\right]^{\alpha/(\alpha+2)}. \tag{2.7}$$

In the following, we will deduce values of alpha fitting with formula (2.7) from SN Ia spectra.

3 Application to supernovae photosphere

We measured the photospheric velocity of 25 SN Ia using the blueshift of the silicium line SiIIλ λ 6347Å 6371Å in their spectra[1]. On the whole, we notice that all the photospheric velocities have the same evolution and, more precisely, the dispersion of velocity drops from 5 690 km/s before the peak brightness to 3 590 km/s a few days later (see Fig 1). This can be explained by a decreasing influence of the initial conditions set by the explosion mechanism. We found that the model of homologous expansion with $f(r/t) \propto v^{-\alpha}$ fits well with the measures. Indeed, by derivating the expression of the photospheric radius (2.7), we can easily devise a linear relation between the logarithm of the photospheric velocity and the logarithm of the time which is observed in the data. It gives us the free parameter for each supernovae (Table 1) and the median for this parameter in our sample is around 8, which is slightly higher than those found in previous work (Chevalier 1982).

---

[1] The spectra were extracted from the database SUSPECT (*http://bruford.nhn.ou.edu/∼suspect/index.html*)

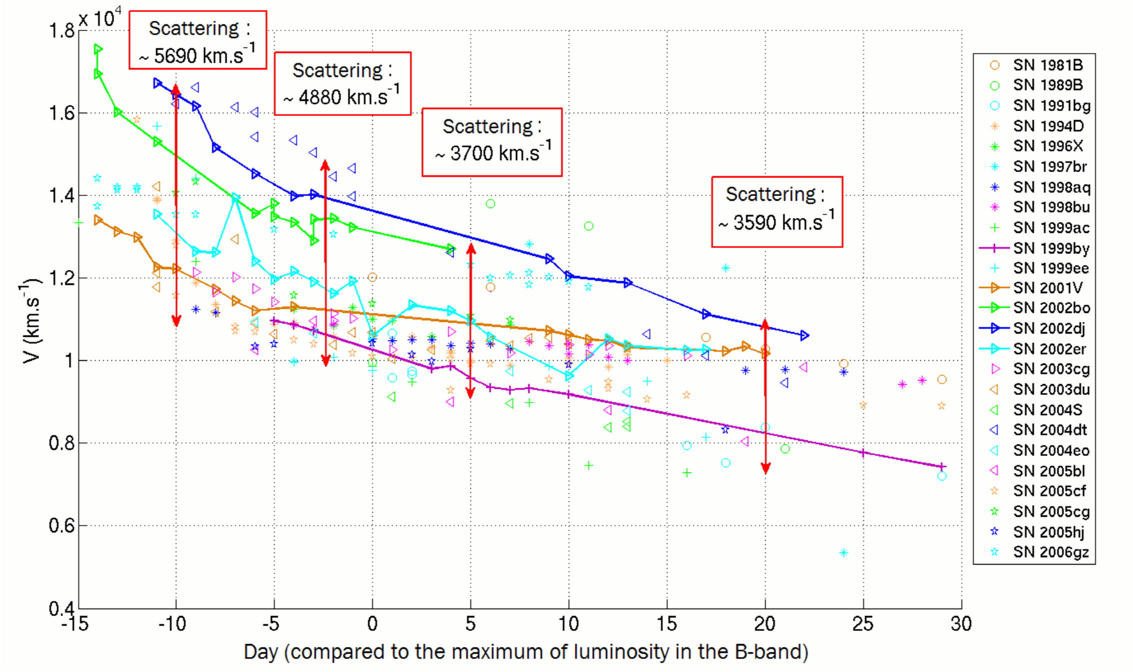

Fig. 1. Evolution of photospheric velocities of 25 SNIa with the time. We estimate the uncertity of the measurement around 200km/s.

4 Conclusion

In this paper we present a self-similar analysis of polytropic gas dynamics. We construct a self-similar solution which describes the asymptotic behavior of this gas. Thanks to a very simple homologous expansion model, we can determine the free parameter which governs the evolution of photospheric velocities of 25 SN Ia. The value of this parameter depends on the past history of the supernova, this may help us to further constrain the current theoretical model based on self-similar solutions.

References

Barenblatt, G. I. & Zeldovich, Y. A., 1972, Ann. Rev. Fluid Mech., 4, 285

Branch, D., Drucker, W. & Jeffery, D. J., 1988, ApJ, 330, L117

Burgan, J. R., Gutierrez, J., Munier, A., Fijalkow, E. & Feix, M. R., 1978, Strongly coupled plasmas, B36, 597

Chevalier, R. A., 1982, ApJ, 258, 790

Coggeshall, S. V., 1991, Phys. Fluids A, 3, 757

Falize, E., 2008, PhD thesis, Observatoire de Paris, France

Falize, E., et al., 2008, J. Phys. Conf. Series, 112, 042015

Pearce, E. C., Colgate, S. A. & Petschek, G., 1988, ApJ, 325, L33

Sedov, L. I., 1959, Mir Edition

Simonsen, V. & Meyer-ter-Vehn, J., 1997, Phys. Fluids, 9, 1462

Stanyukovich, K. P., 1960, Pergamon press

Zeldovich, Y. B. & Raizer, Y. P., 1967, Academic Press (London)

Table 1. References used to get the data concerning the photospheric velocities along with the value of the free parameter α for each SN Ia

Supernovae	Reference / Source	Redshift	Δm_{15}	α
SN 1981B	Branch D. et al., ApJ, 270:123 (1983)	0,00601		6,04
SN 1989B	Barbon R. et al., A&A, 237:79 (1990)	0,00213		4,52
SN 1991bg	Turatto M. et al., MNRAS, 283:1 (1996)	0,00312	1,95	3,60
SN1994D	Patat F. et al., MNRAS, 278:111 (1996)		1,26	8,22
SN 1996X	Salvo M. E. et al., MNRAS, 321:254 (2001)	0,00691	1,31	23,93
SN 1997br	Li W. D. et al., A. J., 117:2709 (1999)	0,0069	1,00	4,31
SN 1998aq	Branch D. et al., Astron. J., 126:1489 (2003)	0,003699	1,12	18,87
SN 1998bu	Spyromilio J. et al., A&A, 426:547 (2004)	0,003	1,01	15,72
SN 1999ac	Garavini G. et al., A. J., 130:2278 (2005)	0,00949	1,30	6,01
SN 1999ee	Hamuy M. et al., A. J., 124:417 (2002)	0,0117	0,94	3,80
SN 1999by	Peter Garnavich / Höflich P. et al., ApJ, 568:791 (2002)	0,00271	1,87	4,68
SN 2001V	Matheson T. et al., A. J., 135:1598 (2008)	0,015	0,99	14,90
SN 2002dj	Pignata G. et al., ArXiv 0805.1089P (2008)	0,00939	1,08	6,04
SN 2002bo	Benetti S. et al., MNRAS, 348:261 (2004)	0,00468	1,13	8,90
SN 2002er	Kotak R. et al., A&A, 436:1021 (2005)	0,00856	1,33	8,21
SN 2003cg	Elias-Rosa N. et al., MNRAS, 369:1880 (2006)	0,004	1,25	12,12
SN 2003du	Stanishev V. et al., A&A, 469:645 (2007) Anupama G. C. et al., A&A, 429:667 (2005)	0,0073	1,02	12,03
SN 2004eo	Pastorello A. et al., MNRAS, 377:1531 (2007)	0,016	1,46	7,77
SN 2004dt	Altavilla G. et al., A&A, 475:585 (2007)	0,01973	1,21	3,41
SN 2004S	Kriscuinas K. et al., A. J., 133:58 (2007)	0,0094	1,14	9,69
SN 2005bl	Taubenberger S. et al., MNRAS, 385:75 (2008)	0,024	1,93	10,90
SN 2005cf	Garavini G. et al., A&A, 427:535 (2007)	0,0065	1,12	8,09
SN 2005hj	Quimby R. et al., ApJ, 660:1083 (2007)	0,0574		10,97
SN 2005cg	Quimby R. et al., ApJ, 636:400 (2006)	0,0313		6,25
SN 2006gz	Hicken M. et al., ApJ, 669:17 (2007)	0,028	0,69	18,78