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MAGNETOHYDRODYNAMICS TURBULENCE: WAVES OR EDDIES?

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Abstract. The dynamics of 3D incompressible MHD turbulence is strongly affected by the presence of a large-scale magnetic field $\mathbf{B_0}$. In a recent work we have investigated through high-resolution direct numerical simulations this dynamics for a driven turbulence. The analysis reveals a non trivial interaction between the 2D state (eddy turbulence) and the 3D modes (wave turbulence). In particular, it is shown that an initial 2D state without energy is unstable: energy is transfered from the 3D modes to the 2D state with an effectiveness larger for stronger B_0. In this case, the final state is a mixture of eddy and Alfvén wave turbulence.

Keywords: Turbulence, magnetohydrodynamics, solar physics, Alfvén waves

1 Introduction

A variety of astrophysical plasmas is well described by incompressible MHD turbulence. It is particularly true in solar physics with coronal loops (*e.g.* Galtier, 1999; Buchlin & Velli, 2007) or coronal holes (*e.g.* Bigot, Galtier & Politano, 2008a). It is therefore important to continue the theoretical investigation of MHD turbulence which started half century ago. The first description proposed by Iroshnikov–Kraichnan (1964, 1965 – hereafter IK) is a heuristic model for incompressible MHD turbulence à la Kolmogorov where the large-scale magnetic field fluctuations are supposed to act on small-scales as a uniform magnetic field, leading to counterpropagating Alfvén wave packets whose interactions with turbulent motions produce a slowdown of the nonlinear energy cascade. The typical transfer time through the scales is then estimated as τ_{eddy}^2/τ_A (instead of τ_{eddy} for Navier-Stokes turbulence), where $\tau_{eddy} \sim \ell/u_\ell$ is the eddy turnover time at characteristic length scale ℓ and u_ℓ is the associated velocity. The Alfvén time is the time of collision between two counterpropagating wave packets and is estimated as $\tau_A \sim \ell/B_0$ where B_0 represents the large-scale magnetic field normalized to a velocity ($\mathbf{B_0} \to \mathbf{B_0}\sqrt{\mu_0\rho_0}$). Hence, the energy spectrum in $k^{-3/2}$ unlike the $k^{-5/3}$ Kolmogorov one for neutral flows. The weakness of the IKs phenomenology is the apparent contradiction between the presence of Alfvén waves and the absence of a real uniform magnetic field: the external field is supposed to be played by the large-scale magnetic field but its main effect – anisotropy – is not included in the description.

Two fundamental evolutions in MHD turbulence have been made during the last two decades. There are both concerned with anisotropy. The first one is the conjecture that the refined times $\tau_{eddy} \sim \ell_\perp/u_{\ell_\perp}$ and $\tau_A \sim \ell_\parallel/B_0$ (where $\perp$ and $\parallel$ are respectively the perpendicular and parallel directions to the mean magnetic field $\mathbf{B_0}$) are balanced at all scales (Goldreich & Sridhar, 1995). It leads to the heuristic $k_\perp^{-5/3}$ energy spectrum as well as the relationships $k_\parallel \sim k_\perp^{2/3}$. Whereas the first relation is a trivial consequence of the conjecture, the second prediction reveals a non trivial character of MHD turbulence. The second fundamental evolution is the possibility to handle the effects of a strong B_0 on the MHD dynamics through a rigorous mathematical treatment of wave turbulence which leads asymptotically to a set of integro-differential equations. The exact solution for Alfvén wave turbulence at zero cross-helicity is a $k_\perp^{-2}$ energy spectrum (Galtier et al., 2000). Note that the form of the energy spectrum in the regime of strong turbulence is still the subject of discussions (*e.g.* Galtier et al., 2005; Boldyrev, 2006) although the relationships $k_\parallel \sim k_\perp^{2/3}$ seems to be often verified, whereas the weak turbulence prediction has been obtained recently by two independents set of direct numerical simulations (Bigot et al., 2008b; Perez & Boldyrev, 2008).

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2 2D versus 3D dynamics

In a recent paper (Bigot & Galtier, 2011) we have investigated through high-resolution direct numerical simulations the MHD dynamics for a driven turbulence and in the presence of a uniform magnetic field. In particular, the temporal and spectral properties of the 2D state (eddy turbulence) are investigated. We have shown that if initially the energy contained in the 2D state is set to zero it becomes shortly non negligible in particular when the intensity of B_0 is strong. For our larger B_0 intensity ($B_0 = 15$, with fluctuations b_{rms} of order one) this energy saturates in the stationary phase around 2/3 of the total energy whereas the energy of the 3D modes remains roughly constant. In all situations, the magnetic energy dominates the kinetic energy but it is shown that at large B_0/b_{rms} and in the decay phase the natural state for the 3D modes is the equipartition whereas the 2D state is magnetically dominated.

From a spectral point of view, when the B_0 intensity is strong enough the $k_\perp$–energy spectra are mainly composed at large scales by the 2D state and at small scales by the 3D modes. This situation is similar to rotating turbulence for neutral fluids (Smith & Waleffe, 1999) where in particular the nonlinear transfers are reduced along the rotating rate. However, a detailed analysis of the temporal evolution of the 2D state energy spectra shows that a direct cascade happens whereas for rotating turbulence an inverse cascade is generally evoked. The same remark holds for energy spectra at fixed $k_\parallel > 0$ where we observe additionally a pinning effect at large $k_\perp$. According to the value of B_0/b_{rms} scalings close to $k_\perp^{-3/2}$, $k_\perp^{-5/3}$ or $k_\perp^{-2}$ are found which are in agreement with different predictions for eddy and wave turbulence (Galtier et al., 2000; Boldyrev, 2006).

The external force seems to be an important parameter for the dynamics. For example in Perez & Boldyrev (2008) a change of spectral slope was reported for the $k_\perp$–energy spectrum when the intensity of B_0 is modified. In this work the external force was applied at $k_\parallel > 0$ and $k_\perp = 1, 2$. In Muller et al. (2003) a forcing which kept the ratio of fluctuations to mean field approximately constant was implemented by freezing modes $k \leq 2$ and a spectral slope close $k_\perp^{-3/2}$ was reported for $B_0 = 5$. We find that the 2D state is essential at large B_0/b_{rms} since it mainly drives the nonlinear dynamics. The fact that the 2D state was imposed by the forcing has certainly an impact on the nonlinear dynamics since it prevents the natural growth of the 2D state energy at small $k_\perp$.

3 Conclusion

Waves and turbulence are ubiquitous in astrophysical plasmas. The recent detections of Alfvén waves and their interpretation (Cargill & De Moortel, 2011) shows that they are probably the main ingredient to understand the solar corona dynamics. From a theoretical point of view we know that turbulence is also the natural state of the coronal plasma. Then, it is fundamental to better understand the evolution of MHD turbulence and to answer the question: is MHD turbulence is by nature made of waves or eddies ? The latest results reveals that both are present which means that we need to understand both regimes and their mutual interaction.

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