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NUMERICAL SIMULATIONS OF MAGNETIC RELAXATION IN ROTATING STELLAR RADIATION ZONES

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Abstract. I detail the results of simulations of magnetic relaxation, as is assumed to happen in stellar radiation zones. While previous studies focused on the description of static equilibria, rotation is here included in the numerical setup which substantially modifies the outcome of the simulations. In particular, magnetic equilibria are found, whose configuration remain stable over diffusive timescales. Their privileged axis are inclined with respect to the rotation axis. The stationary internal flows and the dependence of the final equilibrium state on the rotation rate are described. Implications on stellar evolution are discussed.

Keywords: stars: magnetic fields, magnetohydrodynamics (MHD)

1 Introduction

Spectropolarimetric observations routinely reveal the presence of magnetic fields at the surface of a substantial proportion of early type stars, in particular of the chemically peculiar ones. It is thus clear that magnetism is somehow linked with their evolution as it potentially modifies microscopic transport processes, as well as macroscopic ones by interfering with rotation and meridional flows. Besides, current stellar evolution models including all known hydrodynamic transport processes still present difficulties in explaining many observed features, among which internal solar rotation profile recovered from helioseismology, chemical abundances anomalies in intermediate as well as massive stars (nitrogen enrichments in slowly rotating massive stars, hydrogen in type IIn supernovae progenitors, etc.). However, the details of the interplay between magnetic and hydrodynamic processes remain uncertain, not to say totally ununderstood.
It is clear that these magnetic fields often display large-scale organized features, since observations can in general be interpreted in terms of an inclined (the "oblique rotator") dipole model Bagnulo et al. (2002); Donati & Landstreet (2009) with lower contributions of the higher multipoles, though smaller scale features are mostly unresolved. But the way such a large-scale magnetic equilibrium is reached, starting from a field certainly organized on much smaller scales, still lacks of a coherent theoretical frame.
I here present the results of numerical simulations, in order to model the relaxation process occurring from an initial stochastic magnetic field presumably of primordial origin or being the result of a dynamo having occurred earlier, during the convective pre-main-sequence phase and to decipher the physical processes at work during the formation of large-scale equilibrium magnetic field in a stratified region. In both cases, it is taken for granted that the initial field is organized initially on much smaller scales than the final equilibrium; this latter one appears to be a minimum-energy state in agreement with the hydrodynamical constraints on the system. For the first time, rotation is included in the numerical set-up.

2 Numerical Set-up

2.1 Implementation

Use is made of the STAGGER code Nordlund & Galsgaard (1995); Gudiksen & Nordlund (2005), a high-order finite-difference Cartesian MHD code which uses a "hyper-diffusion" scheme. A modest resolution of 128^3 is used here, and is sufficient since one does not need to resolve turbulent features. The code includes Ohmic as well

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as thermal and kinetic diffusion. Using Cartesian coordinates avoids problems with singularities and simplifies the boundary conditions: periodic boundaries are used here. The rotation is implemented through the Coriolis acceleration only, while the centrifugal acceleration is not taken into account to avoid undesired effects arising from the enhanced rotation rate used in the simulation. However, the hierarchy between sound crossing time (τ_s), Alfvén crossing time (τ_A), rotation period (T) and Ohmic diffusive timescale (τ_η) is still well-preserved: $\tau_s \ll \tau_A, T \ll \tau_\eta$. The magnetic energy is kept fixed throughout the different runs; in contrast the rotation rate changes from one run to another, in order to cover a wide range of values spanning from $T \ll \tau_A$ (this is the case e.g. for a typical Ap star, where $\tau_A \simeq 10$ yrs and $T \simeq 100$ days), to $T \gg \tau_A$ (which can easily be reached for slowest rotating stars, assuming the inner field strength is two orders of magnitude higher than that at the surface).

2.2 *Initial conditions*

The star is modeled as a self-gravitating ball of ideal gas ($\gamma = 5/3$) with radial density and pressure profiles initially obeying the polytropic relation $P \propto \rho^{1+(1/n)}$, with index $n = 3$. This approximates a stably stratified radiation zone of an early-type, main sequence star. The influence of a convective core is neglected: owing the high turbulent diffusivity the large scale field penetrates this region easily – it is assumed to be unaffected.
I start from an initial normal distribution for the magnetic energy distribution: $|B| = B_0 \exp(-r^2/2B_w^2)$ with the scale parameter $B_w = 0.35\ R_*$; $|B|^2$,B_0, R_* and r being respectively the magnetic energy, the central magnetic field intensity, the stellar radius and the radius. This distribution and the initial seed for the random distribution in the direction of the stochastic field are chosen so that the resulting field in the non-rotating case is a simple nested torus, to deal with the simplest case. The influence of rotation on the development of non-axisymmetric configurations resulting from different initial conditions will be explored in a forthcoming work.

3 Results

3.1 *General comments*

Including rotation, a set of stationary MHD equilibria similar to the one obtained in the non-rotating case is obtained. Fig. 1a shows the representation of the perpendicular dipole resulting from the relaxation in the slowly rotating regime. The field lines (perpendicular to $\mathbf{B}$) are drawn. The final magnetic configuration is a twisted torus, with a strong toroidal component inside the star while the magnetic field lines in the vicinity of the stellar surface (not shown here) match with a quasi dipolar, potential field at the surface and are thus constrained to be poloidal in all cases. The differences between the static and the stationary equilibrium with rotation are twofolds: (i) the inclination β between the rotation axis and the magnetic privileged axis (torus revolution axis) $\mathbf{B} = \int_\mathcal{V} \mathbf{r} \times \mathbf{B}\, d\mathcal{V}$ varies as a function of the rotation rates, (ii) the internal flows and the differential rotation regime are modified.

3.2 *Settling of the equilibrium*

The evolution covering approximately 40 rotation periods (up to $t = 600$ in arbitrary units) happens in three consecutive phases: (i) the relaxation itself for $0 \leq t \leq 120$; (ii) the stationary evolution once the field has reached the relaxed state for $120 \leq t \leq 450$; (iii) the diffusive limit for $450 \leq t \leq 600$.

Relaxation phase In this first phase, the system evolves from small scale to large scale coherence. The ideal MHD invariants are no longer conserved, since during this phase the magnetic diffusivity η plays a key role in helping the system to organize into a larger scale equilibrium: larger angular modes structures are first smoothed out since $\delta\mathbf{B}/\delta t \simeq -\eta\, m^2\, \mathbf{B}$ in the limit of high azimuthal numbers m.

Stationary evolution During the second phase, some ideal MHD invariants are indeed conserved: magnetic helicity, and higher order invariants are conserved such as the mass enclosed in magnetic flux surfaces. This last invariant was shown by Chandrasekhar & Prendergast (1958) to be conserved in the ideal MHD, axisymmetric case. A closer analysis (to be published in a separate work) reveals that in the rotating stationary case, the equilibrium can still be identified using the same analytical tools as in the static case. It is found that it is of linear, non-force-free type.

Futhermore, inner flows reach a steady state. I detail here the slowly rotating case. Fig. 1b shows cross sections of the poloidal magnetic field lines (isocontours) and of the angular frequency $\Omega = v_\varphi/(r\ \sin\theta)$ (color scale), toroidally averaged (in frame associated with the magnetic field, here with a quasi-perpendicular inclination). One can readily notice a departure to the Ferraro state (Ferraro 1954), with the presence of two counter rotating poloidal flows. Therefore, though on azimuthal average responsible for a solid-body rotation (as was predicted by Moss 1989), these flows mainly concentrated deep inside the star may be responsible for an extra-mixing mechanism associated with the presence of a large-scale magnetic field. This is to be observed in parallel to the recent results obtained from the VLT-Flames Survey, showing nitrogen enrichments in intrinsically slowly rotating massive stars requiring additional sources of extra-mixing (Brott et al. 2011).

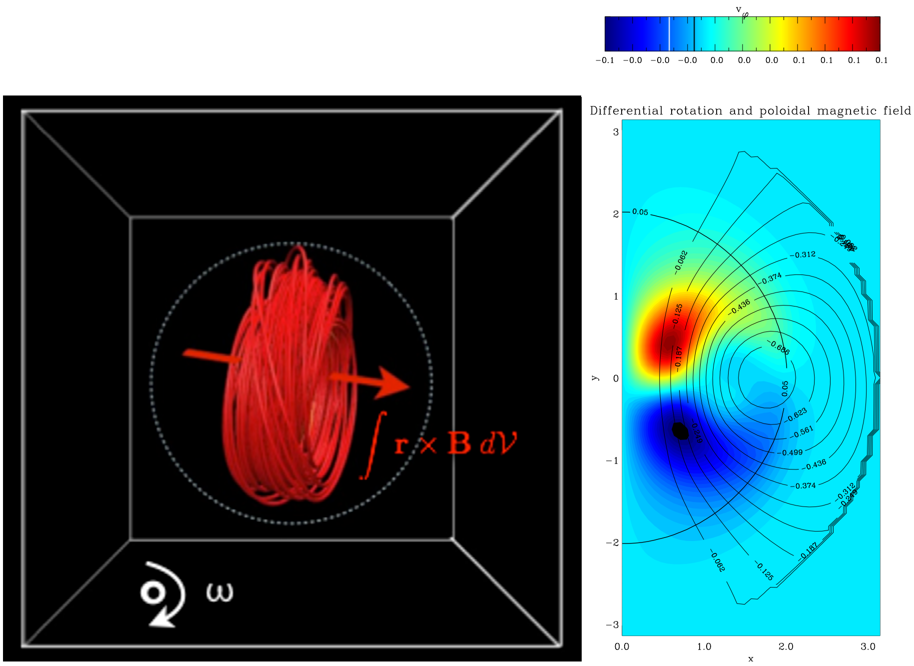

Fig. 1. Left: Three-dimensional representation of magnetic field lines for the perpendicular rotator (dashed line: stellar surface). **Right**: cross-section in the magnetic reference frame (toroidal average) of the azimuthal velocity v_φ (color scale) and the poloidal flux function $\Psi\ r\ \sin\theta$ such that the poloidal field is tangent to Ψ in any point (isocontours). All quantities are normalized to their maximum.

Diffusive limit The third and last phase of the simulation corresponds to the diffusive limit: the neutral line migrates in direction of the surface; the high poloidal field component reaches the surface and the flux is lost from the star. In this limit, the invariants are no longer conserved.

3.3 Final state: magnetic dipole inclination

It is interesting to evaluate the dependency of the inclination angle β, as some tendencies have been reported following recent observations on Ap stars Bagnulo et al. (2002); Mathys (2008). Large inclinations ($\beta \to 90°$) seem to arise in fast rotators ($T < 100\,$d); slow rotators ($100\,\mathrm{d} < T < 1000\,\mathrm{d}$) display a tendency to axisymmetry ($\beta \to 0°$) while for very slow rotators ($T > 1000$ d) β increases again. Fig. 2a shows the evolution of this angle β as a function of the rotation rate imprinted to the model for seven simulations covering seven different rotation rates spanning approximately 8 τ_A. For high rotation rates, the inclination angle goes to 0°, while for low rotation rates, the final state tends to a perpendicular rotator ($\beta \to 90°$). On Fig. 2b is plotted the final inclination angle as a function of the rotation rate. Two distincts behaviours emerge. One below $\Omega = 0.75$, where the final inclination angle decreases linearly from 90° to 0° i.e. from the perpendicular dipole to the aligned one, and above this value a saturated regime with β in the vicinity of 0°.

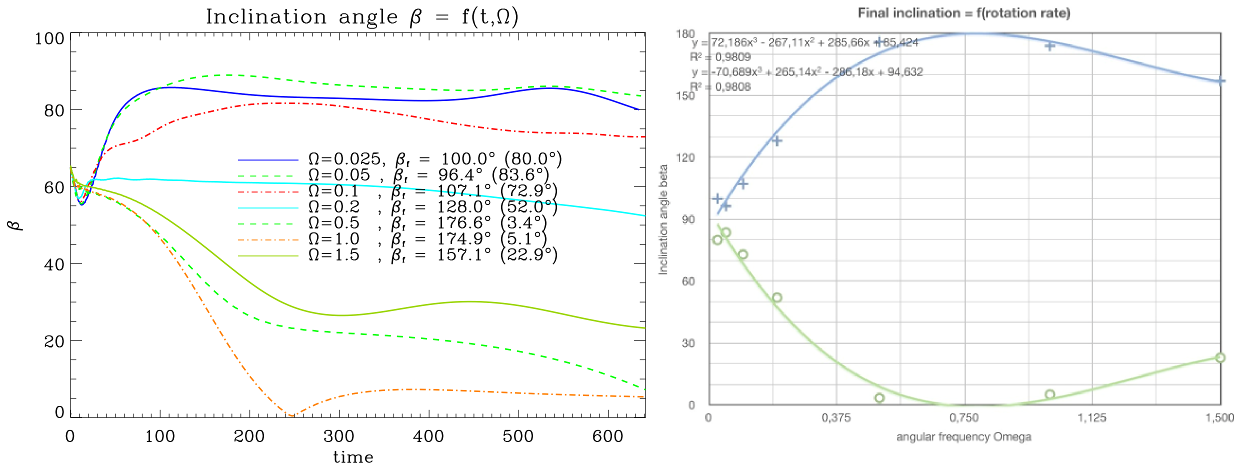

Fig. 2. Left: temporal evolution of the angle β (modulo 180°; actual values given in legend) between the rotation and the magnetic privileged axis. **Right**: final inclination angle β (and modulo 180°) as a function of the rotation rate.

4 Conclusion and perspectives

It has been shown how the numerical relaxation method can be used as a test-case tool to understand magnetic configurations evolution in early-type, main sequence stars. It provides informations about the equilibrium state between differential rotation and meridional flows, which will be important when implementing MHD transport processes in upcoming stellar evolution codes including magnetic fields. In the cases explored here, magnetic dipoles, presenting a toroidal component inside the star result from the relaxation process. Their inclination with respect to the magnetic field depends of the rotation rate: for the simulations performed it is found that slower rotators give perpendicular dipoles while faster rotators give aligned dipoles. Some ideal MHD invariants are shown to be conserved during the relaxed phase, although this still happens in a resistive regime. These invariants are described and allows to identify the type of equilibrium obtained: it shares many similarities with the static equilibria described in recent literature. However, poloidal flows are observed, especially in the slowly rotating case, although the star remains on average in a solid-body rotation regime. These flows might be responsible for an extra source of mixing.

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