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THE EQUILIBRIUM TIDE IN VISCOELASTIC PARTS OF PLANETS

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Abstract. Earth-like planets have viscoelastic mantles, whereas giant planets may have viscoelastic cores. As for the fluid parts of a body, the tidal dissipation of such solid regions, gravitationally perturbed by a companion body, highly depends on the tidal frequency, as well as on the rheology. Therefore, modelling tidal interactions presents a high interest to provide constraints on planet properties, and to understand their history and their evolution. Here, we examine the equilibrium tide in the solid core of a planet, taking into account the presence of a fluid envelope. We explain how to obtain the different Love numbers that describe its deformation. Next, we discuss how the quality factor Q depends on the chosen viscoelastic model. Finally, we show how the results may be implemented to describe the dynamical evolution of planetary systems.

Keywords: planetary systems, dynamical evolution and stability

1 Introduction

Since 1995 a large number of extrasolar planets have been discovered with a large diversity of physical parameters (Santos & et al. 2007). Quite naturally the question arose whether these planets could allow the development of life. Among the conditions which determine the habitability, i.e. the presence of liquid water, many are closely linked with the rotational and orbital elements of the planetary system. How these evolve in time depends mainly on the tidal interactions between the host star and the planet(s); these are very strong in close systems, and they can even modify the structure of the components by internal heating. Provided the system loses no angular momentum, it tends to the state of minimum energy in which the orbits are circular, the rotation of the components is synchronized with the orbital motion, and the spins are aligned. However, in very close systems of star-planet kind, such final state cannot be achieved: instead, the planet spirals toward the star and may eventually be engulfed by it (Hut 1981; Levrard et al. 2009). To predict the fate of a binary system, one has to identify the dissipative processes that achieve the conversion of kinetic energy into heat, from which one may then draw the characteristic times of circularization, synchronization and spin alignment. The extrasolar planets may be classified in two groups: giant planets with a potential rocky core, and telluric planets made of solid and fluid layers. Since tidal mechanism is closely related with the internal structure, one has to investigate its effects on each kind of materials that may compose a planet. The purpose of this study is to determine the tidal dissipation in the solid parts of planets (viscoelastic cores of giant planets & viscoelastic layers of Earth-like planets).

2 The system

Two-layer model. We will consider as a model a two-bodies system where the component A, rotating at the angular velocity Ω, has a viscoelastic core of shear modulus μ, made of ice or rock, surrounded by a fluid envelope, such as an ocean, streching out from core's surface (of mean radius R_c) up to planet's surface (of mean radius R_p). Both core and envelope are considered homogeneous, with constant density ρ_c and ρ_o respectively. This model is represented on the left panel of Fig. 1.

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Configuration. We undertake to describe the tide exerted by B (of mass m_B) on the solid core of A, when moving in an elliptic orbit around A, with eccentricity e, at the mean motion ω. Since no assumption is made on the B's orbit, we need to define an inclination angle I to determine the position of the orbital spin of B with respect to the total angular momentum of the system (in the direction of Z_R) wich defines an inertial reference plane (X_R, Y_R), perpendicular to it. The spin axis of A then presents an obliquity ε with respect to Z_R. Refere to the right panel of Fig. 1 for a synthetic representation of the system configuration.

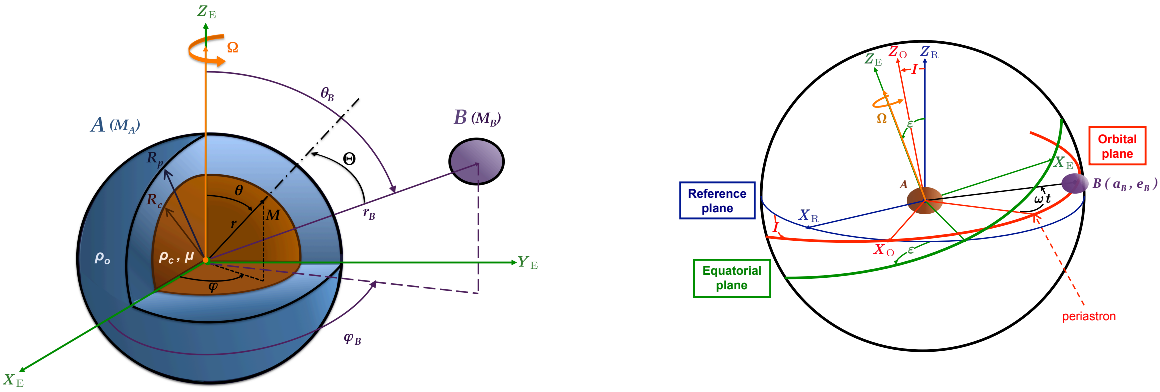

Fig. 1. Left: the system is composed by a two-layer main component A, with an homogeneous and incompressible solid core and an homogeneous static fluid envelope, and a point-mass perturber B orbiting around A. **Right:** B is supposed to move on an elliptical orbit, inclined with respect to the inertial reference plane (X_R, Y_R). The equatorial plane of A (X_E, Y_E) is also inclined with respect to this same reference plane.

We follow the methodology of Dermott (1979).

3 Tidal dissipation

3.1 Definition

In presence of dissipation, there is a lag between the line of centers and the tidal bulge. Then, the response ϕ' of the body to the tidal potential U defines the second order complex Love number $\tilde{k}_2$ whose real part corresponds to the purely elastic deformation, while its imaginary part accounts for dissipation (Biot 1954, see also Tobie 2003 and Henning et al. 2009). U is developed on spherical harmonics (Y_2^m), each term having a large range of tidal frequencies $\sigma_{2,m,p,q} = (2-2p+q)\,\omega - m\Omega$, for $(m, p, q) \in [\![-2, 2]\!] \times [\![0, 2]\!] \times \mathbb{Z}$, resulting from the expansion of U on the Keplerian elements using the Kaula transform (Kaula 1962, see also Mathis & Le Poncin-Lafitte 2009).

This dissipation is quantified by the *quality factor* Q (see for example Tobie 2003):

$$Q^{-1}(\sigma_{2,m,p,q}) = -\frac{\mathcal{I}m\, \tilde{k}_2(\sigma_{2,m,p,q})}{\left|\tilde{k}_2(\sigma_{2,m,p,q})\right|}\,, \quad \text{where} \;\; \tilde{k}_2(\sigma_{2,m,p,q}) = \frac{\phi'(\sigma_{2,m,p,q})}{U(\sigma_{2,m,p,q})}\,. \tag{3.1}$$

Note that expression (3.1) depends on the tidal frequency $\sigma_{2,m,p,q}$.

3.2 Role of the fluid shell

In absence of of a fluid shell surouding the solid core, *i.e.* in the case where the planet is completely solid, the second order Love number, then denoted by $\tilde{k}_2^{\emptyset}$, is expressed by:

$$\tilde{k}_2^{\emptyset}(\sigma_{2,m,p,q}) = \frac{3}{2}\,\frac{1}{1+\bar{\mu}(2, m, p, q)}\,, \tag{3.2}$$

where quantity $\bar{\mu}$, called the *c*omplex effective shear modulus, is linked with the anelasticity and the gravity of the planet's core.

Acting as an overload on the solid core, the fluid shell, previously deformed by the tide, increases the tidal deformation of the core's surface. The second order Love number $\tilde{k}_2$ takes then a different form than in the fully-solid case. Thus, the tidal dissipation rate may be expressed in terms of the complex effective shear modulus $\bar{\mu} \equiv \bar{\mu}_1 + \boldsymbol{i}\,\bar{\mu}_2 = 19\,\mu/(2\,\rho_c\,g_c\,R_c)$, wathever the choosen rheological model, as:

$$Q(\sigma_{2,m,p,q}) = \sqrt{1 + \frac{9}{4\alpha^2 A^2 D^2}\left\{1 + \frac{\left[B + \bar{\mu}_1(\sigma_{2,m,p,q})\right]\left[\frac{2\alpha C}{3} + \bar{\mu}_1(\sigma_{2,m,p,q})\right]}{\bar{\mu}_2(\sigma_{2,m,p,q})}\right\}^2}\,, \tag{3.3}$$

where α, A, B, C and D account for the planet rheology through the ratios of radii R_c/R_p and densities ρ_o/ρ_c.

In 2004, Ogilvie & Lin studied tidal dissipation in rotating giant planets with solid cores, resulting from the excitation of inertial waves in the convective region by the tidal potential. They obtained a decrease of the effective quality factor $Q_{\rm eff} = \left(R_p/R_c\right)^5 Q$, which mesures the dissipation of the whole planet, associated to the fluid equilibrium tide of a fully convective planet (except for a small solid core): from $Q_{\rm eff} = 10^6$ to $Q_{\rm eff} = 10^5$.
Since the composition of giant planets cores is weakly constrained (Guillot 2005), we explore in Fig. 2 a large field of values of the visco-elastic parameters considering the Maxwell rheological model.

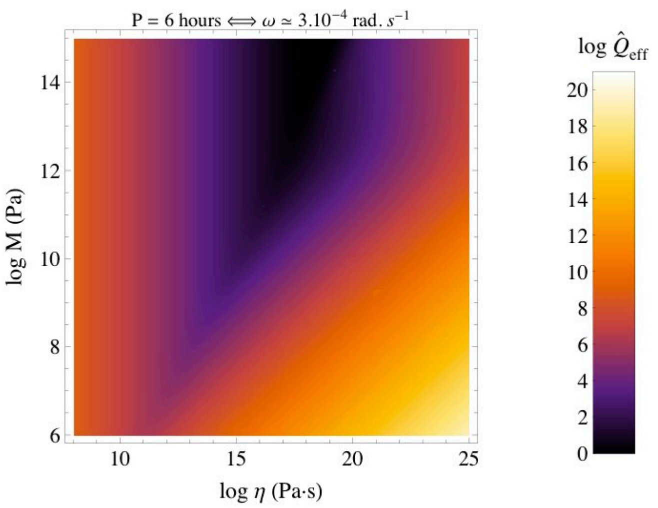

Fig. 2. Dissipation quality factor $Q_{\rm eff}$ in function of the viscoelastic parameters M and η, of a Saturn-like two-layers planet, using the Maxwell model.

The present two-layer model proposes an alternative process to reach such a dissipation (the region above the second contour line $Q_{\rm eff} = 10^5$, in light purple), and even more depending on the viscosity η and the stiffness M (the region above the first contour line $Q_{\rm eff} = 10^4$, in dark purple). These results are obtained for a Saturn-like planet with a planet radius $R_p = 9.140 \times R_\oplus$, a core radius $R_c = 0.2 \times R_p$, a core mass $M_c = 11 \times M_\oplus$ and a fluid envelope density $\rho_o = 0.1 \times \rho_c$ (Guillot et al. 1995; Guillot 1999; Dermott 1979; Tobie 2003).

To explain the tidal dissipation observed in giant planets of our proper Solar System (eg. $Q_{\rm Jupiter} = 3.56 \pm 0.56 \times 10^4$ determinated by Lainey et al. 2009, see also Lainey et al. 2011, for Saturn), all processes have to be taken into account.

4 Dynamical evolution

Due to dissipation, the tidal torque has non-zero average over the orbit, and it induces an exchange of angular momentum between each component and the orbital motion. This exchange governs the evolution of the semi-major axis a, the eccentricity e of the orbit, the inclination I of the orbital plane, of the obliquity ε and that of the angular velocity of each component (see for example Mathis & Le Poncin-Lafitte 2009). Depending on the initial conditions and on the planet/star mass ratio, the system evolves either to a stable state of minimum energy (where all spins are aligned, the orbits are circular and the rotation of each body is synchronized with the orbital motion) or the planet tends to spiral into the parent star. The evolution of the semi-major axis a, of the eccentricity e, of the inclination I, of the obliquity ε and of the angular velocity Ω ($\bar{I}_A$ denotes the moment of inertia of A), is governed by the following equations, derived from Mathis & Le

Poncin-Lafitte (2009) and Remus et al. (2011):

$$
\begin{aligned}
\frac{1}{t_{\rm sync}} &\equiv -\frac{1}{(\Omega-\omega)\,\bar{I}_A}\frac{{\rm d}\left(\bar{I}_A\,\Omega\right)}{{\rm d}t} = \frac{1}{(\Omega-\omega)\;\bar{I}_A}\,\frac{8\pi}{5}\frac{\mathcal{G}M_B^2R_{\rm eq}^5}{a^6}\sum_{m,j,p,q}\left\{\frac{\left|\tilde{k}_2(\sigma_{2,m,j,p,q})\right|}{\hat{Q}(\sigma_{2,m,j,p,q})}\left[\mathcal{H}_{m,j,p,q}(e,I,\varepsilon)\right]^2\right\},\\
\frac{1}{t_{\rm circ}} &\equiv -\frac{1}{e}\frac{{\rm d}e}{{\rm d}t} = \frac{1}{\omega}\,\frac{1-e^2}{e^2}\,\frac{4\pi}{5}\,\frac{\mathcal{G}M_BR_{\rm eq}^5}{a^8}\sum_{m,j,p,q}\left\{\left[(2-2p)\left(1-\frac{1}{\sqrt{1-e^2}}\right)+q\right]\frac{\left|\tilde{k}_2(\sigma_{2,m,j,p,q})\right|}{\hat{Q}(\sigma_{2,m,j,p,q})}\left[\mathcal{H}_{m,j,p,q}(e,I,\varepsilon)\right]^2\right\},\\
\frac{1}{t_{\rm align_A}} &\equiv -\frac{1}{\varepsilon}\frac{{\rm d}\varepsilon}{{\rm d}t} = \frac{1}{\varepsilon\,\sin\varepsilon}\,\frac{{\rm d}\left(\cos\varepsilon\right)}{{\rm d}t} = \frac{1}{\varepsilon\,\sin\varepsilon}\,\frac{1}{\bar{I}_A\,\Omega_A}\,\frac{4\pi}{5}\,\frac{\mathcal{G}M_B^2R_{\rm eq}^5}{a^6}\sum_{m,j,p,q}\left\{(j+2\cos\varepsilon)\,\frac{\left|\tilde{k}_2(\sigma_{2,m,j,p,q})\right|}{\hat{Q}(\sigma_{2,m,j,p,q})}\left[\mathcal{H}_{m,j,p,q}(e,I,\varepsilon)\right]^2\right\},\\
\frac{1}{t_{\rm align_{Orb}}} &\equiv -\frac{1}{I}\,\frac{{\rm d}I}{{\rm d}t} = \frac{1}{I\,\sin I}\,\frac{{\rm d}\left(\cos I\right)}{{\rm d}t} = \frac{1}{I\,\sin I}\,\frac{1}{\omega}\,\frac{1}{\sqrt{1-e^2}}\,\frac{4\pi}{5}\frac{\mathcal{G}M_B^2R_{\rm eq}^5}{a^8}\\
&\times\sum_{m,j,p,q}\left\{\left[j+(2q-2)\cos I\right]\,\frac{\left|\tilde{k}_2^F(\sigma_{2,m,j,p,q})\right|}{\hat{Q}(\sigma_{2,m,j,p,q})}\left[\mathcal{H}_{m,j,p,q}(e,I,\varepsilon)\right]^2\right\},\\
\frac{1}{a}\,\frac{{\rm d}a}{{\rm d}t} &= -\frac{2}{\omega}\,\frac{4\pi}{5}\,\frac{\mathcal{G}M_BR_{\rm eq}^5}{a^8}\sum_{m,j,p,q}\left\{(2-2p+q)\,\frac{\left|\tilde{k}_2^F(\sigma_{2,m,j,p,q})\right|}{\hat{Q}(\sigma_{2,m,j,p,q})}\left[\mathcal{H}_{m,j,p,q}(e,I,\varepsilon)\right]^2\right\},
\end{aligned}
$$

where $\mathcal{H}(e,I,\varepsilon)$ is proportional to the product of Kaula's functions that intervene in the expansion of the tidal potential in spherical harmonics, and $R_{\rm eq}$ is the equatorial radius of A (in general, $R_{\rm eq}\neq R_p$).

5 Conclusion

Our preliminary evaluations confirm the results of Dermott (1979), and they reveal a much higher dissipation in the solid cores of planets than that found by Ogilvie & Lin (2004) for the fluid envelop of a planet having a small solid core. These results seem to be in good agreement with observed properties of Saturn's system (Lainey et al. 2011).

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