SF2A 2012
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COUPLING BETWEEN COROTATION AND LINDBLAD RESONANCES

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Abstract. We consider the classical Elliptic Restricted Three-Body Problem with two bodies (particle and satellite) orbiting a central planet. If we take into account the oblateness of the central body through the classical additional terms up to J_6, the secular terms causing the orbit precessions appear in the disturbing potential leading to the presence of two critical resonant arguments : $\phi = (m+1)\lambda' + m\lambda + \varpi$ and $\phi' = (m+1)\lambda' + m\lambda + \varpi'$, where m is an integer, λ and ϖ the mean longitude and the longitude of the periapsis of the particle, and the primed quantities apply to the satellite. The arguments ϕ' and ϕ respectively describe the Corotation Eccentric Resonance (CER) and the Lindblad Eccentric Resonance (LER). We developed a simple model (the CoraLin model) which encapsulate in a simple adimensional form the coupling between the two resonances. We examine the asymptotic configurations where these resonances are well separated or completely superimposed.

Keywords: Celestial mechanics

1 Introduction

We consider the classical problem of two bodies (the satellites) orbiting a central body (the planet), near a mean motion resonance. The simplest case occurs when the two satellites orbit in a common plane, and have masses μ and μ' possibly non-zero, but much smaller than the planet mass, M. From d'Alembert rules, two critical resonant angles, ϕ and ϕ', appear in the problem, see Eq. (2.1). There is a considerable amount of literature for the case $\mu = 0$ and $e' = 0$ (the planar, restricted and circular three-body problem), which has led to the so-called second fundamental model of resonance associated. For $\mu = 0$ and $e' \neq 0$, and under some simplifying hypotheses, another kind of resonances occurs, called the corotation resonances, akin to the 1:1 behavior of a particle near the L_4 and L_5 Lagrange points. In general, the two resonances associated with the angles ϕ and ϕ' are strongly coupled because they occurs at nearly the same semi-major axis, due to the usually small value of the planet oblateness. Here write the simplest system of differential equations that describes this coupling, using non-dimensional variables that permit applications in a wide variety of situations.

2 Hamiltonian formalism

We consider a system of two satellites of masses μ and μ' revolving around a planet of mass M in a common orbital plane, with $\mu, \mu' \ll M$. The masses μ, μ' and M will denotes at the same times the bodies and their mass. We assume that $\mu \to 0$, and we will call μ the "particle" and μ' the "satellite" that perturbs the particle. We use the classical notations a, e, λ, n and ϖ for the geometric semi-major axis, orbital eccentricity, mean longitude, mean motion and longitude of periapsis of μ. Similar primed notations are used for μ'. We consider a situation where μ and μ' are near a first order mean motion resonance $m+1$:m, where m is an integer such that $n/n' \approx (m+1)/m$. Near such first order resonance, d'Alembert's rules implies that the arguments of resonances are given by :

$$\begin{aligned} \phi =& \quad (m+1)\lambda' - m\lambda - \varpi \\ \phi' =& \quad (m+1)\lambda' - m\lambda - \varpi' \end{aligned} \tag{2.1}$$

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The situations $\dot{\phi} = 0$ and $\dot{\phi}' = 0$ correspond to the exact resonances associated with ϕ and ϕ'. where we can write the equations of motion, after approximate renormalization of the variables (El Moutamid et al *in prep*):

$$\left\{ \begin{array}{lcl} \frac{dJ_c}{d\tau} & = & -\sin(\phi') \\ \frac{d\phi'}{d\tau} & = & J_c - J_L \\ \frac{dh}{d\tau} & = & -(J_c - J_L + D)\cdot k \\ \frac{dk}{d\tau} & = & +(J_c - J_L + D)\cdot h + \epsilon_L, \end{array} \right. \tag{2.2}$$

The corresponding Hamiltonian of the motion is

$$\mathcal{H} = \frac{(J_c - J_L)^2}{2} - \cos(\phi') - DJ_L - \epsilon_L h \tag{2.3}$$

Where J_c is the constant of Jacobi, J_L is proportional to $\frac{e^2}{2}$, $e = (h, k)$ is the the eccentricity vector, D is the distance between the Corotation and the Linblad resonances, and ϵ_L is the forcing value of the eccentricity due to the effect of Lindblad resonance. This is the simplest system that describes the combined effect of the corotation and Lindblad resonances acting on a particle. For this reason, we call it the "coraLin" model. It depends on only two dimensionless parameters, D and ϵ_L. We note that the full width of the unperturbed corotation resonance is $\Delta\chi = \Delta(J_c - J_L) = \pm 2$, and that the unperturbed Lindblad resonance occurs at $\chi = -D$, see Fig. (1)).

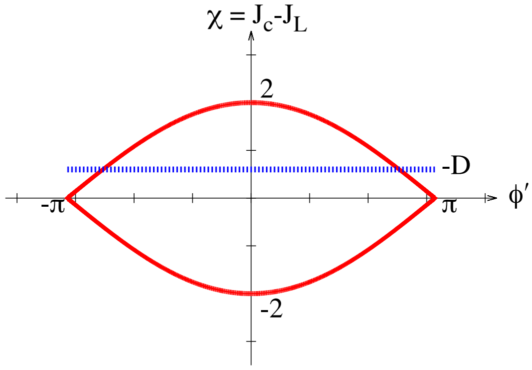

Fig. 1. System combined effect of Corotation (red) and Lindblad (blue).

3 Asymptotic behaviors

If $D = 0$ (the two resonances are superimposed), the system is integrable.
We now consider $|D| \gg 2$ i.e. when the Lindblad resonance occurs far away from the corotation region. There are two cases :

1) The particle is trapped in the corotation radius, the variations of (h, k) in the systems (2.2) are much faster than the variations of J_c and ϕ'. Consequently, the action $\oint hdk$ is adiabatically conserved. Since (h, k) essentially describe a circle centered on the forced value $(-\epsilon_L/(\chi + D), 0)$, this means that (h, k) rapidly move on a circle of constant radius, whose center slowly moves along the Ok axis. In particular, if (h, k) starts at the forced value $(-\epsilon_L/(\chi + D), 0)$, then it will stay at that value as χ slowly changes. In other words, the orbital eccentricity of the particle will permanently adjust itself so that $e = |\epsilon_L/(\chi + D|$ as χ varies.

2) The particle is trapped in the Lindblad radius, the situation is reversed, the variations of (J_c, ϕ') in the systems (2.2) are much faster than the variations of (h, k). Thus $\oint J_c d\phi'$ is adiabatically conserved.

4 Conclusions

We built a dynamic system which describes in a generic way the coupling between the Corotation and Lindblad resonances, we examined the asymptotic cases where the two resonances are almost superimposed, and the case where the two resonances are well separated, leading to a general solution based on adiabatic invariance arguments. Our 'toy model' incorporates all the ingredients of interacting CER/LER, in a non dimensional way. It can be use as a fast numerical tool to explore the probability of capture into the Corotation Resonance, in divers situations (Saturn's satellite, Neptune's arc ...).

The authors thank the organizers of the “Journées de la SF2A 2012”, P. Robutel for the interesting discussions during this work and the EMERGENCE-UPMC project.

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