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S. Boissier, P. de Laverny, N. Nardetto, R. Samadi, D. Valls-Gabaud and H. Wozniak (eds)

INFLUENCE OF THE MEASUREMENT PROCESSING IN THE DETERMINATION OF THE EQUIVALENCE PRINCIPLE VIOLATION SIGNAL FOR THE MICROSCOPE EXPERIMENT

E. Hardy[1], A. Levy[1], G. Métris[2], A. Robert[3], M. Rodrigues[1] and P. Touboul[1]

Abstract. The MICROSCOPE space mission aims at testing the Equivalence Principle (EP) with an accuracy of 10^{-15}. The test is based on the precise measurement delivered by a differential electrostatic accelerometer onboard a drag-free satellite which includes two cylindrical test masses submitted to the same gravitational field and made of different materials. This high precision experiment is compatible with only very little perturbations. But the mathematical process applied to extract the signal at the EP violation frequency introduces numerical effects which perturb the measurement analysis. Aliasing arises from the finite time span of the measurement, and is amplified by possible small irregularities in the sampling due to telemetry losses. Numerical simulations have been run to estimate the projection rate of a perturbation at any frequency on the EP violation frequency and to test its compatibility with the mission specifications. Moreover, different procedures for the data analysis have been considered to select the one minimizing these effects taking into account the uncertainty about the frequencies of the implicated signals. After an overall presentation of the MICROSCOPE mission, this paper will focus on the numerical perturbations introduced during the data processing of the scientific measurement and describe the considered methods to minimize them.

Keywords: Techniques: miscellaneous, Cosmology: miscellaneous

1 Introduction

The Equivalence Principle (EP) is at the basis of General Relativity and states the Universality of Free Fall, that is to say that the acceleration of an object in a gravitational field is independent of its mass and its internal composition. The Universality of Free Fall has been tested throughout the centuries with an improving accuracy thanks to the Lunar Laser Ranging method or sophisticated torsion-balances. The latter have led to a record accuracy of a few 10^{-13} (Schlamminger et al. (2008)). However, the accuracy of these on-ground experiments is limited by the numerous perturbations of the terrestrial environment. In addition, some unification theories which try to merge gravitation with the three other fundamental interactions expect a violation of the EP below 10^{-14} (Damour et al. (2002)). Being performed in space, the MICROSCOPE mission will be able to test the Equivalence Principle with an accuracy of 10^{-15}.

To reach the high precision objective of the MICROSCOPE experiment, it is necessary to determine and reduce at best every perturbation. This paper focuses on the numerical effects which perturb the observation of the EP signal during the measurement processing. The time span of the measurement is necessarily finite and may not equate with a whole number of the perturbations period. That is why these perturbations are spectrally spread. A perturbation at any frequency can therefore have a component with a given projection rate at the EP frequency. Moreover, the projection rates are amplified by the accidental measurement losses. The projection values have to be estimated by taking into account the uncertainties on the considered signal frequencies in order to check their compatibility with the mission specifications.

After a general overview of the MICROSCOPE space mission, this paper will focus on the influence of the observation window which introduces the numerical effects. The influence of the measurement losses and the different methods investigated to manage them will then be presented.

[1] ONERA, 92322 Châtillon, France
[2] Université de Nice Sophia-Antipolis, Centre National de la Recherche Scientifique (UMR 6526), Observatoire de la Côte d'Azur, Géoazur, 06130 Grasse, France
[3] CNES, 31401 Toulouse, France

2 The MICROSCOPE space mission

MICROSCOPE is a 200 kg satellite developed by CNES to orbit around the Earth for a 18 months mission. The onboard payload is composed of two differential electrostatic accelerometers developed by ONERA, each one being composed of two imbricated cylindrical test masses. The masses positions are detected thanks to a capacitive method and a control loop with electrostatic actuation keeps the masses motionless at the centre of the accelerometer cage, so that they both follow the same trajectory. For one of the accelerometer, the two masses are made of different materials (titanium and platinum). A difference measured between the forces applied to maintain them on the same trajectory would therefore indicate a violation of the Universality of Free Fall (a direct consequence of the EP). The second accelerometer is composed of two test masses with the same composition and enables to assess the experiment accuracy.

There are several advantages to perform the experiment in space. The experiment is not limited by the free fall duration: it can last for several orbits. Moreover, the environment is much less disturbed than on Earth particularly because a drag-free system compensates for perturbations common to the two test masses (the so called common mode), enabling to decrease the dynamics of the individual signals. Lastly, there is no large mass close to the experiment and the Earth's gravity gradient effects are small and easy to correct.

The frequency and phase of the signal to be detected are well defined, corresponding to the Earth gravity signal. The satellite pointing can either be inertial or spinning. In the first case, the Earth gravity field is modulated by the orbital frequency and the signal frequency is: $f_{EP_i} = f_{orb}$. In the second case, the signal frequency f_{EP_s} is the sum of the spin and orbital frequencies: $f_{EP_s} = f_{orb} + f_{spin}$. In comparison, the signal frequency is increased and thus closer to the minimum of the instrumental noise.

3 Influence of the observation window

3.1 Determination of the projection rate

The Fourier transform of a sine signal of infinite duration is composed of only one spectral line: the total energy is concentrated at the sine frequency. However, the Fourier transform applied on a finite duration which does not correspond to an exact number of periods will lead to a spread spectrum.

The measurement provided by the MICROSCOPE instrument is necessarily of finite duration. That is why the perturbation signals are subjected to this spectral spread. Therefore, a perturbation at a frequency different from the EP frequency may have a component at this frequency.

Our objective is to determine the value of the projection rate of a perturbation at any frequency on the EP violation frequency to compare the result with the mission specifications. Let S_{obs} be our observation signal and S_{EP} the model, the equation to be solved in order to estimate the EP violation term δ is:

$$S_{obs} = \delta S_{EP}$$

The Least Squares estimated solution is:

$$\hat{\delta} = \frac{\langle S_{EP}, S_{obs}\rangle}{\langle S_{EP}, S_{EP}\rangle}$$

Adding a disturbing signal S_d, the parameter to be estimated will be modified by:

$$\Delta\delta = \frac{\langle S_{EP}, S_d\rangle}{\langle S_{EP}, S_{EP}\rangle}$$

The signals S_{EP} and S_d are now considered to be sine signals:

$$S_{EP} = A_{EP} sin(\omega_{EP} t + \phi_{EP})$$

$$S_d = A_d sin(\omega_d t + \phi_d)$$

The Least Squares Method leads to the same solution when applying a discrete Fourier Transform to each side of the equation. The scalar product can therefore be computed in the Fourier domain: this method is used for the numerical simulations. The scalar product can equally be computed in the temporal domain. To correspond to the discrete measurement, the scalar product is discrete and defined on a finite number of frequencies. However, it is possible to approximate it with a continuous integral for an analytical evaluation.

Table 1. Projection rate and special specifications for some of the linear combinations of f_{orb} and f_{spin} in spinning pointing.

Frequency	Projection rate	Specification
f_{orb}	9.8×10^{-6}	10^{-4}
$f_{spin} - 2f_{orb}$	3.6×10^{-5}	3.3×10^{-4}
$2f_{orb}$	2.3×10^{-5}	2.5×10^{-4}
$f_{spin} - f_{orb}$	3.2×10^{-5}	3.3×10^{-4}
$3f_{orb}$	4.8×10^{-5}	5×10^{-4}
f_{spin}	3.2×10^{-5}	3.3×10^{-4}
$f_{spin} + f_{orb}$	1	1
$f_{spin} + 2f_{orb}$	7.9×10^{-5}	10^{-3}

The resulting projection rate has been plotted as a function of the perturbation frequency f_d in figure 1, using a rectangular window. This window has been chosen because it provides worst case results in comparison with apodisation windows. The results depend on the two signals phases ϕ_{EP} and ϕ_d. The curve therefore represents a maximisation of the projection rate as a function of these two parameters, in order to reach the worst case. The projection rate oscillates as a function of the perturbation pulsation ω_d, between null and maximal values limited by an envelope which varies as the inverse of the measurement duration.

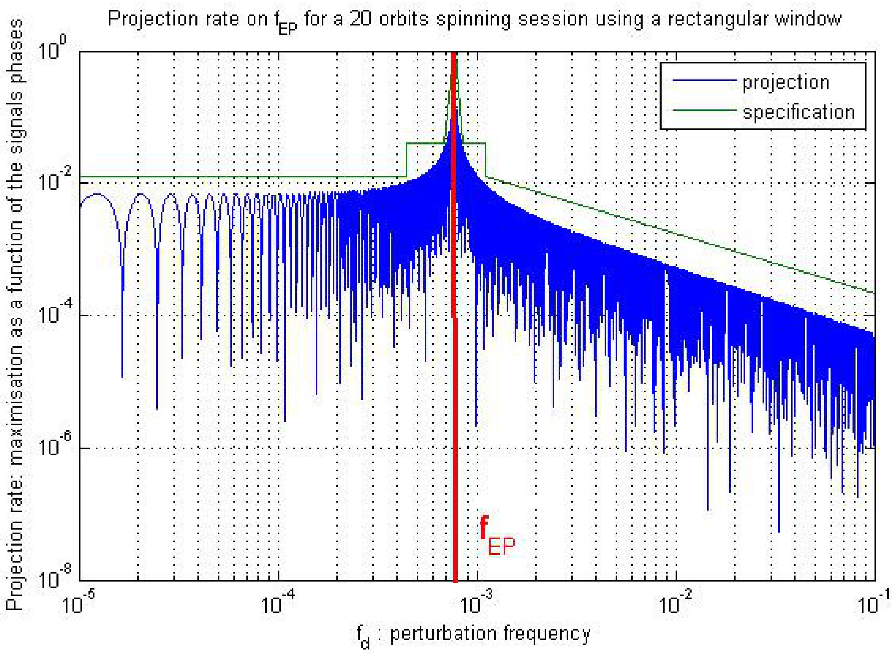

Fig. 1. Projection rate of a pertubation signal on the Equivalence Principle signal as a function of the perturbation frequency.

3.2 Choice of the measurement duration and spin frequency

The inertial and spinning motions of the satellite induce important perturbations. Most of the largest perturbations are associated to fundamental frequencies: multiples of the orbital frequency in inertial pointing and linear combinations of the orbital and the spin frequencies in spinning pointing. Very restrictive specifications, presented in the third column of table 1, have therefore been defined for the projection rate of the signals at the discrete frequencies impacted by these effects. We define the duration of the analysis window T, the orbital period T_{orb} and the spin period T_{spin}. If the relation $T = k_1 T_{orb} = k_2 T_{spin}$, k_1 and k_2 being integers, holds, then the linear combinations of f_{spin} and f_{orb} correspond to the minima of the projection rate curve in figure 1. In case of inertial pointing, we choose $k_1 = 120$ corresponding to sessions of about 8.3 days. In case of rotating pointing, we choose $k_1 = 20$ corresponding to sessions of about 1.4 days; this sets $T_{spin} = \frac{k_1}{k_2} T_{orb}$. k_2 will be chosen between 70 and 100 in order to get a spin frequency larger than the orbital frequency and an EP frequency closer to the minimum of the instrument noise (this justifies the shorter integration time for sessions with a rotating pointing).

However, the frequencies of the implicated signals are not perfectly known. The uncertainty on the orbit determination causes an error on the orbital frequency. Moreover, the inertial and the spinning pointing cannot be exactly realised. Because of the uncertainties, the reality does not exactly match with the ideal case leading to non null projection rates. Column 2 of table 1 presents the worst projection rates computed using shifts of 2×10^{-8} rad/s on the orbital frequency with respect to the ideal case, 3×10^{-8} rad/s on the spin frequency and 1×10^{-8} rad/s on the inertial pointing realisation, and demonstrates the compatibility with the specifications.

4 Measurement losses

During the data transmission from the satellite to the ground station, a very small part of the data can be accidentally lost. The data loss may happen during the data transmission from the satellite to the ground station. Part of the data could be recollected during the next fly by, but it is impossible to guarantee a 100% recollection. The experience of the PICARD mission provides a good estimation of the losses frequency, since the MICROSCOPE satellite will follow nearly the same orbit and use the same station network. For 10 months, about 100 measurement losses happened, so the probability of such an event to happen during an orbit is about 2%; their duration ranges from a second to a few hours. An other source of data loss arises from the crackings caused by the gas contained in the six thrust tanks. The worst case is dependant on the gas pressure and the affected tanks surface and corresponds to about 40 crackings shorter than one second per orbit.

Measurement losses tend to increase the projection rate of the perturbations on the EP violation signal. For example monochromatic signals of frequencies j_1/T and j_2/T, j_1 and j_2 being integers, are orthogonal (their discrete scalar product is null) in case of regular sampling but this is no longer true in case of irregular sampling (or equivalently regular sampling with gaps). Different procedures for the data processing have been determined depending on the duration of the measurement loss.

4.1 Short measurement losses (shorter than a minute)

The projection rate in case of missing data can be computed by replacing the lost data with null values. The blue curve of figure 2 and the second column of table 2 show that the specifications are no longer satisfied when using a model of the distribution of the tank crackings. To overcome this difficulty, a simple solution consists in replacing the lost data by the mean value of the signal before and after the measurement loss. The green curve of figure 2 and the third column of table 2 show that this method is compatible with the specifications. The conclusions relating to the spinning mode are also valid for the inertial mode.

The tank crackings are numerous - about 40 per orbit - but shorter than one second. For longer measurement losses duration due to data transmission failure, the same procedure is used, but only one measurement loss whose duration is shorter than a minute is accepted per 20 orbits.

4.2 Long measurement losses (longer than a minute)

The probability that a measurement loss longer than a minute appears during a spinning mode session is weak, because the duration of the session is only 20 orbits. But for the inertial mode sessions, whose duration is 120 orbits, the probability to have at least one measurement loss longer than a minute reaches 16%. It is therefore necessary to have a method to deal with longer measurement losses.

The selected method consists in the elimination from the measurement of the entire orbit which contains the data loss, so that the duration of each data portion corresponds to a whole number of orbits. The data portions are then stick together again. The time span of the measurement for a 120 orbits inertial session is therefore reduced by a few orbits, which is still acceptable regarding the noise level. To test the compatibility of this method with the specifications, we have determined the worst case for the measurement losses duration and disposition: regular distributed losses. Analytical analysis confirmed by numerical simulations has shown that in this case the amplitude of the envelope of the projection rate as a function of the perturbation frequency

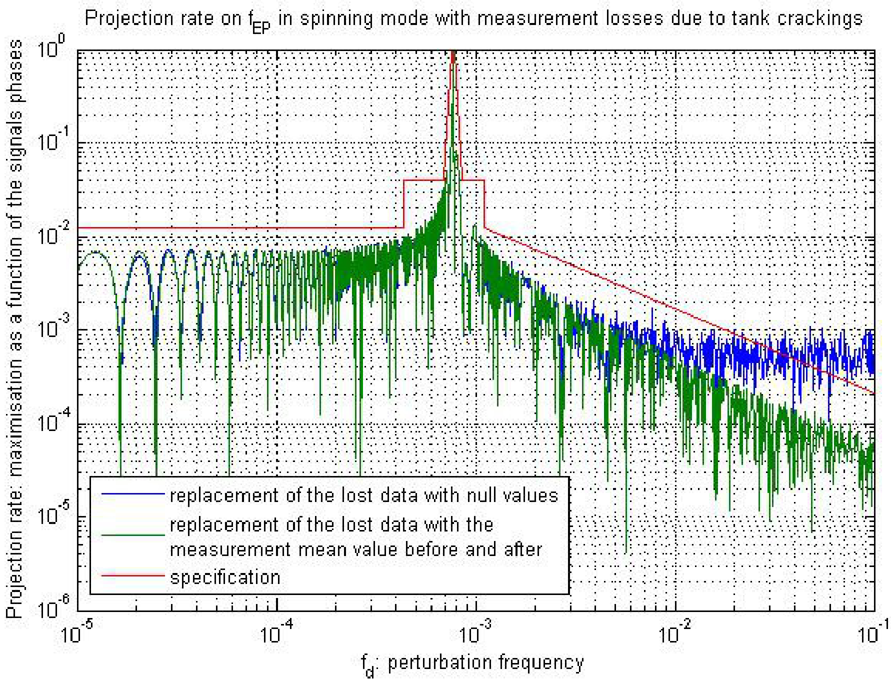

Fig. 2. Projection rate of a pertubation signal on the Equivalence Principle signal as a function of the perturbation frequency with a measurement losses scheme corresponding to the model of the tanck crackings.

Table 2. Projection rate and special specifications for some of the linear combinations of f_{orb} and f_{spin} in spinning pointing with a measurement losses scheme corresponding to the model of the tanck crackings. Method 1: replacement of the lost data with null values; method 2: replacement of the lost data by the mean value of the signal before and after the measurement loss.

Frequency	Projection rate: method 1	Projection rate: method 2	Specification
f_{orb}	7.4×10^{-4}	9.8×10^{-6}	10^{-4}
$f_{spin} - 2f_{orb}$	4.2×10^{-4}	3.6×10^{-5}	3.3×10^{-4}
$2f_{orb}$	6.0×10^{-4}	2.3×10^{-5}	2.5×10^{-4}
$f_{spin} - f_{orb}$	3.8×10^{-4}	3.2×10^{-5}	3.3×10^{-4}
$3f_{orb}$	1.8×10^{-4}	4.8×10^{-5}	5×10^{-4}
f_{spin}	6.1×10^{-4}	3.2×10^{-5}	3.3×10^{-4}
$f_{spin} + f_{orb}$	1	1	1
$f_{spin} + 2f_{orb}$	5.5×10^{-4}	8.0×10^{-5}	10^{-3}

is proportional to the inverse of the duration of one subsession and not to the inverse of the total duration of the session.

To guarantee a failure probability lower than 1%, it is necessary to be able to deal with three measurement losses longer than a minute. The worst case therefore corresponds to four subsessions of 30 orbits. The results of the numerical simulation presented in figure 3 are compatible with the specification. The efficiency of the method is therefore validated.

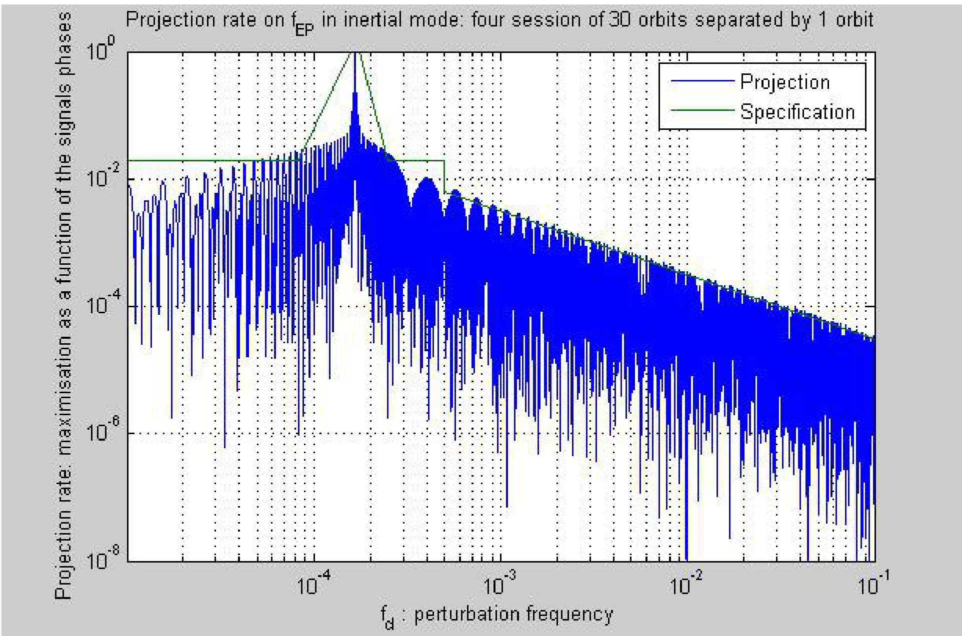

Fig. 3. Projection rate of a pertubation signal on the Equivalence Principle signal as a function of the perturbation frequency: four 30 orbits sessions separated by one orbit.

5 Conclusions

For the success of the MICROSCOPE space mission, a crucial problem is to limit the perturbations which appear during the measurement analysis. Because of the finite duration of the measurement window, a perturbation at any frequency can have a projection at the EP frequency. It is therefore necessary to adjust the frequencies corresponding to the main perturbations, mainly the orbital and the spin frequency, to get a minimal projection. However, the projection effects are amplified by the frequencies uncertainties. Numerical simulations of the projection rate taking into account these uncertainties have proved the result to be compatible with the specifications. The measurement losses increase the projection rate on the EP frequency. To deal with numerous very short losses (shorter than a second) or one loss up to a minute, the missing data are replaced by the mean value of the measurement before and after the interruption. For measurement losses longer than a minute, the inertial session is separated in several independent subsessions. Thanks to these procedures, the success probability of the mission reaches a level compatible with the specification. The rejection can still be considerably improved by using apodisation windows, like the Blackman or the Hann window, but the specifications are reached even in the worst case of a rectangular window.

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