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COMPRESSIBLE TURBULENCE: A DIFFERENT PHYSICS ?

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Abstract. A possible phenomenological view is proposed for compressible hydrodynamic turbulence under isothermal closure. In the inertial zone a cascade for total energy is expected to take place due to the i) fluctuations transported by the fluid itself in subsonic case, leading to a -5/3 spectrum for density weighted turbulent velocity field ($\delta\rho^{1/3}\delta\mathbf{v}$) and ii) fluctuations transported by the sonic waves in the supersonic case, leading to a -2 spectrum for turbulent velocity field ($\delta\mathbf{v}$). A generalisation can be anticipated in polytropic case pursuing this given approach.

Keywords: Compressible, interstellar medium, phenomenology, solar wind, turbulence.

1 Introduction

Turbulence in compressible fluids (neutral or charged) is extremely complicated to be understood clearly (analtically or numerically). In astrophysics we have, however, the media like the solar plasma or the interstellar clouds which are (highly) compressible and turbulent (Elmegreen & Scalo 2004; Carbone 2012). It is therefore needed that the corresponding physics be governed by the compressible turbulence. The present article is prepared in order to give a complementary view with respect to our article of last year in this proceedings. While the article Galtier & Banerjee (2011a) presented the spectral aspects of compressible hydrodynamic turbulence (under isothermal condition) revealed from a derived exact relation (Galtier & Banerjee 2011b) and using Kolmogorov's phenomenology, here we shall try to understand those spectra by proposing a different phenomenology (of course the idea is underlying in the said exact relation) for three dimensional fully developped turbulence. For incompressible neutral fluids, turbulence is described by the phenomenology of Kolmogorov which is based on the famous concept of Richardson's energy cascade that takes place owing to the successive fragmentation of the fluid vortices (or eddies) to smaller vortices. In case of incompressible MHD fluids, the admitted phenomenology is based on the Alfvén effect where the scaling law (or the cascading of energy) is caused by the deformations of the fluctuating Alfvén modes due to sporadic interactions of two oppositely propagating fluctuating Alfvén modes. In the first case we expect a -5/3 energy (total or kinetic) spectrum (which is the famous Kolmogorov spectrum) and for the second case we predict a -3/2 energy spectrum (which is known as the Iroshnikov–Kraichnan spectrum). The situation becomes far more complicated for compressible turbulence. For neutral fluids, we can no more think of pure eddies as the compressible fluid velocity field is not solenoidal. Similarly, in case of compressible MHD turbulence, we have to abandon the idea of Alfvén effect because for compressible MHD fluids the linear wave modes consist of two compressional modes (magnetosonic waves) with Alfvén mode which is an incompressible mode. The only existing phenomenology for compressible case is using the Burger's equation which is a one-dimensional equation and predicts a -2 velocity (v^2) spectrum for supersonic turbulence (for neutral fluids) using the notion of shock formation (Frisch 1995). This phenomenology does not describe the subsonic regime. Recent numerical simulations (Kritsuk et al. 2007) predict a -5/3 spectrum for subsonic turbulence and a -2 spectrum for supersonic turbulence for three dimensional (3D) compressible isothermal turbulence. A satisfactory phenomenological description for 3D compressible fluid turbulence (and so for MHD) is yet to be developped. In the following section we shall give a possible phenomenological approach (without using directly the shock formation) for understanding the spectra of different regimes (subsonic and supersonic).

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2 Phenomenology for compressible hydrodynamic turbulence

We start with our derived exact relation which is written as

$$-2\varepsilon = \langle(\nabla' \cdot \mathbf{v}')(\mathbf{R}-\mathbf{E})\rangle + \langle(\nabla \cdot \mathbf{v})(\tilde{\mathbf{R}}-\mathbf{E}')\rangle + \nabla_{\mathbf{r}} \cdot \left\langle \left[\frac{\delta(\rho\mathbf{v})\cdot\delta\mathbf{v}}{\mathbf{2}} + \delta\rho\delta\mathbf{e} - \mathbf{C_s^2}\bar{\delta}\rho\right]\delta\mathbf{v} + \bar{\delta}\mathbf{e}\delta(\rho\mathbf{v})\right\rangle , \quad (2.1)$$

the first two terms of right hand side present the source terms (exclusive for compressible turbulence) whereas the third one represents the flux term. We note that the source terms are having $(\nabla\cdot\mathbf{v})$ (or $\nabla'\cdot\mathbf{v}'$) as coefficient. Neglecting the dispersive effect of the flow field, we can show (by the basic equations) that $(\nabla\cdot\mathbf{v})$ propagates with a constant phase velocity C_S (sound speed in the corresponding turbulent medium). The source terms can then be written as

$$S(r) \sim \left\langle (\nabla\cdot\mathbf{v})(\frac{\mathbf{1}}{\mathbf{2}}\rho'\mathbf{v}'.\delta\mathbf{v})\right\rangle \sim \langle(\nabla\cdot\delta\mathbf{v})(\rho'\mathbf{v}'.\delta\mathbf{v})\rangle . \quad (2.2)$$

We can remark that in the flux terms the energy density fluctuations are transported by the fluid velocity fluctuations (represented by the $\frac{1}{2}[\delta(\rho\mathbf{v}).\delta\mathbf{v}]\delta\mathbf{v}]$ term) whereas the source terms represent the transport of the scalar $(\rho\mathbf{v}\cdot\delta\mathbf{v})$ by $(\nabla\cdot\mathbf{v})$ where the fluctuating quantity is $\delta\mathbf{v}$.

In describing a possible phenomenology for isothermal compressible hydrodynamic turbulence we use the following points:

i) For a given length scale l, there is a competition between the flux and the source terms regarding the transport of their respective fluctuating scalars (as defined above) between two points separated by l.

ii) Considering turbulence to be a phenomenon without memory, the transport which will cover the distance l later (i.e. with a greater characteristic time) will determine the characteristic non-linear energy transfer time for that length scale l.

iii) In this case, we can define two characteristic times $\tau_l(= l/v_l)$ and $\tau_C(= l/C_S)$ where $v_l \sim \mid \delta\mathbf{v} \mid$.

iv) We assume that the time rate of average total energy is a scale invariant.

2.1 *Subsonic turbulence* ($\delta v < C_S$):

In this case, the transport by flux terms, being slower of the two, governs the effective energy transfer and so the characteristic time is given by $\tau_{NL} = \tau_l$. The source terms during that time cover a longer distance $l_C = \tau_l C_S$ and so is forgotten by the receiver point of our concern. This consideration leads to the following dimensional analysis

$$\mathrm{flux} \sim \varepsilon \Rightarrow \frac{\rho_l v_l^3}{l} \sim \varepsilon \Rightarrow E_w(k) \sim \varepsilon^{2/3}k^{-5/3} , \quad (2.3)$$

where $w \equiv \rho^{1/3}v$ (Kritsuk et al. 2007; Federrath et al. 2010).

2.2 *Supersonic turbulence* ($\delta v > C_S$):

In this case, the source terms being slower govern the inter-scale energy transfer of the system and the characteristic time is $\tau_{NL} = \tau_C$. Utilising the exact relation we find

$$\mathrm{source} \sim \varepsilon \Rightarrow (\rho v)\frac{v_l^2}{l} \sim \varepsilon \Rightarrow E_v(k) \sim \varepsilon k^{-2} , \quad (2.4)$$

as seen in some simulations (Federrath et al. 2010).

2.3 *Sonic scale* ($\delta v = C_S$):

In this scale $v_l = C_S$. So the characteristic times are also equal. As it is a given scale (k given), we cannot expect a power law for that. But evaluating properly the $E_w(k)$ and $E_v(k)$ and then equalising them for $l = l_s$ we can estimate the sonic scale (l_s).

Schematically, hence we can write the compressible turbulence phenomenology as follows ($M_S \equiv v_l/C_S$)

$$\varepsilon \sim \left\{ \frac{\rho_l v_l^3}{l}, \frac{M_S}{\tau_C}(\rho v)v_l \right\} , \quad (2.5)$$

where the two expressions represent the average energy transfer rate respectively in the subsonic and the supersonic regime of compressible hydrodynamic turbulence.

3 Conclusions

The above phenomenology gives a possible description of compressible hydrodynamic turbulence under isothermal closure. The above approach however leaves some open questions which need to be answered for a thorough comprehension of compressible turbulence:

i) Does the total energy (kinetic energy + compressible energy) flux rate remain scale invariant in compressible case ?

ii) Are the one point quantities like (ρv) or $(\rho' v')$ come to be really inert (as we considered in our approach) in the dimension analysis for the fluctuating (or turbulent) quantities ?

iii) As reported recently by Wagner et al. (2012) that for compressible turbulence the existence and range of the inertial zone is dependent on the forcing criteria. Is the universality destroyed due to this problem ?

One can anyways attempt to give an analogical view as that of the above for polytropic closures (which is more realistic for astrophysical context) where the complexity increases considerably. In case of compressible MHD fluids, one can intuitively understand the degree of complexity depending upon the possible regimes which associates the subalfvénic and superalfvénic regimes too in addition to the subsonic and supersonic regimes. It is however expected that the recent numerical simulations (Kritsuk et al. 2009) along with a recent derived exact relation in compressible MHD turbulence (Banerjee & Galtier 2012) will help us to obtain a clearer view for the corresponding phenomenology.

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