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SYMMETRY BREAKING BETWEEN SASI SPIRAL MODES

R. Kazeroni[1], J. Guilet[2] and T. Foglizzo[1]

Abstract. The accretion shock formed during the collapse of massive stars is subject to the Standing Accretion Shock Instability (SASI). Spiral modes of SASI can redistribute angular momentum and spin-up a neutron star born from a non-rotating progenitor. If the asymmetries in the progenitor are initially small, two counter-rotating spiral modes with similar amplitudes emerge. In the non-linear regime of SASI, the symmetry between these modes may be broken and a strong spiral mode dominates the dynamics. We study here the timescale for symmetry breaking in order to evaluate the favorable conditions leading to angular momentum redistribution by the SASI. We perform 2D numerical simulations of a simplified setup in cylindrical geometry. These simulations show that a symmetry breaking occurs only if the initial radius of the shock wave is large enough compared to the radius of the neutron star. Furthermore, in the regime where symmetry breaking occurs, we observe stochastic variations, which require a statistical approach. A path towards an analytical description of the timescale for symmetry breaking is proposed.

Keywords: hydrodynamics, instabilities, shock waves, stars: neutron, stars: rotation, supernovae: general

1 Introduction

Hydrodynamic instabilities play an important role in the neutrino-driven mechanism which may explain the explosion of massive stars. By generating non-radial motions of matter below the shock wave during the first second of the collapse of the massive star, instabilities are able to trigger an asymmetric explosion. The Standing Accretion Shock Instability (SASI, Blondin et al. 2003) causes global shock oscillations and induces large scale asymmetries ($l \sim 1$,2). 3D simulations showed that SASI spiral modes can dominate the dynamics of the flow below the shock (Blondin & Mezzacappa 2007; Iwakami et al. 2008; Fernández 2010; Hanke et al. 2013; Abdikamalov et al. 2014). The spiral modes of SASI can redistribute angular momentum (Blondin & Mezzacappa 2007; Foglizzo et al. 2012). In the case where the progenitor is non-rotating, the SASI spiral modes have the potential to explain pulsar spin periods of a few hundreds of milliseconds (Guilet & Fernández 2014). This angular momentum redistribution by the SASI can occur only if the symmetry between SASI spiral modes rotating in opposite directions breaks in the non-linear regime. In this proceeding, our aim is to characterize the timescale for symmetry breaking between SASI spiral modes and to determine whether it is short enough to take place before an explosion sets in. In Sect. 2 we introduce our 2D numerical simulations in cylindrical geometry, and obtain constraints on the symmetry breaking mechanism in Sect. 3.

2 Method

The physical system consists of a standing accretion shock in a steady-state flow. As in Yamasaki & Foglizzo (2008) we restrict our simulation domain to the equatorial plane of the progenitor for the sake of simplicity. We use a cylindrical geometry, which has the advantage of allowing the study of non-axisymmetric modes in 2D simulations. The shock wave initially stands at a radius $r_{\rm sh}$ from the center. The accreting matter is described by a perfect gas with an adiabatic index $\gamma = 4/3$. Above the shock, the gas flows inwards radially. The gas decelerates through the stationary shock and accretes on the hard surface of the neutron star at the radius r_*. The gravity is Newtonian and self-gravity is neglected. We neglect heating to avoid convective motions

[1] Laboratoire AIM, CEA/DSM-CNRS-Université Paris Diderot, IRFU/Service dAstrophysique, CEA-Saclay F-91191, France
[2] Max-Planck-Institut fur Astrophysik, Karl-Schwarzschild-Str. 1, D-85748 Garching, Germany

and include cooling to mimic neutrino emission by electron capture with the approximation $\mathcal{L}_0 \propto p^{3/2}\,\rho$, as in Blondin & Mezzacappa (2007) where p and ρ denote the pressure and the density. The cooling is dominant only in a narrow region close to the accretor. The flow is supersonic above the shock with an incident Mach number $\mathcal{M}_1 = 5$ at $r = r_{\rm sh}$ and subsonic below the shock. The two solutions are connected by the Rankine-Hugoniot jump conditions for an ideal gas, neglecting the dissociation of iron.

Our computational domain does not include the proto neutron star. Its surface corresponds to the inner boundary of the domain. We use cylindrical coordinates $(r,\,\phi)$ with $r \in [r_*,\,3\,r_{\rm sh}]$ and $\phi \in [0,\,2\,\pi]$. We impose reflexive conditions at the inner boundary. The flow at the outer boundary is given by the upstream steady state solution. The initial condition corresponds to the stationary flow obtained by solving the time-independent continuity, Euler and energy equations. To perform time-dependent hydrodynamic calculations, we employ a version of the Godunov code RAMSES (Teyssier 2002; Fromang et al. 2006) with a constant grid and no AMR. The numerical resolution is 600 to 1000 cells in the radial direction and 1000 to 1600 cells in the azimuthal direction. High resolution is required to resolve properly the dynamics of the flow in the vicinity of the proto neutron star with steep gradients. We define the radii ratio $R = r_{\rm sh}/r_*$. The initial value of R is chosen by adjusting the cooling normalization constant. As in Fernández & Thompson (2009) we use a cutoff in entropy in the cooling function to turn off the cooling in the first few cells outside r_* to avoid the divergence of the numerical solution. The cooling function is such that:

$$\mathcal{L} = \mathcal{L}_0\,\exp\left[-\left(\frac{s}{k\,s_{\rm min}}\right)^2\right] \tag{2.1}$$

$s = (\gamma-1)^{-1}\ln\left(p/\rho^\gamma\right)$ is an entropy function, $s_{\rm min}$ its value at $r = r_*$ and k a real number chosen to introduce only minimal modification to the postshock steady state flow. To test the robustness of our code, we computed the growth rates and the frequencies of the unstable modes of the SASI in the linear regime and obtained a very satisfactory agreement with the values of Yamasaki & Foglizzo (2008) computed with a perturbative analysis: the discrepancies are less than 2% for the frequencies and less than 8% for the growth rates.

We let the steady state flow relax on the numerical grid for a hundred dynamical timesteps and then introduce two entropy perturbations at pressure equilibrium in the supersonic flow to trigger SASI spiral modes $m = 1$ and $m = -1$. The linear phase of SASI lasts approximately 2 SASI oscillation periods and the simulation finishes at $t = 1s$. Without any perturbations, our code is perfectly spherically symmetric, the SASI does not develop and the shock remains circular. Moreover, if the two perturbations have exactly the same amplitude, the symmetry does not break and we obtain a stationary sloshing mode in the non linear regime which can be seen as a sum of the two counter-rotating spiral modes. In order to study the symmetry breaking we vary the relative perturbation amplitudes of the two spiral modes and define the initial asymmetry by $\epsilon = (A_{\rm r}^2 - A_{\rm l}^2)/(A_{\rm r}^2 + A_{\rm l}^2)$ where $A_{\rm r}$ and $A_{\rm l}$ respectively stand for the amplitudes of the shock displacement associated to the modes $m = 1$ and $m = -1$. We also vary the radii ratio R. This ratio selects the unstable modes of SASI and affects their growth rates (Foglizzo et al. 2007). We neglect the initial rotation of the progenitor in our setup which would favour the prograde SASI mode (Yamasaki & Foglizzo 2008). Finally we have set two different methods to estimate the timescale for symmetry breaking. The first one is based on the time evolution of the angular momentum flux at the inner boundary. This flux is very close to zero before the symmetry breaking and start to deviate from zero when one of the spiral modes dominates the dynamics. The second method uses the triple point that forms in the shock wave (Blondin & Mezzacappa 2007). We track the triple point in our simulations and compute the time evolution of its rotation rate. This rotation rate evolves rather randomly before the symmetry breaking and becomes almost constant afterwards. The two methods are consistent within a SASI oscillation period which is sufficient for our study.

3 Characterization of the symmetry breaking

We perform a total of 80 simulations varying R and ϵ such that $R = \{1.67,\,2,\,2.22,\,2.5,\,3,\,4\}$ and $10^{-3} \le |\epsilon| \le 1$. For large values of $|\epsilon|$, a strong spiral mode is triggered in the linear phase of SASI and saturates in the non-linear regime. For lower values ($|\epsilon| \le 0.2 - 0.3$) the initial asymmetry is too small to lead to a symmetry breaking in the linear phase during which spiral modes and sloshing mode of a given index m grow at the same rate. By varying R, our simulations show that symmetry breaking is not a systematic behaviour of the SASI. Indeed, we do not obtain any symmetry breaking when $R \le 2$. For these values of R, we show that sloshing modes dominate the dynamics of SASI in the non-linear regime. Even if we consider an extreme case: $\epsilon = 1$ meaning that only one of the two spiral modes is triggered and dominates the linear regime, a sloshing mode dominates the non-linear regime as in the left panel of Fig. 1. Our simplified setup shows that if $R \leq 2$ and if we assume that initial asymmetries in the progenitor are weak, then there is no sign of symmetry breaking and SASI is unable to redistribute angular momentum.

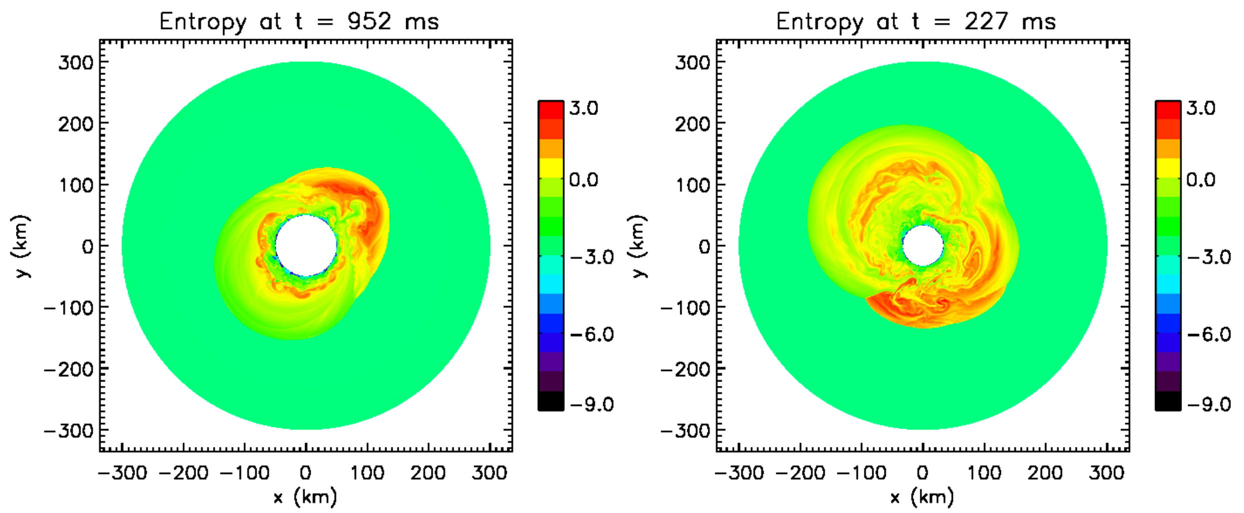

Fig. 1. Left: Snapshot of the entropy at $t = 952$ ms for $R = 2$ and $\epsilon = 1$. A sloshing motion dominates the non-linear regime despite a strong initial asymmetry. **Right:** Snapshot of the entropy at $t = 227$ ms for $R = 3$ and $\epsilon = 0.1$. The symmetry breaking has already occured.

We now focus on the case $R > 2$ for which we obtain systematically a symmetry breaking. For $R = 2.22$ and weak initial asymmetry, the timescale for symmetry breaking is comparable to the critical time of 1 s after bounce (around 30 SASI oscillation periods) whereas for $R = \{2.5, 3, 4\}$ the symmetry breaking occurs within 3 to 10 SASI oscillations in the non-linear phase as in the right panel of Fig. 1. However, for low values of the initial asymmetry $|\epsilon| \leq 0.2$, there is no clear trend between the level of initial asymmetry and the timescale for symmetry breaking. More precisely, a decrease in ϵ does not increase this timescale which is rather chaotic. For some cases where $|\epsilon| \leq 0.1$, the sign of ϵ does not determine the direction of rotation of the shock wave. We have extensively tested the code to check that this non-deterministic phenomenon is not due to a numerical artefact. We propose a physical interpretation to this feature of the SASI. We suggest that parasitic instabilities that grow on an unstable mode, and which can explain the SASI saturation amplitude (Guilet et al. 2010) might modify in a stochastic way the level of asymmetry between the spiral modes. This might be enough, if initially $|\epsilon| \leq 0.1$, to change the dominating spiral mode before a symmetry breaking occurs. A statistical approach is therefore required to address the issue of a timescale for symmetry breaking between spiral modes in order to get a general picture in this parameter space. Fig. 2 shows the number of SASI oscillations to reach the symmetry breaking for an initial asymmetry $|\epsilon|$ and for $R = \{2.22, 2.5, 3, 4\}$. We do not mention cases for $R = \{1.67, 2\}$ here because they do not lead to a symmetry breaking. The size of the error bars is 2 SASI oscillations and corresponds to the precision of our methods to evaluate the timescale. For $R = \{2.22, 2.5\}$ and $|\epsilon| < 0.2$ the variations of the number of SASI oscillations to reach symmetry breaking are greater than the error bars. This illustrates the stochasticity of symmetry breaking for weak initial asymmetries.

Finally, we propose a physical mechanism for the symmetry breaking, which is based on the effect of the rotation induced by the spiral modes on their growth rate. Guilet & Fernández (2014) showed that the rotation induced by spiral modes scales as $A_{\rm r}^2 - A_{\rm l}^2$, the rotation below the shock being in the same direction as the spiral mode of largest amplitude. Moreover, Yamasaki & Foglizzo (2008) showed that the growth rates of unstable modes vary linearly with the angular momentum, prograde modes begin favored and retrograde modes being damped. If the rotation induced by the spiral modes has a similar effect on the growth rate, then the growth of the spiral mode of largest amplitude would be favored thus potentially leading to a symmetry breaking. This model needs to be further developed to be compared to the results of the numerical simulations.

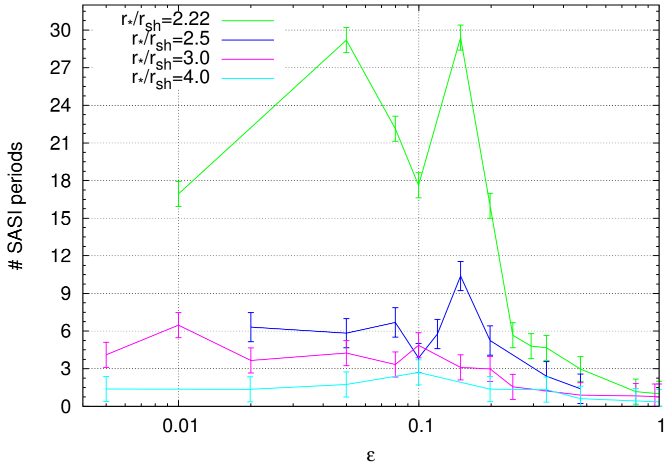

Fig. 2. Number of SASI oscillations before reaching the symmetry breaking with respect to the initial asymmetry ϵ. From top to bottom are shown ratios $R = 0.45, 0.4, 0.33, 0.25$. The size of the error bars is 2 SASI oscillations.

4 Conclusion

We have used a simplified model (Blondin & Mezzacappa 2007; Foglizzo et al. 2007; Yamasaki & Foglizzo 2008; Fernández & Thompson 2009) to study the SASI in the non-linear regime with 2D numerical simulations on a cylindrical domain. We varied the radii ratio R and the initial asymmetry ϵ of the perturbations to study the timescale for symmetry breaking. Our simulations show that the symmetry breaking between SASI spiral modes occurs only for some values of R. Moreover, when it occurs, the symmetry breaking is affected by a non-deterministic phenomenon which requires a statistical approach for small initial asymmetries.

Characterizing the symmetry breaking is essential to understand the conditions under which a strong spiral mode dominates the non-linear regime and redistributes angular momentum. Our study proposes a path towards an analytical description of the symmetry breaking.

The spin of the neutron star at birth can be either dominated by the conservation of the initial angular momentum of the pulsar or by the redistribution of angular momentum due the SASI spiral modes. The question of the threshold between these two regimes will be addressed in future work in which initial rotation will be included in our setup.

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