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HYDRODYNAMICAL SCALING LAWS TO STUDY TIDAL DYNAMICS IN PLANETARY SYSTEMS

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Abstract. Tidal dissipation in stars and planets intrinsically depends on the nature of their tidal response, which is directly linked to their internal structure and rheology. Indeed, solids and fluids do not behave in the same way, the response of the second being highly resonant. This study, focused on viscous friction acting on tidal waves, uses a local model to provide scaling laws allowing to better understand the physics of dissipation in the fluid regions of celestial bodies. It shows how the nature and properties of low-frequency tidal gravito-inertial waves change with fluid parameters such as viscosity, thermal diffusivity, rotation and stratification. Besides, the scaling laws derived from the local model are applied to the tidal dynamics of a two-body system that highlights the impact of the characteristics of dissipation.

Keywords: hydrodynamics, waves, turbulence, planet-star interactions

1 Introduction

Since the theoretical calculations carried out by Lord Kelvin (Kelvin 1863), who was the first to consider a tidally deformed celestial body, tides have occupied a central place in the study of planetary systems. In this context, gravitational tides have been studied mainly thanks to the developments made by Love and the corresponding Love numbers (Love 1911). Then, Goldreich brought a major contribution in the 1960's with the introduction of the tidal quality factor Q, which is a general way to take into account tidal dissipation in celestial mechanics (Goldreich & Soter 1966). This is of great importance since tides drive the secular orbital/spin evolution of stars, planets and satellites by converting their mechanical kinetic energy into internal heating (Laskar et al. 2012; Bolmont et al. 2014).

However, the mechanisms driving tidal dissipation are not the same in solids and fluids. They depend on the nature of the materials that compose a body and on the structure of this later. Studies dealing with the effects of gravitational perturbations in rocky cores and planets (see for example Efroimsky & Lainey 2007; Efroimsky 2012; Remus et al. 2012b) show that the Q factor of these solid bodies varies regularly with the forcing frequency, which obviously does not match with the case of fluid bodies. Indeed, numerous works published during the last decades (Zahn 1966, 1975; Ogilvie & Lin 2004, 2007; Remus et al. 2012a, for stars and the envelopes of giant planets) attest of the resonant response of fluid bodies to tidal perturbations, this behavior being synonym of a strong dependence of the Q factor on the tidal frequency and of an erratic evolution of the orbital dynamics (Auclair-Desrotour et al. 2014).

Such a behavior is explained by dissipation mechanisms, like the viscous friction, thermal diffusion and Ohmic diffusion (in the presence of a magnetic field), acting on fluid tidal waves. This work focuses on viscous friction within a low-frequency range and does not take into account magnetic aspects. So, high-frequency acoustic waves are left aside, like Alfvén waves which propagate in magnetized fluid regions, and gravito-inertial waves only remain. These laters predominate the tidal response of stars, the external envelope of giant planets and the fluid layers of rocky planets and satellites like the Earth or Europa.

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Inertial waves are driven by rotation, gravity waves by stratification, and their restoring forces are the Coriolis acceleration and the buoyancy respectively. Their characteristic frequencies are the so-called inertial frequency 2Ω, Ω being the spin frequency of the body, and the Brunt-Väisälä frequency N corresponding to the radial variations of the specific entropy. Gravito-inertial waves result from their coupling in stably stratified rotating fluid regions. Thus, to study their complex dissipation by using a reduced local model appears as an interesting way to explore its behavior over large domains of parameters (see also Ogilvie 2005; Jouve & Ogilvie 2014). Such studies are complementary of those carried out with complex global models.

The aim of the present work is to propose a method to investigate the dependence of tidal dissipation on the fluid parameters. Using a fluid box in which rotation, stratification, viscosity and thermal diffusivity are taken into account, scaling laws describing viscous friction on tidal waves are obtained (Auclair-Desrotour, Mathis, Le Poncin-Lafitte in preparation for the complete derivation). Next, these scaling laws are used to illustrate, through the concrete example of a planet-satellite system, how the quality factor Q and the evolution of orbital dynamics are linked to the fluid parameters (see Auclair-Desrotour et al. 2014).

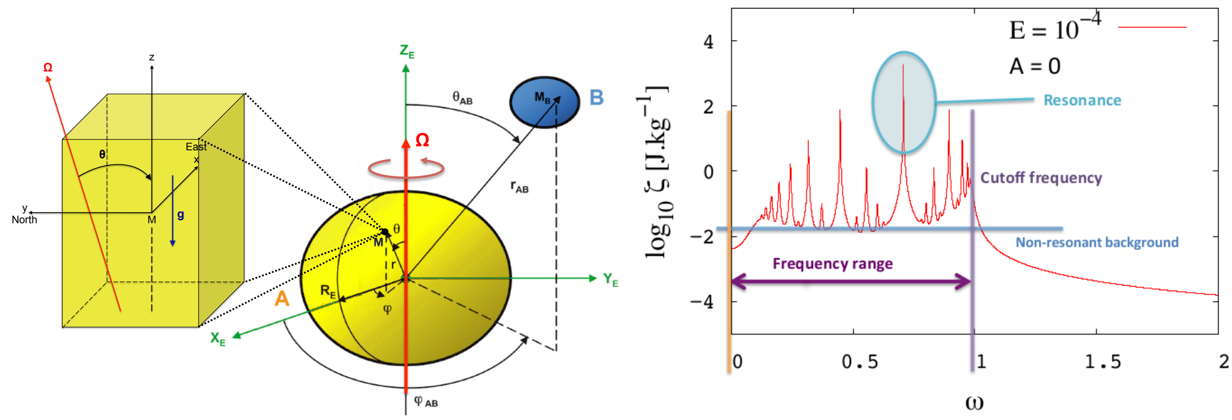

Fig. 1. Left: The local analytical model: a Cartesian fluid section of a rotating fluid body A tidally excited by a perturber B. The control parameters are the viscosity ν, thermal diffusivity κ and frequencies 2Ω and N of the fluid. **Right:** A typical dissipation spectrum computed from the local model. ζ is the energy dissipated by viscous friction per mass unit over a rotation period in the box; $\omega = \chi/2\Omega$ is the tidal frequency of the perturbation normalized by the inertial frequency 2Ω; $E = 2\pi^2\nu/\left(\Omega L^2\right)$ is the Ekman number and $K = 2\pi^2\kappa/\left(\Omega L^2\right)$ the normalized thermal diffusivity. Here $K = 0$ and $A = 0$, which means that the waves are purely inertial and viscously damped (top left blue zone in Fig. 2).

2 Hydrodynamical scaling laws

The model used here generalizes the first local model presented in Ogilvie & Lin (2004). Consider a local section of a fluid region in a rotating star, planet or satellite tidally excited by a perturber. It is a rotating Cartesian box of side length L inclined with respect to the spin axis of the body $\boldsymbol{\Omega}$ by a colatidude θ (Fig. 1). The coordinate z corresponds to the radial direction, x and y to the azimutal and latitudinal ones. In the box, the fluid is supposed homogeneous of density ρ, kinetic viscosity ν and thermal diffusivity κ. Three dimensionless control parameters are identified: $A = (N/2\Omega)^2$ giving the nature of the waves ($A \ll 1$ for inertial waves, $A \gg 1$ for gravity ones), the so-called Ekman number $E = 2\pi^2\nu/\left(\Omega L^2\right)$ weighting the terms of viscous diffusion with respect to the Coriolis terms, and $K = 2\pi^2\kappa/\left(\Omega L^2\right)$ which is a equivalent of E for thermal diffusion. The Prandlt number $Pr = E/K$ compares the viscous and thermal diffusions (see Fig. 2).

Decomposing variables into Fourier series allows to compute an analytical expression for the energy ζ per mass unit dissipated by viscous friction over a rotation period ($T = 2\pi/\Omega$). This gives access to the properties of the dissipation spectrum (Fig. 1), which has the shape of a batch of resonances located between ω_- and ω_+ (see Gerkema & Shrira 2005, for their expression). Four different asymptotical behaviors are identified. They are represented in Fig. 2 and illustrated by the corresponding spectra. Scaling laws for the viscous friction are derived analytically in each regime for the positions ω_{mn}, widths at mid-height l_{mn} and heights H_{mn} of resonances ($m, n \in \mathbb{Z}$), the number of peaks $N_{\rm kc}$, the height of the non-resonant background $H_{\rm bg}$ (that

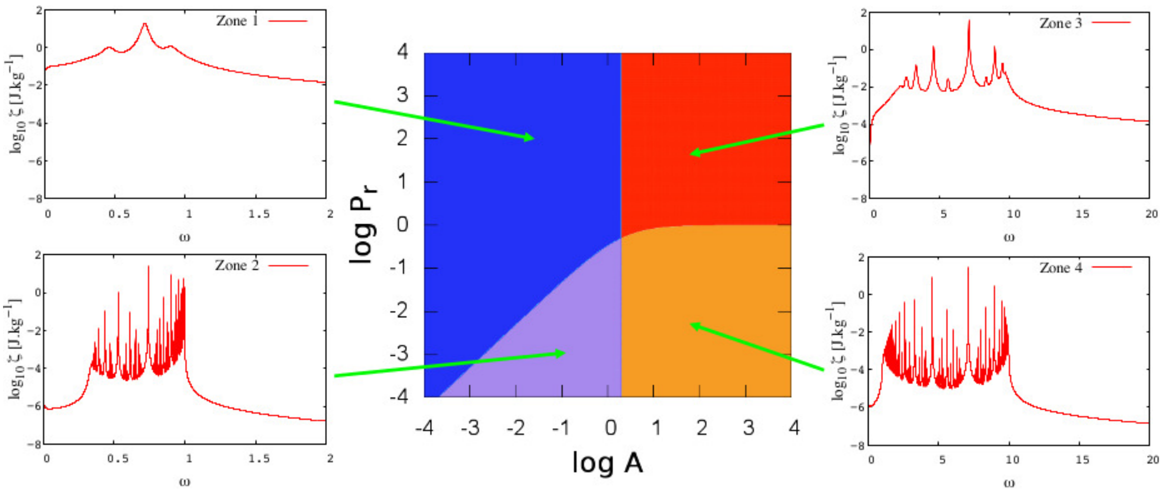

Fig. 2. Asymptotical behaviors of the tidal waves. Zones colored in blue and purple correspond to inertial waves, the two other to gravity waves ; zones colored in blue and red correspond to the case where viscous diffusion predominates over thermal diffusion, the two zones below corresponding to the opposite case.

corresponds to the so-called equilibrium tide) and the sharpness ratio $\Xi = H_{11}/H_{\rm bg}$ of the spectrum. Ξ gives the relative contrast between the resonances and the background (Table 1).

Table 1. Scaling laws for the properties of the energy viscously dissipated for the different regimes (we define $A_{11} \equiv 2\cos^2\theta$ and $Pr_{11} \equiv A/(A + A_{11})$). **Top left:** Inertial waves dominated by viscosity. **Top right:** Gravity waves dominated by viscosity. **Bottom left:** Inertial waves dominated by heat diffusion. **Bottom right:** Gravity waves dominated by heat diffusion. F is the amplitude of the forcing.

Domain	$A \ll A_{11}$		$A \gg A_{11}$	
$Pr \gg Pr_{11}$	$\frac{\chi_{mn}}{2\Omega} \propto \frac{n}{\sqrt{m^2+n^2}} \cos\theta$	$N_{\rm kc} \propto E^{-1/2}$	$\frac{\chi_{mn}}{2\Omega} \propto \frac{m}{\sqrt{m^2+n^2}} \sqrt{A}$	$N_{\rm kc} \propto A^{1/4} E^{-1/2}$
	$l_{mn} \propto E$	$H_{mn} \propto F^2 E^{-1}$	$l_{mn} \propto E$	$H_{mn} \propto F^2 E^{-1}$
	$H_{\rm bg} \propto F^2 E$	$\Xi \propto E^{-2}$	$H_{\rm bg} \propto F^2 E A^{-1}$	$\Xi \propto A E^{-2}$
$Pr \ll Pr_{11}$	$\frac{\chi_{mn}}{2\Omega} \propto \frac{n}{\sqrt{m^2+n^2}} \cos\theta$	$N_{\rm kc} \propto A^{-1/2} K^{-1/2}$	$\frac{\chi_{mn}}{2\Omega} \propto \frac{m}{\sqrt{m^2+n^2}} \sqrt{A}$	$N_{\rm kc} \propto A^{1/4} K^{-1/2}$
	$l_{mn} \propto AK$	$H_{mn} \propto F^2 A^{-2} E K^{-2}$	$l_{mn} \propto K$	$H_{mn} \propto F^2 E K^{-2}$
	$H_{\rm bg} \propto F^2 E$	$\Xi \propto A^{-2} K^{-2}$	$H_{\rm bg} \propto F^2 E A^{-1}$	$\Xi \propto A K^{-2}$

3 Impact on tidal dynamics

As described above, the properties of the dissipation directly impact the long-term evolution of planetary systems. This point is easily illustrated through the case of a two-bodies coplanar system, for example a satellite orbiting circularly around a planet (e.g. the Mars-Phobos case studied by Efroimsky & Lainey 2007). If we introduce in the dynamical equations a frequency-dependent tidal quality factor (e.g. Mathis & Le Poncin-Lafitte 2009), which is proportional to the inverse of ζ in our local model, the semi-major axis a of the system evolves erratically (Fig. 3). Instead of falling on the planet regularly with an increasing velocity as in the case corresponding to Kaula's constant Q model (Kaula 1964), the satellite only comes nearer to it, jumping from a position to an other each times it meets a peak of resonance. Under some assumptions detailed in Auclair-Desrotour et al. (2014), the amplitude of a jump Δa can be written as a function of the frequency ω_p, width at mid-height l_p and sharpness ratio $\Xi_p = H_p/H_{\rm bg}$ of a peak,

$$\frac{\Delta a}{a} \approx \frac{2l_{\rm p}}{3\sqrt{\sqrt{2}-1}\,(1+\omega_{\rm p})}\left[\sqrt{\Xi_p}-1\right]^{\frac{1}{2}}. \tag{3.1}$$

Each of these characteristics are now given explicitly as functions of the internal parameters of the fluid, A, E and K thanks to scaling laws obtained in Table 1. For example, the variations of the width l_{11} and sharpness Ξ of the main dissipation resonance with the Ekman number E are represented in Fig. 3.

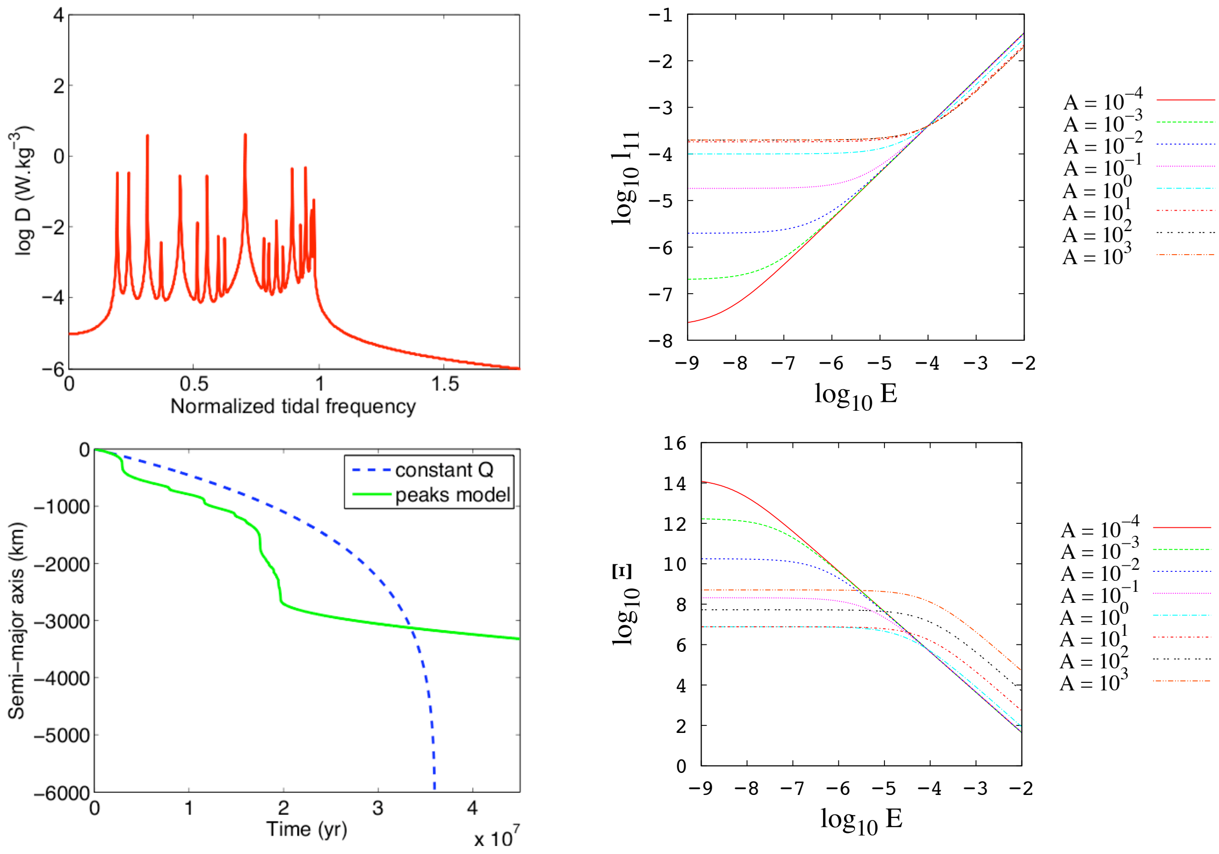

Fig. 3. Top left: A synthetical dissipation spectrum generated using the local model of the fluid box. D is proportional to ζ. **Bottom left:** The corresponding evolution of the semi-major axis a of the fluid planet-satellite system (for details, see Auclair-Desrotour et al. 2014). **Top right:** Width at mid-height of the main resonance of ζ as a function of the Ekman number $E = 2\pi^2\nu/\left(\Omega L^2\right)$ for different values of $A = (N/2\Omega)^2$. **Bottom right:** Sharpness of the main resonance of ζ as a function of the Ekman number E for different values of A.

4 Conclusions

The properties of fluid tidal dissipation have a direct impact on the secular dynamics of planetary systems. Indeed, dissipation resonances cause local rapid changes of orbital parameters, that are tightly related to the widths and heights of the peaks. In this context, our local model provides scaling laws that describe the evolution of the complex and resonant tidal dissipation as a function of fluid parameters. Moreover, It seems to be an interesting qualitative tool to unravel the complex physics of dissipation. In a near future, our method could be extended to magnetized fluid regions. It would allow to study Alfvén waves in addition to gravito-inertial waves and to refine the map of the asymptotical behaviors (Fig. 2) with new regimes. This will contribute to improve tidal dissipation modeling in studies of the dynamical evolution of planetary systems.

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