SF2A 2014
J. Ballet, F. Bournaud, F. Martins, R. Monier and C. Reylé (eds)

TIDAL FRICTION IN ROTATING TURBULENT CONVECTIVE STELLAR AND PLANETARY REGIONS

S. Mathis[1,2], P. Auclair-Desrotour[1,3], M. Guenel[1] and C. Le Poncin-Lafitte[4]

Abstract. Turbulent friction in stellar and planetary convection zones is one of the key physical mechanisms that drive the dissipation of the kinetic energy of tidal flows in stars and planets hosting companions. This friction acting both on the equilibrium tide and on tidal inertial waves thus deeply impacts the dynamics of the spin of the host star/planet and the orbital architecture of the surrounding system. It is thus very important to obtain robust prescription for this friction. In the current state-of-the-art, it is modeled by a turbulent viscosity coefficient using mixing-length theory. However, none of the existing prescriptions take into account the action of the possibly rapid rotation that strongly affects convective flows. In this work, we propose such a new prescription that takes into account rotation and discuss the possible implication for tidal dissipation in rotating stars and planets.

Keywords: hydrodynamics, waves, turbulence, planet-star interactions

1 Introduction and context

Tidal friction is one of the mechanisms that drive the evolution of planetary systems (e.g. Hut 1981; Laskar et al. 2012; Mathis & Remus 2013; Ogilvie 2014). In this context, tidal friction in the turbulent convective envelopes of low-mass stars and giant planets and the cores of telluric planets must be carefully evaluated. In the present state-of-the-art, the turbulent friction acting on tidal flows in these regions (e.g. Ogilvie & Lin 2004, 2007; Remus et al. 2012) is modeled thanks to an effective turbulent viscosity coefficient (Zahn 1966). This corresponds to the assumptions that it can be described through a viscous force while we have a scale-separation between tidal and turbulent convective flows. The properties of the turbulent viscosity thus described the effective efficiency of the couplings between turbulence and tidal flows. Therefore, it depends on the frequency of the forcing as well as on the dynamical parameters that impact stellar and planetary convection (Zahn 1966; Goldreich & Keeley 1977; Goodman & Oh 1997; Penev et al. 2007; Ogilvie & Lesur 2012).

Among them, rotation is one of the parameters that must be taken into account. Indeed, the Coriolis acceleration strongly affects the dynamics of turbulent convective flows (e.g. Brown et al. 2008; Julien et al. 2012; Barker et al. 2014). Therefore, it is absolutely necessary to get a robust prescription for the turbulent friction applied on tidal waves by rotating convection in stars and planets as a function of their angular velocity that evolves during their evolution. To reach this objective, properties of rotating turbulent flows such as their characteristic velocities and length scales must be known if we wish to model this friction using the mixing-length framework (Zahn 1966). In this context, the work by Stevenson (1979) is particularly interesting since they are derived in the asymptotic regimes of slow and rapid rotation. Moreover, these asymptotic scaling laws have been now confirmed by Barker et al. (2014) using high-resolution non-linear 3-D Cartesian simulations of turbulent convection. In this work, we thus propose a new prescription for tidal friction in rotating turbulent convective regions that takes rotation into account using the results obtained by Stevenson (1979) and Barker et al. (2014).

[1] Laboratoire AIM Paris-Saclay, CEA/DSM - CNRS - Université Paris Diderot, IRFU/SAp Centre de Saclay, F-91191 Gif-sur-Yvette Cedex, France
[2] LESIA, Observatoire de Paris, CNRS UMR 8109, UPMC, Univ. Paris-Diderot, 5 place Jules Janssen, 92195 Meudon, France
[3] IMCCE, Observatoire de Paris, UMR 8028 du CNRS, UPMC, 77 Av. Denfert-Rochereau, 75014 Paris, France
[4] SYRTE, Observatoire de Paris, UMR 8630 du CNRS, UPMC, 77 Av. Denfert-Rochereau, 75014 Paris, France

2 Prescription for the friction in rotating turbulent convective layers

2.1 State of the art

The first study of the friction applied by turbulent convection on tidal flows was achieved by Zahn (1966) in the case of stars (see also Zahn 1989). In his work, he examined the coupling between turbulence and the large-scale equilibrium tide induced by the hydrostatic adjustment of the star due to the tidal perturber (e.g. Remus et al. 2012). His approach was based on three main assumptions. First, he assumed that the friction applied by turbulence can be described thanks to a viscous force involving an eddy-viscosity $\nu_{\rm T}$. Second, he assumed a scale-separation between tidal and turbulent convective flows. Finally, the characteristic velocity and length scale of turbulent convection, respectively $V_{\rm c}$ and $L_{\rm c}$, were described using the mixing-length theory. In this framework, he proposed the following prescription for the eddy-viscosity:

$$\nu_{\rm T;NR} = \frac{1}{3} V_{\rm c} L_{\rm c}\, f\left(\frac{P_{\rm tide}}{T_{\rm c}}\right). \tag{2.1}$$

In this expression, NR stands for *Non-Rotating convection* and f is a function that describes the loss of efficiency of tidal friction in the case of rapid tide when $P_{\rm tide} \ll T_{\rm c}$, $P_{\rm tide}$ and $T_{\rm c}$ being respectively the tidal period and the characteristic convective turn-over time (see Zahn 1966; Goldreich & Keeley 1977; Goodman & Oh 1997; Penev et al. 2007; Ogilvie & Lesur 2012, for detailed discussions of f).

However, as pointed above (rapid) rotation strongly affects turbulent convective flows (e.g. Chandrasekhar 1953; Brown et al. 2008; Julien et al. 2012; Barker et al. 2014). Therefore, $V_{\rm c}$, $L_{\rm c}$, and as a consequence $\nu_{\rm T}$, vary with rotation. In the present state-of-the-art, we are thus in a situation where the action of the Coriolis acceleration is taken into account in the physical description of tidal flows (e.g. Ogilvie & Lin 2004, 2007; Remus et al. 2012) while it is ignored in the one of the turbulent friction while the angular velocity of celestial bodies varies along their evolution (e.g. Gallet & Bouvier 2013, for solar-type stars).

2.2 Modelling and assumptions

To study the modification of the turbulent friction applied on tidal flows in rotating stellar and planetary convective regions, we use theoretical results first derived by Stevenson (1979) and confirmed by high-resolution numerical simulations computed by Barker et al. (2014) in Cartesian geometry. We thus choose to consider a local Cartesian set-up with a box centered around a point M in a rotating convective zone (see fig. 1); (M, x, y, z) is the associated reference frame. We introduce the angular velocity Ω of the studied body. The box has a characteristic length L and is assumed to have an homogeneous density ρ. Its vertical axis is inclined with an angle θ with respect to the rotation axis.

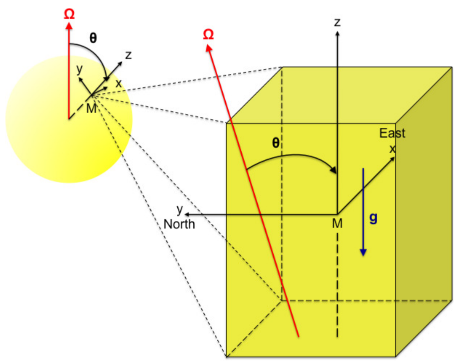

Fig. 1. The local Cartesian model. The spin $\vec{\Omega}$ is represented by the red arrow and the gravity $\vec{g}$ by the blue one.

Next, we introduce the control parameters of the system:

- the convective Rossby number defined as in Stevenson (1979)

$$R_{\rm o}^{\rm c} = \left(\frac{V_{\rm c}\,(\Omega=0)}{L_{\rm c}\,(\Omega=0)\, 2\Omega|\cos\theta|}\right) = \frac{T_\Omega}{T_{\rm c}\,(\Omega=0)}, \tag{2.2}$$

where we introduce the characteristic convective turn-over time $T_{\rm c} = \dfrac{L_{\rm c}}{V_{\rm c}}$ and the dynamical one $T_\Omega = \dfrac{1}{2\Omega}$; $R_{\rm o}^{\rm c} \ll 1$ and $R_{\rm o}^{\rm c} \gg 1$ correspond to rapid and slow rotation regimes respectively;

- the Ekman number
$$E = \frac{2\pi^2 \nu_{\rm T}}{\Omega L^2}, \tag{2.3}$$
which compares the respective strength of the viscous force and of the Coriolis acceleration.

2.3 The new eddy-viscosity prescription

As in previous works, which do not take into account the action of rotation on convection (see sec. 2.1), we assume i) that the turbulent friction on tidal velocities can be modeled through a viscous force involving an eddy-viscosity coefficient and ii) a scale-separation between turbulent convective and tidal flows.

To derive this coefficient as a function of rotation, we have to know the variation of $V_{\rm c}$ and $l_{\rm c}$ as a function of Ω and to verify that the mixing-length approach, which is generally used in stellar and planetary models, can be assumed in our context. In this framework, this is the great interest of the work by Barker et al. (2014). They demonstrated that scaling laws obtained by Stevenson (1979) for $V_{\rm c}$ and $L_{\rm c}$ as a function of $R_{\rm o}^{\rm c}$ using such a mixing-length formalism is robust and verified when computing high-resolution Cartesian numerical simulations of rotating turbulent convective flows in a set-up corresponding to the one studied here for $\theta \approx 0$.

We can thus generalize the prescription proposed in eq. (2.1) to the rotating case by writing*

$$\nu_{\rm T;RC} = \frac{1}{3} V_{\rm c}\left(R_{\rm o}^{\rm c}\right) L_{\rm c}\left(R_{\rm o}^{\rm c}\right) f\left(\frac{P_{\rm tide}}{T_{\rm c}}\right), \tag{2.4}$$

where RC stands for *Rotating Convection.* To get $V_{\rm c}\left(R_{\rm o}^{\rm c}\right)/V_{\rm c}\left(\Omega=0\right)$ and $L_{\rm c}\left(R_{\rm o}^{\rm c}\right)/L_{\rm c}\left(\Omega=0\right)^\dagger$, we use the scaling laws that have been derived by Stevenson (1979) and verified by Barker et al. (2014):

- in the *slow rotation* regime ($R_{\rm o}^{\rm c} \gg 1$), we have
$$\frac{V_{\rm c}\left(R_{\rm o}^{\rm c}\right)}{V_{\rm c}\left(\Omega=0\right)} \approx \left(1 - \frac{1}{242\left(R_{\rm o}^{\rm c}\right)^2}\right) \quad \text{and} \quad \frac{L_{\rm c}\left(R_{\rm o}^{\rm c}\right)}{L_{\rm c}\left(\Omega=0\right)} \approx \left(1 + \frac{1}{82\left(R_{\rm o}^{\rm c}\right)^2}\right)^{-1}; \tag{2.5}$$
- in the *rapid rotation* regime ($R_{\rm o}^{\rm c} \ll 1$), we have
$$\frac{V_{\rm c}\left(R_{\rm o}^{\rm c}\right)}{V_{\rm c}\left(\Omega=0\right)} \approx 1.5(R_{\rm o}^{\rm c})^{1/5} \quad \text{and} \quad \frac{L_{\rm c}\left(R_{\rm o}^{\rm c}\right)}{L_{\rm c}\left(\Omega=0\right)} \approx 2(R_{\rm o}^{\rm c})^{3/5}. \tag{2.6}$$

We also define a first Ekman number computed with the turbulent viscosity prescription where the modification of turbulent friction by rotation is ignored ($E_{\rm NR}$) or taken into account ($E_{\rm RC}$), i.e.

$$E_{\rm NR} = \frac{2\pi^2 \nu_{\rm T;NR}}{\Omega L^2} \quad \text{and} \quad E_{\rm RC} = \frac{2\pi^2 \nu_{\rm T;RC}}{\Omega L^2}. \tag{2.7}$$

In fig. 2, we plot $\nu_{\rm T;RC}/\nu_{\rm T;NR}$ and the corresponding ratio for the Ekman number $E_{\rm RC}/E_{\rm NR}$ as a function of $R_{\rm o}^{\rm c}$.

The small-dashed green and continuous blue lines correspond to the slow- and rapid-rotation asymptotic regimes respectively (the red long-dashed line corresponding to the non-rotating case). In the regime of rapidly rotating convective flows ($R_{\rm o}^{\rm c} \ll 1$), the turbulent friction decreases by several orders of magnitude with a scaling $\nu_{\rm T;RC}/\nu_{\rm T;NR} \propto \left(R_{\rm o}^{\rm c}\right)^{4/5} \propto \Omega^{-4/5}$. It can be understood coming back on the action of (rapid) rotation on the convective instability and turbulence. The Coriolis acceleration tends to stabilize the flow and thus the degree of turbulence decreases with increasing rotation as well as the corresponding eddy-viscosity.

---

*It must be pointed that the *isotropic* eddy-viscosity must be considered as a qualitative quantity because rapid rotation leads to strongly anisotropic turbulent flows (e.g. Julien et al. 2012).

†As Stevenson (1979), we define L_c as the smallest length-scale characterizing the dominant convective mode with a wave vector k_c.

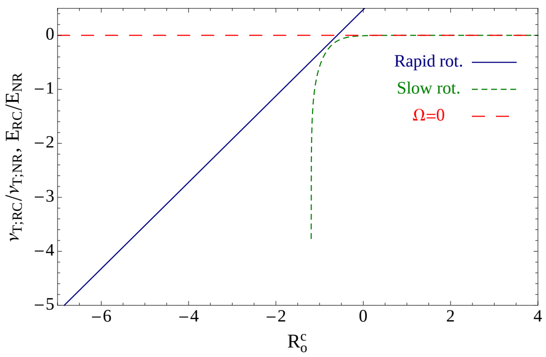

Fig. 2. The ratio $\nu_{T;RC}/\nu_{T;NR}$ (and E_{RC}/E_{NR}) as a function of R_o^c. The small-dashed green and continuous blue lines correspond to the slow- and rapid-rotation asymptotic regimes respectively. The red long-dashed line corresponds to the non-rotating case.

3 Consequences for the linear tidal dissipation

The linear response of planetary and stellar convection zones to tidal perturbations is constituted by the superposition of a non-wave like displacement, the equilibrium tide, and of tidally excited inertial waves, the dynamical tide. The restoring force of inertial waves is the Coriolis acceleration. Because of their dispersion relation $\chi = 2\Omega\, k_z/|\vec{k}|$, where χ is their frequency and $\vec{k}$ their wave number, they propagate only if $\chi \in [-2\Omega, 2\Omega]$. To understand the impact of rapid rotation on the turbulent friction derived in the previous section on both the equilibrium and dynamical tides, we now consider the linear response of the Cartesian set-up studied here (cf. fig. 1) to a periodic forcing. We follow the local analytical approach, which has been introduced by Ogilvie & Lin (2004) and Auclair-Desrotour et al. (2014b) to understand tidal dissipation in convective regions, with taking into account here the inclination angle θ between the spin ($\vec{\Omega}$) and the gravity ($\vec{g}$) at M (see fig. 1).

Because of the form of the forced velocity field, the tidal dissipation spectrum and the corresponding energy dissipated per rotation period (ζ) is a complex resonant function of the normalized tidal frequency ($\omega \equiv \chi/2\Omega$). It corresponds to resonances of the inertial waves that propagate in planetary and stellar convection zones. An example of such resonant spectra is represented in fig. 3 (left panel) for $E = 10^{-4}$ and $\theta = 0$. Following Auclair-Desrotour et al. (2014b), we characterize ζ by the following characteristics:

- the non-resonant background of the dissipation spectra H_{bg}; it scales as $H_{bg} \propto E$;
- the number of resonant peaks N_{kc}; it scales as $N_{kc} \propto E^{-1/2}$;
- their width at half-height l_{mn}; it scales as $l_{mn} \propto E$;
- their height H_{mn}; it scales as $H_{mn} \propto E^{-1}$;
- the sharpness of the spectrum defined as $\Xi = H_{11}/H_{bg}$; it scales as $\Xi \propto E^{-2}$.

From now on, X_{RC} is a quantity evaluated with E_{RC} (i.e. with $\nu_{T;RC}$) while X_{NR} is computed using E_{NR} (i.e. with $\nu_{T;NR}$).

We can thus deduce interesting conclusions from obtained results both on the equilibrium and dynamical tides.

- The *equilibrium tide*: in our local Cartesian set-up, it is represented by the non-resonant background H_{bg}. Using Eq. (2.7), we thus deduce that its efficiency scales as $\Omega^{-9/5}$ in the regime of rapid rotation. This loss of efficiency of the equilibrium tide in rapidly rotating convective regions is illustrated in fig. 3 where we plotted the ratio $H_{bg;RC}/H_{bg;NR}$ as a function of R_o^c.
- The *dynamical tide:* we use scaling laws obtained for the resonances of tidal inertial waves. We deduce that as soon as studied convective regions are in the regime of rapid rotation, their number and height respectively increase as $N_{kc} \propto \Omega^{9/10}$ and $H_{mn} \propto \Omega^{9/5}$ while their width decreases as $l_{mn} \propto \Omega^{-9/5}$. The

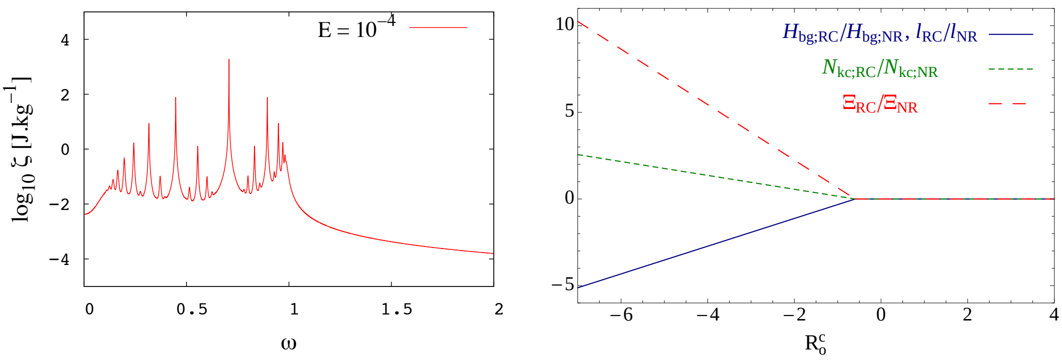

Fig. 3. Left: the complex variation of tidal dissipation in convective layers as a function of the normalized tidal frequency $\omega \equiv \chi/2\Omega$ for $E = 10^{-4}$ and $\theta = 0$. **Right:** variation of $H_{\rm bg;RC}/H_{\rm bg;NR}$, $l_{\rm RC}/l_{\rm NR}$, $N_{\rm kc;RC}/N_{\rm kc;NR}$, and $\Xi_{\rm RC}/\Xi_{\rm NR}$ as a function of $R_{\rm o}^{\rm c}$ in logarithmic scales.

sharpness of ζ is increased as $\Xi \propto \Omega^{18/5}$. This variation of the properties of the resonant tidal dissipation spectra is illustrated in fig. 3 (right panel) where we plot the ratios $N_{\rm kc;RC}/N_{\rm kc;NR}$, $l_{\rm RC}/l_{\rm NR}$, and $\Xi_{\rm RC}/\Xi_{\rm NR}$ as a function of $R_{\rm o}^{\rm c}$. As demonstrated by Auclair-Desrotour et al. (2014a), this has important consequences for the evolution of the spin of the body and of the orbits of the companions. For example, the relative migration induced by a resonance scales as $\Delta a/a \equiv l_{mn}\, \Xi^{1/4} \propto \Omega^{-9/10}$.

4 Conclusions

Thanks to the results obtained by Barker et al. (2014) on the scalings of velocities and length scales in rotating turbulent convection zones, we proposed a new prescription for the eddy-viscosity coefficient that allows to describe the linear tidal friction in such regions as a function of the convective Rossby number ($R_{\rm o}^{\rm c}$). In their work, Barker et al. (2014) indeed confirmed scalings as a function of rotation that have been first derived by Stevenson (1979) using mixing-length theory. Using these results, we straightforwardly derived our new prescription for the turbulent friction that depends on rotation and that generalizes previous studies where its action was ignored. We then demonstrated that the eddy-viscosity is decreased by several orders of magnitude in the rapidly rotating regime that leads to a deep modification of the tidal dissipation spectrum. As demonstrated by Auclair-Desrotour et al. (2014a), it must be taken into account in the simulation of the dynamical evolution of planetary systems. Indeed, the angular velocity of their components vary along their evolution because of applied tidal (and electromagnetic) torques that are themselves function of the rotation rate. In a forthcoming work, we will apply our new prescription to the evolution of star-planet and planet-moon systems and of multiple stars. Direct non-linear interactions and couplings between tidal waves and turbulent convection will also be examined.

References

Auclair-Desrotour, P., Le Poncin-Lafitte, C., & Mathis, S. 2014a, A&A, 561, L7

Auclair-Desrotour, P., Mathis, S., & Le Poncin-Lafitte, C. 2014b, ArXiv e-prints

Barker, A. J., Dempsey, A. M., & Lithwick, Y. 2014, ArXiv e-prints

Brown, B. P., Browning, M. K., Brun, A. S., Miesch, M. S., & Toomre, J. 2008, ApJ, 689, 1354

Chandrasekhar, S. 1953, Royal Society of London Proceedings Series A, 217, 306

Gallet, F. & Bouvier, J. 2013, A&A, 556, A36

Goldreich, P. & Keeley, D. A. 1977, ApJ, 211, 934

Goodman, J. & Oh, S. P. 1997, ApJ, 486, 403

Hut, P. 1981, A&A, 99, 126

Julien, K., Knobloch, E., Rubio, A. M., & Vasil, G. M. 2012, Physical Review Letters, 109, 254503

Laskar, J., Boué, G., & Correia, A. C. M. 2012, A&A, 538, A105

Mathis, S. & Remus, F. 2013, in Lecture Notes in Physics, Berlin Springer Verlag, Vol. 857, Lecture Notes in Physics, Berlin Springer Verlag, ed. J.-P. Rozelot & C. . Neiner, 111–147

Ogilvie, G. I. 2014, ARA&A, 52, 171

Ogilvie, G. I. & Lesur, G. 2012, MNRAS, 422, 1975

Ogilvie, G. I. & Lin, D. N. C. 2004, ApJ, 610, 477

Ogilvie, G. I. & Lin, D. N. C. 2007, ApJ, 661, 1180

Penev, K., Sasselov, D., Robinson, F., & Demarque, P. 2007, ApJ, 655, 1166

Remus, F., Mathis, S., & Zahn, J.-P. 2012, A&A, 544, A132

Stevenson, D. J. 1979, Geophysical and Astrophysical Fluid Dynamics, 12, 139

Zahn, J. P. 1966, Annales d'Astrophysique, 29, 489

Zahn, J.-P. 1989, A&A, 220, 112