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THE TIME TRANSFER FUNCTIONS: AN EFFICIENT TOOL TO COMPUTE RANGE, DOPPLER AND ASTROMETRIC OBSERVABLES

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Abstract. Determining range, Doppler and astrometric observables is of crucial interest for modelling and analyzing space observations. We recall how these observables can be computed when the travel time of a light ray is known as a function of the positions of the emitter and the receiver for a given instant of reception (or emission). For a long time, such a function–called a reception (or emission) time transfer function–has been almost exclusively calculated by integrating the null geodesic equations describing the light rays. However, other methods avoiding such an integration have been considerably developped in the last twelve years. We give a survey of the analytical results obtained with these new methods up to the third order in the gravitational constant G for a mass monopole. We briefly discuss the case of quasi-conjunctions, where higher-order enhanced terms must be taken into account for correctly calculating the effects. We summarize the results obtained at the first order in G when the multipole structure and the motion of an axisymmetric body is taken into account. We present some applications to on-going or future missions like Gaia and Juno. We give a short review of the recent works devoted to the numerical estimates of the time transfer functions and their derivatives.

Keywords: subject, verb, noun, apostrophe

1 Observables computed from the Time Transfer Functions

Many observations in the Solar System rest on the measurement of the travel time of light rays. Modelling the light propagation requires a mathematical tool defined as follows. Assume that space-time is covered by a single system of coordinates $x^0 = ct, \boldsymbol{x} = (x^i)$, where $i = 1, 2, 3$. Consider a light ray emitted at time t_A at a point of spatial coordinates $\boldsymbol{x}_A$ and received at time t_B at a point of spatial coordinates $\boldsymbol{x}_B$. Here, light rays are null geodesic paths (light propagating in a vacuum). The light travel time $t_B - t_A$ may be regarded as a function of the variables $\boldsymbol{x}_A, t_B, \boldsymbol{x}_B$, so that one can write

$$t_B - t_A = \mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B). \tag{1.1}$$

$\mathcal{T}_r$ may be called the "(reception) time transfer function" (TTF)*. As we shall see below, the interest of this function is not confined to the range experiments: knowing $\mathcal{T}_r$ is sufficient for modelling observations based on the Doppler-tracking or the gravitational bending of light (astrometry).

1.1 Radioscience observables

Suppose that the above-mentioned signal is exchanged between two observers $\mathcal{O}_A$ and $\mathcal{O}_B$. The range observable can directly be computed from the TTF as it is defined as the difference of proper time between the reception and emission of the signal

$$R(\tau_B) = c(\tau_B - \tau_A) = c(\tau_B - \tau_A(t_A = t_B - \mathcal{T})). \tag{1.2}$$

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*In this communication, we generally omit the term "reception" for the sake of brevity. Note that similar results can be derived from the "(emission) time transfer function" $\mathcal{T}_e$ defined by $t_B - t_A = \mathcal{T}_e(t_A, \boldsymbol{x}_A, \boldsymbol{x}_B)$.

Let ν_A and ν_B be the frequencies of the signal as measured at $(ct_A, \boldsymbol{x}_A)$ by $\mathcal{O}_A$ and at $(ct_B, \boldsymbol{x}_B)$ by $\mathcal{O}_B$, respectively. The ratio ν_B/ν_A is given by (see, e.g., (see, e.g., Teyssandier et al. 2008, and Refs. therein)

$$\frac{\nu_B}{\nu_A} = \frac{[(g_{00} + 2g_{0i}\beta^i + g_{ij}\beta^i\beta^j)^{1/2}]_{x_A}}{[(g_{00} + 2g_{0i}\beta^i + g_{ij}\beta^i\beta^j)^{1/2}]_{x_B}} \frac{(k_0)_{x_B}}{(k_0)_{x_A}} \frac{1 + (\beta^i \widehat{k}_i)_{x_B}}{1 + (\beta^i \widehat{k}_i)_{x_A}}, \tag{1.3}$$

where the quantities $g_{\alpha\beta}$ are the components of the metric, $\beta^i_{x_A} = [dx^i_A/cdt]_{t_A}$ and $\beta^i_{x_B} = [dx^i_B/cdt]_{t_B}$ are the coordinate velocities divided by c of $\mathcal{O}_A$ at time t_A and $\mathcal{O}_B$ at time t_B, respectively. The quantities $\widehat{k}_i$ are defined by $\widehat{k}_i = k_i/k_0$, where the k_α are the covariant components of the vector k^μ tangent to the light ray described by affine parametric equations. One has (Le Poncin-Lafitte et al. 2004)

$$\left(\widehat{k}_i\right)_A = c\frac{\partial \mathcal{T}_r}{\partial x^i_A}, \qquad \left(\widehat{k}_i\right)_B = -c\frac{\partial \mathcal{T}_r}{\partial x^i_B}\left[1 - \frac{\partial \mathcal{T}_r}{\partial t_B}\right]^{-1}, \qquad \frac{(k_0)_B}{(k_0)_A} = 1 - \frac{\partial \mathcal{T}_r}{\partial t_B}. \tag{1.4}$$

Substituting these relations in (1.3) yields ν_B/ν_A in terms of the derivatives of the TTF as follows

$$\frac{\nu_B}{\nu_A} = \frac{[(g_{00} + 2g_{0i}\beta^i + g_{ij}\beta^i\beta^j)^{1/2}]_{x_A}}{[(g_{00} + 2g_{0i}\beta^i + g_{ij}\beta^i\beta^j)^{1/2}]_{x_B}} \frac{1 - \frac{\partial \mathcal{T}_r}{\partial t_B} - c\beta^i_{x_B}\frac{\partial \mathcal{T}_r}{\partial x^i_B}}{1 + c\beta^i_{x_A}\frac{\partial \mathcal{T}_r}{\partial x^i_A}}, \tag{1.5}$$

a formula which can also be inferred without using (1.3), as it is shown in Hees et al. (2012).

1.2 Astrometric observables

Let $\{\lambda_{\underline{\alpha}}, \underline{\alpha} = 0, 1, 2, 3\}$ be an orthonormal comoving tetrad attached to $\mathcal{O}_B$ ($\lambda_{\underline{0}}$ coincides with the unit 4-velocity vector of $\mathcal{O}_B$). The direction of the light ray as measured by $\mathcal{O}_B$ is defined by a unit vector proportional to the orthogonal projection of k^μ on the rest frame of $\mathcal{O}_B$ at x_B. The spatial components $n^{\underline{i}}$ of this vector in the basis $\{\lambda_{\underline{i}}\}$ is given by (see, e.g. Brumberg 1991)

$$n^{\underline{i}} = -\left(\frac{\lambda^0_{\underline{i}} + \lambda^j_{\underline{i}}\widehat{k}_j}{\lambda^0_{\underline{0}} + \lambda^j_{\underline{0}}\widehat{k}_j}\right)_{x_B}, \tag{1.6}$$

where $\lambda^\mu_{\underline{\alpha}}$ denote the components of the 4-vector $\lambda_{\underline{\alpha}}$ in the natural basis associated to the coordinates (x^μ). It follows from (1.4) that each $n^{\underline{i}}$ can be expressed in terms of the derivatives of the TTF.

An analogous conclusion can be drawn for the angular separation ϕ between two point-like sources S and S' as measured by $\mathcal{O}_B$ at x_B. Indeed, one has (Teyssandier & Le Poncin-Lafitte 2006)

$$\sin^2\frac{\phi}{2} = -\frac{1}{4}\left[\frac{\left(g_{00} + 2g_{0k}\beta^k + g_{kl}\beta^k\beta^l\right)g^{ij}(\widehat{k}_i - \widehat{k}'_i)(\widehat{k}_j - \widehat{k}'_j)}{(1 + \beta^m\widehat{k}_m)(1 + \beta^l\widehat{k}'_l)}\right]_B, \tag{1.7}$$

where the quantities $\widehat{k}_i$ and $\widehat{k}'_i$ are related to the light rays arriving from S and S', respectively.

2 A survey of the method proposed to compute the TTFs

Two approaches exist to determine the light propagation in metric theories of gravity. The most widespread method consists in solving the null geodesic equations. Analytical solutions have been developed within the first post-Newtonian (1pN) or first post-Minkowksian (1pM) approximation dealing with static monopoles (Shapiro 1964), static mass multipole moments (Kopeikin 1997), moving monopoles (Kopeikin & Schäfer 1999; Klioner 2003b), moving multipole moments (Kopeikin & Makarov 2007),. . . After the pioneering papers by Richter & Matzner (1983) and Brumberg (1987), an analytical solution has been derived within the 2pM approximation for a static monopole, with a metric containing three arbitrary post-Newtonian parameters (Klioner & Zschocke 2010). Finally, the gravitational deflection of the image of a star when observed at a finite distance from a static monopole has been obtained up to the 2pM order in Ashby & Bertotti (2010). On the other hand, a numerical treatment based on a shooting method has been proposed in San Miguel (2007).

The other approach enables to determine the TTFs without integrating the null geodesic equations. Initially grounded on Synge's world function (see John (1975) for the Schwarzschild space-time, and then Linet & Teyssandier (2002), Le Poncin-Lafitte et al. (2004) for much more general cases), this approach is now based on the direct determination of the TTFs (Teyssandier & Le Poncin-Lafitte 2008).

3 Post-Minkowskian expansion of the TTF

We assume that the metric may be expanded in a series in powers of the gravitational constant G:

$$g_{\mu\nu}(x, G) = \eta_{\mu\nu} + \sum_{n=1}^{\infty} g_{\mu\nu}^{(n)}(x, G), \tag{3.1}$$

where $\eta_{\mu\nu} = \text{diag}\,\{1, -1, -1, -1\}$ is the Minkowski metric and $g_{\mu\nu}^{(n)}(x, G)$ stands for the term of order G^n. Then, it may be supposed that $\mathcal{T}_r$ is represented by an asymptotic expansion in a series in powers of G:

$$\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \frac{R_{AB}}{c} + \sum_{n=1}^{\infty} \mathcal{T}_r^{(n)}(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B), \tag{3.2}$$

where $R_{AB} = |\boldsymbol{x}_B - \boldsymbol{x}_A|$ and $\mathcal{T}_r^{(n)}$ stands for the perturbation term of order G^n. It is shown in Teyssandier & Le Poncin-Lafitte (2008) that each $\mathcal{T}_r^{(n)}$ can be expressed by an iterative procedure as a line integral whose the integrand involves only the terms $g_{\mu\nu}^{(k)}$ and $\mathcal{T}_r^{(l)}$ such that $k \leq n-1$, $l \leq n-1$, with an integration taken along the straight line passing through x_B defined by

$$x^\alpha = z^\alpha(\lambda), \quad z^0(\lambda) = x_B^0 - \lambda R_{AB}, \quad z^i(\lambda) = x_B^i - \lambda(x_B^i - x_A^i), \quad 0 \leq \lambda \leq 1. \tag{3.3}$$

So, computing the TTFs never requires the knowledge of the real null geodesics followed by the photons.

4 Application to static, spherically symmetric space-times

The procedure outlined in section 3 allows the determination of the TTF and the direction of light propagation in a static spherically symmetric space-time at any order in G (Teyssandier 2014). This determination can also be obtained by an iterative solution of an integro-differential equation derived from the null geodesic equations (Linet & Teyssandier 2013). Denoting by M the mass of the central body and assuming the metric to be a generalization of the Schwarzschild ds^2 written in the form

$$ds^2 = \left(1 - \frac{2m}{r} + 2\beta\frac{m^2}{r^2} - \frac{3}{2}\beta_3\frac{m^3}{r^3} + \cdots\right)(dx^0)^2 - \left(1 + 2\gamma\frac{m}{r} + \frac{3}{2}\epsilon\frac{m^2}{r^2} + \frac{1}{2}\gamma_3\frac{m^3}{r^3} + \cdots\right)d\boldsymbol{x}^2, \tag{4.1}$$

where $r = |\boldsymbol{x}|$, $m = GM/c^2$ and the coefficients $\beta, \beta_3, \gamma, \epsilon, \gamma_3$, are post-Newtonian parameters equal to 1 in general relativity, the two methods lead to expressions† as follow for the first three terms in Eq. (3.2):

$$\mathcal{T}^{(1)}(\boldsymbol{x}_A, \boldsymbol{x}_B) = \frac{(1+\gamma)m}{c}\ln\left(\frac{r_A + r_B + R_{AB}}{r_A + r_B - R_{AB}}\right), \tag{4.2}$$

$$\mathcal{T}^{(2)}(\boldsymbol{x}_A, \boldsymbol{x}_B) = \frac{m^2}{r_A r_B}\frac{R_{AB}}{c}\left[\kappa\frac{\arccos \boldsymbol{n}_A.\boldsymbol{n}_B}{|\boldsymbol{n}_A \times \boldsymbol{n}_B|} - \frac{(1+\gamma)^2}{1 + \boldsymbol{n}_A.\boldsymbol{n}_B}\right], \tag{4.3}$$

$$\mathcal{T}^{(3)}(\boldsymbol{x}_A, \boldsymbol{x}_B) = \frac{m^3}{r_A r_B}\left(\frac{1}{r_A} + \frac{1}{r_B}\right)\frac{R_{AB}}{c(1 + \boldsymbol{n}_A.\boldsymbol{n}_B)}\left[\kappa_3 - (1+\gamma)\kappa\frac{\arccos \boldsymbol{n}_A.\boldsymbol{n}_B}{|\boldsymbol{n}_A \times \boldsymbol{n}_B|} + \frac{(1+\gamma)^3}{1 + \boldsymbol{n}_A.\boldsymbol{n}_B}\right], \tag{4.4}$$

where $\boldsymbol{n}_A = \boldsymbol{x}_A/r_A$, $\boldsymbol{n}_B = \boldsymbol{x}_B/r_B$ and $\kappa = 2(1+\gamma) - \beta + 3/4\varepsilon$, $\kappa_3 = 2\kappa - 2\beta(1+\gamma) + (3\beta_3 + \gamma_3)/4$.

Equation (4.2) is equivalent to the well-kown formula due to Shapiro and (4.3) recovers the expression already obtained in Teyssandier & Le Poncin-Lafitte (2008), and then in Klioner & Zschocke (2010). On the other hand, (4.4) is a recent result and shows the fecondity of the new procedures.

It follows from Eqs. (4.2)-(4.4) that at least for $n \leq 3$, an enhancement of the contribution proportional to $(1+\gamma)^n$ appears in configurations of quasi-conjunction, i.e. when the unit 3-vectors $\boldsymbol{n}_A$ and $\boldsymbol{n}_B$ are almost opposite $(1 + \boldsymbol{n}_A.\boldsymbol{n}_B \sim 0)$. A result inferred in Ashby & Bertotti (2010) by an 'asymptotic reasoning' is thus rigorously confirmed. The 2pM enhanced term in (4.3) will be required for analyzing data in future missions like for example BepiColombo (Iess et al. 2009), as it may be seen on Figs. 2 and 3 in Hees et al. (2014b). The

†Note that owing to the static character of the metric, $\mathcal{T}_r$ does not depend on t_B. So we may remove the index r.

3pM enhanced contribution from the Sun may reach 30 ps for a light ray grazing the Sun (see Table 1 in Linet & Teyssandier 2013). Taking this contribution into account will therefore be necessary for modelling space mission proposals like ODYSSEY (Christophe et al. 2009), LATOR (Turyshev 2009) or ASTROD (Braxmaier et al. 2012), designed to measure the 1pN parameter γ at the level of 10^{-7}-10^{-8}.

The light deflection has been calculated and discussed within the 2pM approximation in Klioner & Zschocke (2010), Ashby & Bertotti (2010) and Teyssandier (2012). The enhanced 2pM term, proportional to $(1+\gamma)^2$, can reach 16 microarcsecond (μas) for a ray grazing Jupiter (see right of Fig. 2) and is therefore required in the analysis of Gaia data (see, e.g., de Bruijne 2012). In Linet & Teyssandier (2013) and Hees et al. (2014b), it is noted that for a ray grazing the Sun, the 2pM and 3pM enhanced contributions amount to 3 milliarcsecond (mas) and 12 μas, respectively. The last value is to be compared with the 2pM contribution due to the 2pN parameter κ, as illustrated on the left of Fig. 2.

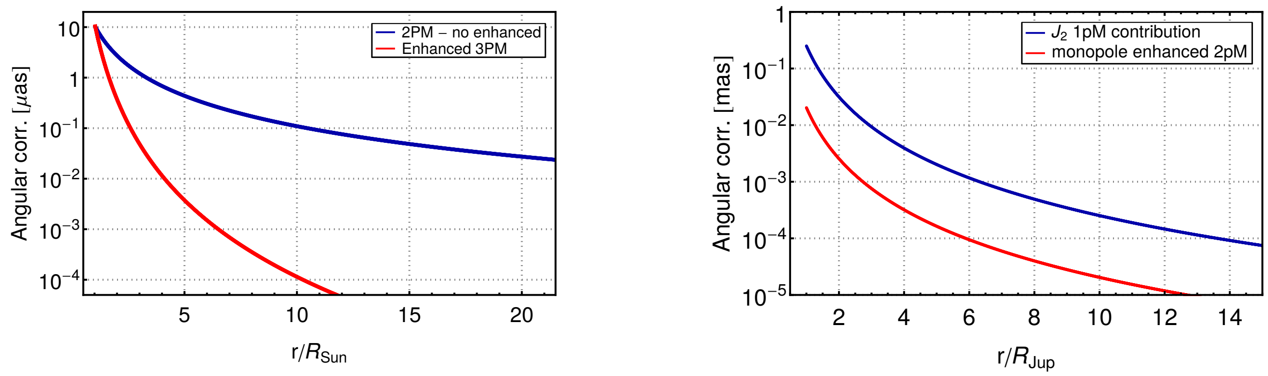

Fig. 1. Left: Contribution of the 2pM term proportional to κ and the 3pM enhanced term on the light deflection for a Sun grazing ray. – Right: Contribution of Jupiter J_2 at 1pM order and contribution of the enhanced 2pM Jupiter monopole term on the deflection of a Jupiter grazing light ray.

5 Effects due to the asphericity and/or the motion of bodies

The gravitational potential of an axisymmetric body is parametrized amongst others by its mass multipole moments J_n. Using a property previously established in Kopeikin (1997) and recovered later (see Teyssandier et al. 2008, and Refs. therein), explicit formulas for the contributions of each J_n to the TTF and its first derivatives have been given in Le Poncin-Lafitte & Teyssandier (2008). Thus, it becomes possible to calculate the influence of any J_n on the gravitational light deflection. These results generalize the expressions previously obtained in various papers for $n=1$ and $n=2$ (see, e.g, Klioner 2003a; Kopeikin & Makarov 2007, and Refs. therein). Recall that the Jupiter J_2 must be taken into account in the analysis of Gaia (see Crosta & Mignard 2006, and references therein) or VLBI observations (see the right of Fig. 1) since it produces a deflection amounting to 240 μas for a grazing light ray. A similar conclusion holds for the Juno mission (see Anderson et al. (2004)) since it is shown in Hees et al. (2014a) that the influence of the quadrupole moment of Jupiter reaches the level of the cm for the range and the level of 10 μm/s for the Doppler (see left of Fig. 2). Some of these effects will be relevant in the data reduction since the expected accuracies for Juno are of 10 cm on the range and 1 μm/s on the Doppler.

The procedure outlined in section 3 noticeably facilitates the determination of the TTF of a uniformly moving axisymmetric body within the 1pM approximation. Denote by $\tilde{\mathcal{T}}_r^{(1)}$ the 1pM TTF corresponding to the body at rest. When this body is uniformly moving with a coordinate velocity $\boldsymbol{v}=c\boldsymbol{\beta}$, it is shown in Hees et al. (2014a) that the 1pM TTF can be written as

$$\mathcal{T}_r^{(1)}(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \Gamma(1-\boldsymbol{N}_{AB}.\boldsymbol{\beta})\tilde{\mathcal{T}}_r^{(1)}(\boldsymbol{R}_A + \Gamma R_{AB}\boldsymbol{\beta}, \boldsymbol{R}_B) , \tag{5.1}$$

where $\Gamma = (1-\beta^2)^{-1/2}$ is the Lorentz factor and

$$\boldsymbol{R}_X = \boldsymbol{x}_X - \boldsymbol{x}_p(t_0) + \frac{\Gamma^2}{1+\Gamma}\boldsymbol{\beta}\left[\boldsymbol{\beta}.(\boldsymbol{x}_X - \boldsymbol{x}_p(t_0))\right] - \Gamma\boldsymbol{v}(t_B - t_0) , \tag{5.2}$$

with $\boldsymbol{x}_p(t_0)$ being the position of the deflecting body at an arbitrary time t_0 usually chosen between $t_B - R_{AB}/c$ and t_B. This recent and general result is particularly simple. The first derivatives of the right-hand side of (5.1) are easily calculated. For a moving monopole, using Eq. (4.2) for $\tilde{\mathcal{T}}_r^{(1)}$ and Eq. (5.1) gives

$$\mathcal{T}_r^{(1)}(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = (1+\gamma)m\Gamma(1-\boldsymbol{\beta}.\boldsymbol{N}_{AB}) \ln \frac{|\boldsymbol{R}_A + \Gamma R_{AB}\boldsymbol{\beta}| + R_B + \Gamma R_{AB}(1-\boldsymbol{\beta}.\boldsymbol{N}_{AB})}{|\boldsymbol{R}_A + \Gamma R_{AB}\boldsymbol{\beta}| + R_B - \Gamma R_{AB}(1-\boldsymbol{\beta}.\boldsymbol{N}_{AB})}. \tag{5.3}$$

This formula recovers the expression obtained in Kopeikin & Schäfer (1999) and Klioner (2003a) using longer calculations. A low velocity expansion of this result is obtained in Bertone et al. (2014). To finish, let us mention that using a similar method but a symmetric trace free (STF) decomposition of the gravitational potential, Soffel & Han (2014) have also determined the expression of the TTF produced by a moving body with arbitrary static multipoles, but their result is only valid in the slow velocity approximation.

In Hees et al. (2014a), these results are applied in the context of the Juno mission to discuss the effects of the mass and the quadrupole moment of Jupiter when the motion of this planet is taken into account. The effect of the motion of Jupiter's monopole is represented on the right of Fig. 2. This contribution is smaller than the expected Juno Doppler accuracy and can safely be ignored in the reduction of the observations. Nevertheless, it is important to point out that this numerical estimate depends highly on the geometry of the probe orbit and should be reassessed in the context of other space missions. In particular, this contribution depends on the quantity $\boldsymbol{\beta}.\boldsymbol{N}_{AB}$ and on the presence of conjunctions (which is not the case for Juno owing its polar orbit, but will be the case in other missions). The deflection of light produced by the motion of Jupiter monopole is of the order of 0.04 μas for a grazing light ray and can safely be ignored for current observations.

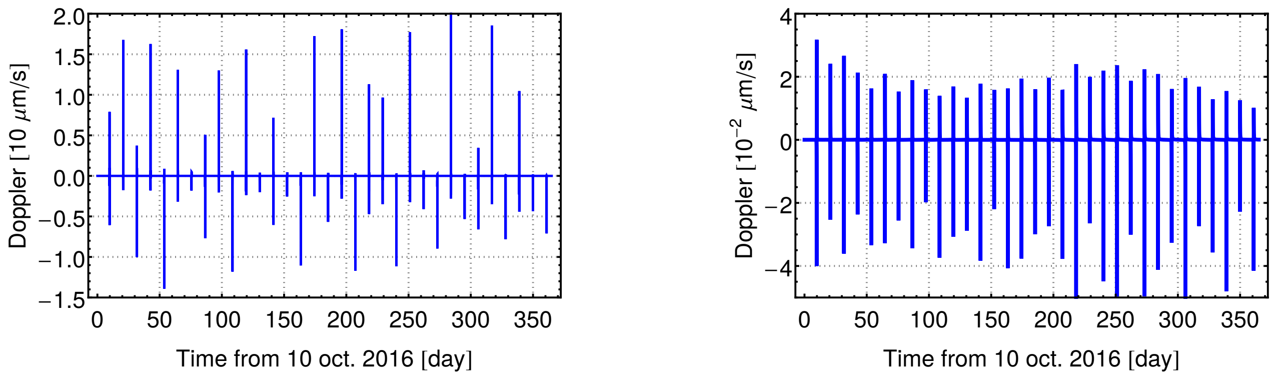

Fig. 2. Left: Effect of Jupiter J_2 on a Doppler link between Juno and Earth. Right: Effect of Jupiter's velocity on a Doppler link between Juno and Earth.

6 Numerical determination of the TTFs and their derivatives

The TTF formalism lends itself well to the numerical simulations of the light propagation in curved space-time. This is useful when no analytical expressions can be found or when systematic comparisons of the propagation of light in different space-times are discussed. This approach is fully developed within the 2pM approximation in Hees et al. (2014b). The iterative procedure mentioned in Sect. 3 gives

$$\mathcal{T}_r^{(1)} = \int_0^1 n\left[z^\alpha(\lambda); g_{\alpha\beta}^{(1)}, R_{AB}\right] d\lambda, \tag{6.1}$$

$$\frac{\partial \mathcal{T}_r^{(1)}}{\partial x^i_{A/B}} = \int_0^1 n_{A/B}\left[z^\alpha(\lambda); g_{\alpha\beta}^{(1)}, g_{\alpha\beta,\sigma}^{(1)}, \boldsymbol{x}_A, \boldsymbol{x}_B\right] d\lambda, \tag{6.2}$$

for $\mathcal{T}_r^{(1)}$ and its first derivatives, and then

$$\mathcal{T}_r^{(2)} = \int_0^1 \int_0^1 l\left[z^\alpha(\mu\lambda); g_{\alpha\beta}^{(2)}, g_{\alpha\beta}^{(1)}, g_{\alpha\beta,\sigma}^{(1)}, \boldsymbol{x}_A, \boldsymbol{x}_B\right] d\mu\, d\lambda\,, \tag{6.3}$$

$$\frac{\partial \mathcal{T}_r^{(2)}}{\partial x^i_{A/B}} = \int_0^1 \int_0^1 l_{A/B}\left[z^\alpha(\mu\lambda); g^{(2)}_{\alpha\beta}, g^{(2)}_{\alpha\beta,\sigma}, g^{(1)}_{\alpha\beta}, g^{(1)}_{\alpha\beta,\sigma}, g^{(1)}_{\alpha\beta,\sigma\delta}, \boldsymbol{x}_A, \boldsymbol{x}_B\right] d\mu\, d\lambda\ , \quad (6.4)$$

for $\mathcal{T}_r^{(2)}$ and its first derivatives, where the functions n, n_A, n_B, l, l_A and l_B can be explicitly written (see Hees et al. 2014b). All the integrations are taken over the straight line defined by Eqs. (3.3).

This kind of procedure avoids the numerical integration of the full set of geodesic equations, which is unnecessarily time consuming since we are only concerned by a single 'time function'. It has been successfully applied to simulate range, Doppler and astrometric observations within some alternative theories of gravity in order to find signatures differing from the predictions of general relativity (see Hees et al. 2012, 2014c, 2015), and more recently to compute the propagation of light in the field of arbitrarily moving monopoles, when no analytical solution is available (Hees et al. 2014a).

7 Conclusion

This survey shows that the TTF formalism is a powerful tool for computing the range, Doppler and astrometric (VLBI) observables involved in Solar System experiments. The iterative method summarized in section 3 is very effective in deriving analytical and numerical solutions. The simplicity of this method relies mainly on the fact that one never has to determine the real trajectory of the photon in order to perform an explicit calculation of the TTF. We have reviewed some of the analytical expressions derived using this formalism. This method has been successfully applied to determine the light propagation in a static spherically symmetric space-time up to the 3pM order and a generic procedure enabling to compute higher order terms has been developed. It has also been applied to determine the influence of the motion and asphericity of bodies on the light propagation. The result is obtained by simple calculations. We have assessed the influence of different terms in the observation of space missions like Gaia or Juno. Finally, the TTF formalism turns out to be very well adapted to the numerical simulations of the effects observable in the Solar System.

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