SF2A 2015
F. Martins, S. Boissier, V. Buat, L. Cambrésy and P. Petit (eds)

EQUILIBRIUM OF SELF-GRAVITATING TORI IN SPHERICAL GRAVITATIONAL AND DIPOLAR MAGNETIC FIELDS

A.Trova[1], V. Karas[1], P. Slaný[2] and J. Kovář[2]

Abstract. We investigate a new model for equilibria of self-gravitating fluid tori with electric charge that are embedded in gravitational potential and a dipolar magnetic field produced by the central mass. We find that the shape and the vertical structure of the massive torus are influenced by effects of self-gravity which were neglected in our previous work (Slaný et al. 2013). We show the impact of self-gravity on the morphology of figures of equilibrium, depending on the rotation of the fluid and the strength of the magnetic field.

Keywords: hydrodynamics – tori: rotation – gravitation – methods: numerical – magnetic field.

1 Introduction

In active galactic nuclei (AGN), the study of equilibrium of toroidal figure of perfect fluid are important to understand the physics and structure of accretion discs (Kozlowski et al. 1978; Abramowicz et al. 1978). This subject has been treated in great detail (Stuchlík et al. 2000; Font & Daigne 2002; Kucáková et al. 2011; Slaný et al. 2013; Kovář et al. 2014). AGN are composed of dusty tori and a central compact body that is frequently associated with a supermassive black hole (the mass typically $M \simeq 10^6$–$10^9 M_\odot$ (Krolik 2004; Eckart et al. 2005)). At a distance of 10^4–10^5 gravitational radii ($R_g \equiv GM/c^2 \approx 1.5(M/M_\odot)$ km) these tori become self-gravitating. At the same time this distance is large enough to reduce the effects of General Relativity (essential near the center) to negligible level (Shlosman & Begelman 1987; Huré 1998). Then we can use the Newtonian limit to study the vertical and radial structures of these objects (Frank et al. 1985).

In this paper we describe an approximate model where self-gravity of the torus material, central mass effect and non-vanishing electric charge density interact to define the radial and vertical structure of an equilibrium configuration. The idea is to use the same method as Slaný et al. (2013) and add the term of self-gravity. In section 2, we give the basic equations and assumptions of our work. In section 3, we use these equations to build a toy-model for equilibrium of tori surrounded by a central mass which produce a dipolar magnetic field and a spherical gravitational field. The section 4 is dedicated to the conclusion.

2 Basic equations and hypothesis

2.1 Equilibrium equation

The Bernoulli equation governs the tori equilibrium in the Newtonian limit. It is given by

$$-\frac{1}{\rho_{\mathrm{m}}}\overrightarrow{\nabla}P - \overrightarrow{\nabla}\Psi_{Sg} - \overrightarrow{\nabla}\Psi_c - \overrightarrow{\nabla}\Phi + \frac{\overrightarrow{\mathcal{L}}}{\rho_{\mathrm{m}}} = 0, \tag{2.1}$$

where P, Ψ_{Sg}, Ψ_c, Φ, $\mathcal{L} = q\rho_{\mathrm{m}} v_\phi \overrightarrow{e_\phi}$ and ρ_{m} are the pressure, the self-gravitating potential of the torus, the central mass potential, the rotational potential, the Lorentz force and the mass-density, respectively. The orbital

[1] Astronomical Institute, Academy of Sciences, Boční II, Prague, CZ-141 31, Czech Republic
[2] Institute of Physics, Faculty of Philosophy and Science, Silesian University in Opava Bezručovo nám. 13, CZ-746 01 Opava, Czech Republic

Test 1 $d_{\mathrm{t}}=0$	$b=1.44$, $c=-0.5$ and $r_c=25 \rightarrow e\sim 0.439$ and $a\sim 7.33\times 10^{-2}$
Test 2 $d_{\mathrm{t}}=0.1$	$b=1.44$, $c=-0.5$ and $r_c=25 \rightarrow e\sim 0.409$ and $a\sim 0.147$
Test 3 $d_{\mathrm{t}}=0.5$	$b=1.44$, $c=-0.5$ and $r_c=25 \rightarrow e\sim 0.285$ and $a\sim 0.443$.

Table 1. Description of the three tests for equatorial tori..

velocity $v_\phi=\frac{K_2}{R}$ for a constant angular velocity ($K_2=$ const), q is the specific charge. After a first integration, we can write

$$\frac{P}{\rho_{\mathrm{m}}}+\Psi_{Sg}+\Psi_c+\Phi+M=\mathrm{Const} \tag{2.2}$$

where M' is the "magnetic potential" given by $\vec{\nabla}M'=-\frac{\vec{\mathcal{L}}}{\rho_{\mathrm{m}}}$.

2.2 Hypothesis

We assumed that the fluid is stationary, axially symmetrical and symmetric with respect to the mid-plane. We are working with an orbital velocity profile with constant angular momentum. Then the centrifugal potential is given by $\Phi=\frac{K_2^2}{2R^2}$. The fluid is incompressible, $\rho_{\mathrm{m}}=$ const and is embedded in an external dipolar magnetic field, which is given, in cylindrical coordinates, by

$$B_R=\frac{3\mu ZR}{\sqrt{R^2+Z^2}^5},\quad B_\phi=0,\quad B_Z=\frac{\mu(2Z^2-R^2)}{\sqrt{R^2+Z^2}^5}. \tag{2.3}$$

Lastly we work with a specific charge distribution, $q(R,Z)$, described in Slaný et al. (2013) (named family II). In cylindrical coordinates (R,θ,Z),

$$q(R,Z)=C\left(\frac{R}{\sqrt{R^2+Z^2}}\right)^3 \tag{2.4}$$

with $C=$ Const.

3 Influence of self-gravity

Normalization of equation (2.1) is given by

$$a\tilde{P}=-d_{\mathrm{t}}\tilde{\Psi}_{\mathrm{Sg}}-\tilde{\Psi}_c-b\tilde{\Phi}-e\tilde{M}+c, \tag{3.1}$$

with a, b, d_{t}, e and c constants which depend on various parameters,such as the central mass, the torus mass, the rotation law, the specific charge and the radius of pressure maximum r_c. In particular, the constant d_{t} is the ratio of the torus mass with the central mass. It represents the strength of self-gravity. It is the main parameter of our work. We are going to vary with this parameter to see the influence of the self-gravity on the morphology of the equilibrium solutions. The tori equilibrium exists only if it can exist a maximum of pressure (mathematical conditions on the function $\tilde{P}$ given by equation 3.1). According to this conditions, we have two possibilities. The equilibrium can exist (1) for equatorial tori, and (2) for off-equatorial tori.

3.1 Equatorial tori

The method is to select a value for the maximum of pressure ($R=r_c, Z=0$) and check if the mathematical conditions are valid. Next, we have to impose a value for the constants b, c and d_{t}. The value of e are given by the conditions and a is the maximum of pressure. To see the influence of self-gravity, we vary the parameter d_{t}. We decided to choose three values $d_{\mathrm{t}}=0$ (no self-gravity) and $d_{\mathrm{t}}=0.1$ and $d_{\mathrm{t}}=0.5$. The values of other constants are in table 1.

The map, corresponding to the parameters described in table 1, are represented in figure 1. $d_{\mathrm{t}}=0$ is plotted at the left, $d_{\mathrm{t}}=0.1$ in the middle and $d_{\mathrm{t}}=0.5$ at the right. We can see that there is no change in the morphology of the solution. The pressure field has a toroidal shape for each value of d_{t}. The shape, the vertical and radial structure change with the strength of the self-gravity. The maximum of pressure raises with the value of d_{t}, see value of a in table 2.

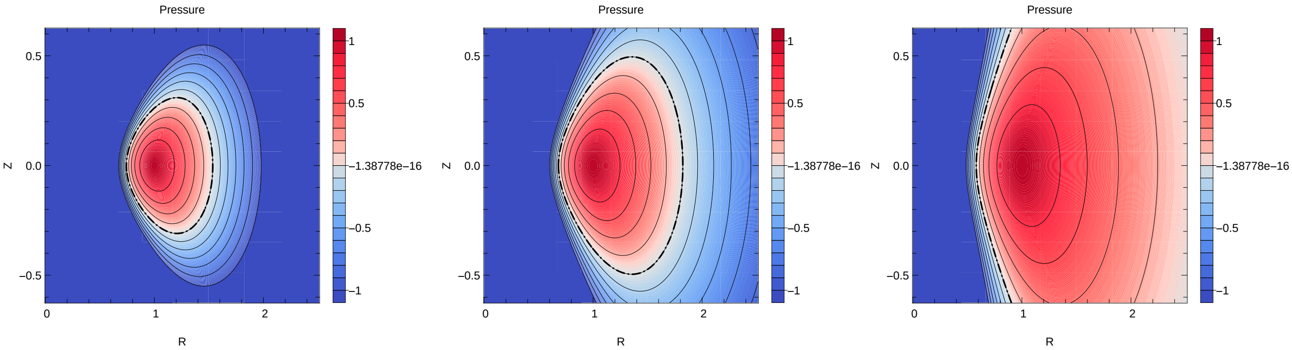

Fig. 1. Map of normalized pressure given by equation (3.1) with the parameters described in table 1. $d_{\mathrm{t}} = 0$ corresponds to the map at the left, $d_{\mathrm{t}} = 0.1$ in the middle, and $d_{\mathrm{t}} = 0.5$ on the right.

Test 1 $d_{\mathrm{t}} = 0$	$c = -0.2$ and $r_c = 20 \rightarrow b = 0.471$, $e \sim -1.12$ and $a \sim 0.114$
Test 2 $d_{\mathrm{t}} = 0.1$	$c = -0.2$ and $r_c = 20 \rightarrow b = 0.495$, $e \sim -1.21$ and $a \sim 0.151$
Test 3 $d_{\mathrm{t}} = 0.5$	$c = -0.2$ and $r_c = 20 \rightarrow b = 0.590$, $e \sim -1.58$ and $a \sim 0.299$.

Table 2. Description of the three tests for off-equatorial tori.

3.2 *Off-equatorial tori*

We set-up the same test and use the same method for off-equatorial tori too. We choose a location for the maximum of pressure ($R = r_c, Z = z_c \neq 0$), and check if the conditions are satisfied. Next we set the constant c. The parameters b and e are given by the conditions of existence of the solutions. As before a is the maximum of pressure and is given by equation (3.1). All these variables are given in table 2.

The map, corresponding to the table 2 and equation (3.1), is plotted in figure 2, $d_{\mathrm{t}} = 0$ at the left, $d_{\mathrm{t}} = 0.1$ in the middle and $d_{\mathrm{t}} = 0.5$ at the right. The same effects, as for equatorial tori, appear in this case. The maximum of pressure raises with the value of d_{t} (see value of a in table 2), the vertical and radial structure change too. The difference with the previous case is the change in the morphology of the pressure field. We can see that for $d_{\mathrm{t}} = 0$, the field has two lobes above and below the equatorial plane. But for $d_{\mathrm{t}} = 0.1$ and $d_{\mathrm{t}} = 0.5$ the two lobes are linked with each other across the equatorial plane.

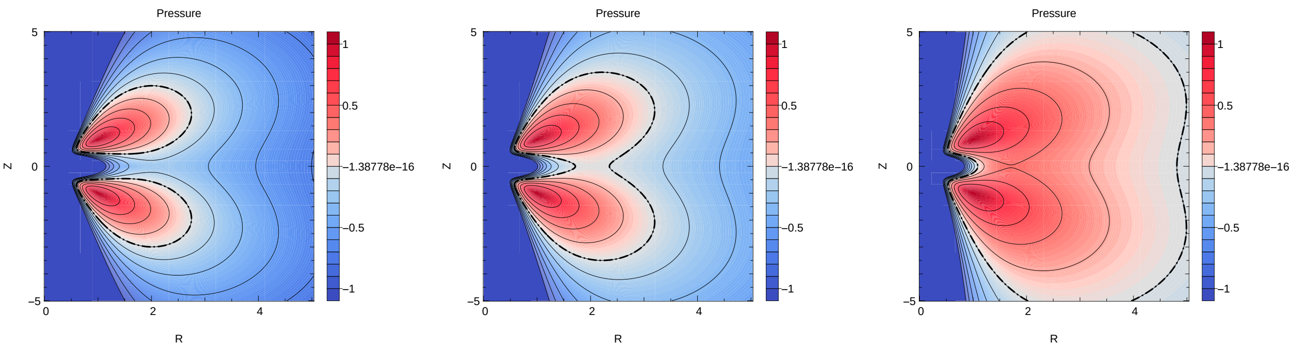

Fig. 2. same as 1 but for off-equatorial tori and for parameters described in 2.

4 Conclusions

In this paper, we analyse the impact of the self-gravity on the conditions of existence of charged fluid tori and their morphology. We consider these tori as a fluid, whose particles carry electrical charges. The fluid is considered as perfect and incompressible. The latter is embedded in the gravitational potential and the dipolar magnetic field due to the central mass and the gravitational force produced by itself (the self-gravity). Our study follow the work done by Slaný et al. (2013) where they neglect the self-gravity. We add the self-gravity term

to the equilibrium equation to find stationary tori configurations and see the impact of this self-gravitational force on the morphology of solutions.
We found different interesting results. We saw that the morphology of tori are similar to the non-selfgravitating case. We found the toroidal configuration and the toroidal off-equatorial configurations (see figure 1 and the graphic at the left of figure 2). A new morphology appears. The two toroidal off-equatorial objects are linked by the mid -plane (see graphics in the middle and at the left of figure 2). An other effect of the self-gravity is that the maximum of pressure seems to raise with the value of $d_{\rm t}$ and the torus becomes thicker, which makes sense because higher gravity implies higher pressure to balance the forces. It will be interesting to test other form for the specific charged (Trova et al. in prep) and to elaborate this model, with a non approximated self-gravity. We can use a SCF (self-consistent field) method (Ostriker & Mark 1968; Blinnikov 1975; Hachisu 1986) which consist to find by an iteration scheme the equilibrium configuration. In this case, the self-gravitational potential is calculated with Poisson's equation. The benefit of this is the possibility to study various equations of state too. Finally it will be interesting to add the magnetic field produced by the torus itself.

References

Abramowicz, M., Jaroszynski, M., & Sikora, M. 1978, A&A, 63, 221

Blinnikov, S. I. 1975, Soviet Ast., 19, 151

Eckart, A., Schödel, R., & Straubmeier, C. 2005, The black hole at the center of the Milky Way

Font, J. A. & Daigne, F. 2002, MNRAS, 334, 383

Frank, J., King, A. R., & Raine, D. J. 1985, Accretion power in astrophysics

Hachisu, I. 1986, ApJS, 61, 479

Huré, J.-M. 1998, A&A, 337, 625

Kovář, J., Slaný, P., Cremaschini, C., et al. 2014, Phys. Rev. D, 90, 044029

Kozlowski, M., Jaroszynski, M., & Abramowicz, M. A. 1978, A&A, 63, 209

Krolik, J. 2004, Nature, 429, 29

Kucáková, H., Slaný, P., & Stuchlík, Z. 2011, J. Cosmology Astropart. Phys., 1, 33

Ostriker, J. P. & Mark, J. W.-K. 1968, ApJ, 151, 1075

Shlosman, I. & Begelman, M. C. 1987, Nature, 329, 810

Slaný, P., Kovář, J., Stuchlík, Z., & Karas, V. 2013, ApJS, 205, 3

Stuchlík, Z., Slaný, P., & Hledík, S. 2000, A&A, 363, 425