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NUMERICAL SIMULATIONS OF ZERO-PRANDTL-NUMBER THERMOHALINE CONVECTION

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Abstract. Thermohaline (or fingering) convection has been used to explain chemical anomalies at the surface of red giant stars. However, recent numerical simulations suggest that the efficiency of thermohaline convection is lower than expected, and thus not sufficient to explain the observations.

One of the uncertainties of these simulations is that they have been performed in a parameter range for the Prandtl number (i.e. the ratio between viscosity and thermal diffusivity) which is far from what can be found in stellar interiors.

Using the small-Péclet-number approximation, we are able for the first time to perform simulations of thermohaline convection in a parameter domain which is relevant for stellar physics. In the present paper, we discuss the validity of this approximation and compare our results with previous simulations and models.

Keywords: Hydrodynamics, instabilities, turbulence, stars: evolution, stars: interiors

1 Introduction

Thermohaline convection (also known as fingering convection) is a mixing process which occurs in flows with stable thermal stratification and unstable chemical stratification. Originally studied in oceanography when a layer of warm salty water lays above a layer of cold fresh water (see for example Stern 1960), it has been introduced in an astrophysical context for the first time by Ulrich (1972).

In stellar interiors, the unstable chemical gradient necessary to raise the instability can have various causes. In particular, it can happen in ^{3}He-shell-burning regions of evolved stars (Charbonnel & Zahn 2007; Denissenkov 2010). It can also be due to heavy infalling material from a planetary system (Vauclair 2004; Garaud 2011; Deal et al. 2013) or a more evolved companion (Stancliffe et al. 2007). Another possibility is the accumulation of heavy elements such as iron thanks to radiative levitation (Théado et al. 2009; Vauclair & Théado 2012; Zemskova et al. 2014).

Thermohaline convection is characterised by three dimensionless numbers: (i) the so-called "density ratio" $R_0 = -{N_\mathrm{T}}^2/{N_\mu}^2$, where N_T and N_μ are the thermal and chemical Brunt-Väisälä frequencies measuring thermal and chemical stratification, (ii) the diffusivity ratio $\tau = \kappa_\mu/\kappa_\mathrm{T}$, where κ_μ and κ_t are the chemical and thermal diffusivities, and (iii) the Prandtl number $Pr = \nu/\kappa_\mathrm{T}$, where ν is the viscosity. According to Baines & Gill (1969), thermohaline convection occurs when

$$1 < R_0 < \frac{1}{\tau}. \tag{1.1}$$

The stellar regime is characterised by very low values of the diffusivity ratio τ and the Prandtl number, along with high values of the density ratio (see Garaud et al. 2015). Because of the very low Prandtl number, the flows are very turbulent and numerical simulations of astrophysical thermohaline convection is computationally expensive.

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Recent simulations in 2D (Denissenkov 2010) and 3D (Traxler et al. 2011; Brown et al. 2013) have been performed with Prandtl numbers down to 10^{-2}. Brown et al. (2013) also proposed an analytical model for thermohaline convection with a free parameter they calibrate thanks to their numerical simulations. Although there is a good agreement between the model and the numerical results, the explored parameter domain is still far from what can be found in stellar interiors.

This is our main motivation to apply the small-Péclet-number approximation (SPNA, see Lignières 1999). Assuming that because of the very high thermal diffusivity, advection is unable to modify the thermal background, the SPNA consists in a Taylor expansion in the Péclet number $Pe = UL/\kappa_{\rm T}$, where U and L are typical velocity and length scales. This approximation has been successfully used to investigate the very-high-thermal-diffusivity limit of the transport generated by the shear instability in stellar radiative zones (Prat & Lignières 2013, 2014).

The purpose of this paper is to apply the SPNA to the problem of thermohaline convection in order to reach the zero-Prandtl limit likely to describe the stellar regime. We first describe our configuration in Sect. 2, and present our results in Sect. 3. Finally, we discuss the consequences of this work and conclude in Sect. 4.

2 Configuration

In this paper, we study thermohaline convection in the Boussinesq approximation. It means that density fluctuations are neglected, except in the buoyancy term. We choose a configuration with uniform background temperature and mean molecular weight gradients $\mathrm{d}T_0/\mathrm{d}z$ and $\mathrm{d}\mu_0/\mathrm{d}z$. The dimensionless equations for the velocity $\vec{v}$ and temperature and mean molecular weight fluctuations θ and μ' are

$$\vec{\nabla}\cdot\vec{v} = 0, \tag{2.1}$$

$$\phi\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = -\vec{\nabla}P + Ra_\mu(R_0\theta - \mu')\vec{e}_z + \Delta\vec{v}, \tag{2.2}$$

$$\frac{\partial\theta}{\partial t} + \vec{v}\cdot\vec{\nabla}\theta + v_z = \frac{1}{\tau}\Delta\theta, \tag{2.3}$$

$$\frac{\partial\mu'}{\partial t} + \vec{v}\cdot\vec{\nabla}\mu' + v_z = \Delta\mu', \tag{2.4}$$

where $\phi = \kappa_\mu/\nu = \tau/Pr$ is the inverse of the Schmidt number, $Ra_\mu = \beta(\mathrm{d}\mu_0/\mathrm{d}z)L^4/(\nu\kappa_\mu)$ the chemical Rayleigh number, $\beta = (\partial\ln\rho/\partial\mu)_{P,T}$ the chemical contraction coefficient and L a typical length. These equations have been obtained by using L, L^2/κ_μ, κ_μ/L, $L\mathrm{d}T_0/\mathrm{d}z$, $L\mathrm{d}\mu_0/\mathrm{d}z$ and $\rho_0\nu\kappa_\mu/L^2$ as length, time, velocity, temperature, mean molecular weight and pressure units.

This adimensionalisation is different from the one used for example by Brown et al. (2013), which is not appropriate to investigate the limit of very high thermal diffusivities. In particular, they use a thermal diffusive time as time unit, which tends to zero when Pr tends to zero. In contrast, all our unit scales remain finite in this limit. Thus, we can use the SPNA, which modifies Eqs. (2.2) and (2.3):

$$\phi\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = -\vec{\nabla}P + Ra_\mu(R_0\tau\psi - \mu')\vec{e}_z + \Delta\vec{v}, \tag{2.5}$$

$$v_z = \Delta\psi, \tag{2.6}$$

with $\psi = \theta/\tau$. This removes the numerical issue posed by the large factor $1/\tau$ in the right-hand side of Eq. (2.3), and the stability is now given by the number $r = R_0\tau$, which according to Eq. (1.1), must verify

$$0 < r < 1. \tag{2.7}$$

We used the code Snoopy (Lesur & Longaretti 2011) to perform direct numerical simulations solving these equations. It uses a Fourier colocation method in all directions with periodic boundary conditions for fluctuations and a Runge-Kutta time scheme. A typical run is 256^3 grid points distributed over 128 cores.

3 Results

We performed first full Boussinesq simulations with decreasing Prandtl numbers down to $6.67\cdot10^{-3}$ and one SPNA simulation (corresponding to $Pr = 0$), all with the same values $r = 2/3$ and $\phi = 1$. The turbulent

diffusion coefficient is defined as

$$D_\mu = -\frac{\langle \mu' v_z \rangle}{\mathrm{d}\mu_0/\mathrm{d}z}, \tag{3.1}$$

where $\langle\rangle$ denotes the temporal and spatial average. This coefficient is related to the chemical Nusselt number Nu_μ, used notably in Traxler et al. (2011) and Brown et al. (2013), thanks to the relation

$$Nu_\mu = 1 + \frac{D_\mu}{\kappa_\mu}. \tag{3.2}$$

One of our simulations was performed at $Pr = 1/3$, $\tau = 1/3$ and $R_0 = 2$, which is close to one simulation of Traxler et al. (2011). We found a Nusselt number of 9.75, in agreement with their value of 9.5 ± 0.3. The results of our simulations are plotted in Fig. 1. It shows that the SPNA is able to describe well the transport by thermohaline convection in the limit of very small Prandtl numbers. One can also see that the corresponding

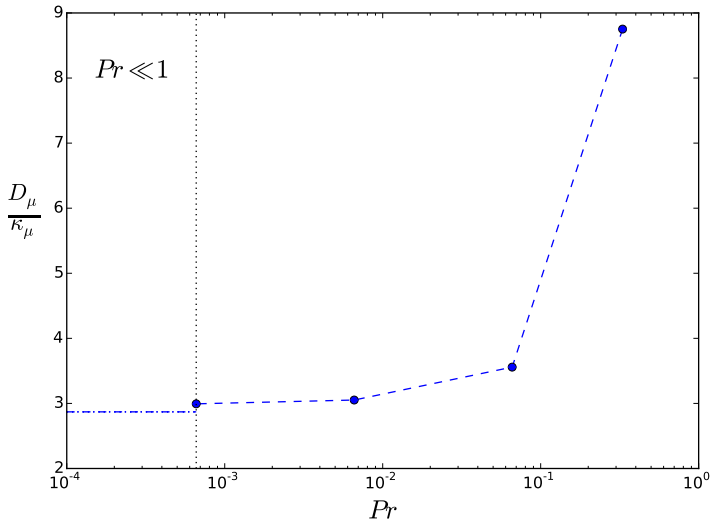

Fig. 1. Turbulent diffusion coefficient as a function of the Prandtl number at $r = 2/3$ and $\phi = 1$. Blue dots are full Boussinesq simulations and the dash-dotted line is a SPNA simulation.

asymptotic regime is almost already reached at $Pr \sim 10^{-2}$, which is the smallest value used by Brown et al. (2013).

Then we performed two series of SPNA simulations, one at constant $\phi = 1$ and various values of r, the other at constant $r = 3.33 \cdot 10^{-3}$ and various values of ϕ. Both results are plotted in Fig. 2, along with the predictions of Brown et al. (2013). The first interesting remark is that our simulations globally follow the trends given by Brown et al. (2013). However, there are sometimes significant discrepancies, especially for very small values of r or when ϕ is greater than one. The latter case is not really an issue, because we expect ϕ to be smaller than one in stellar radiative zones. These discrepancies may be due to the saturation mechanism, which is the most uncertain part of the model of Brown et al. (2013).

4 Conclusions

This study is the first application of the SPNA to thermohaline convection and our results validate its use in this context. Thanks to this approximation, we are able to reach the regime of very low Prandtl numbers, and thus to perform simulations with realistic stellar parameters. Simulations at very small r and ϕ are still challenging, but managable.

As we have seen in the previous section, our results are in relatively good agreement with the predictions of Brown et al. (2013). However, there are significant discrepancies, in particular for very small values of r and ϕ, for which we find a transport coefficient larger than the one given by Brown et al. (2013). For typical models of red giants, r and ϕ can reach values from 1 down to 10^{-5} and 10^{-4}, respectively (Lagarde 2012).

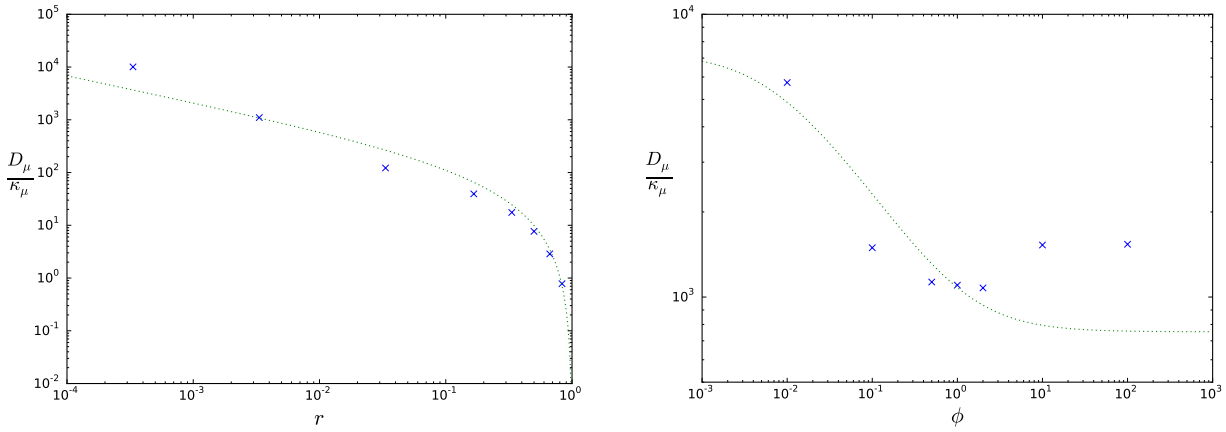

Fig. 2. Left: Turbulent diffusion coefficient as a function of r at $\phi = 1$. **Right:** The same as a function of ϕ at $r = 3.33 \cdot 10^{-3}$. The dotted lines correspond to the model of Brown et al. (2013).

The discrepancies between our simulations and Brown et al. (2013) thus occur in a regime which is relevant for red giants, and could possibly lead to a much higher mixing coefficient in the corresponding regions. This could help to reduce the disagreement between observed chemical abundances of red giants and predictions of stellar evolution using Brown et al. (2013), as highlighted by Wachlin et al. (2014). A more extensive study of the diffusion coefficient in this regime is thus needed.

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