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ASTEROSEISMIC HARE & HOUND EXERCISES: THE CASE OF β CEPHEI STARS

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Abstract. The β Cephei pulsating stars present a unique opportunity to test and probe our knowledge of the interior of massive stars. The information that we can get depends on the quality and number of observational constraints, both seismic and classical ones. The asteroseismology of β Cephei stars proceeds by a forward approach, which can result in multiple solutions, without clear indication on the level of confidence. We seek a method to derive confidence intervals on stellar parameters and investigate how these latter behave depending on the seismic data accessible to the observer. We realise forward modelling with the help of a grid of pre-computed models. We also use Monte-Carlo simulations to build confidence intervals on the inferred stellar parameters. We apply and test this method in a series of hare and hound exercises on a subset of theoretical models simulating observed stars. Results show that a set of 5 frequencies (with knowledge of their associated angular degree) yields precise seismic constraints. Significant errors on the determination of the extent of the central mixed region may result when the theoretical models do not present the same chemical mixture as the observed star.

Keywords: Asteroseismology, Stars: variables: general

1 Introduction

There are now more and more evidence for the presence of extra-mixing at the edge of the convective core in main-sequence B stars, revealed by constraints from eclipsing binaries or required to fit stellar cluster observation with help of isochrones (see e.g. Ribas et al. 2000; Gallart et al. 2005, respectively). However, the efficiency and exact nature of the processes at work remain an open issue for stellar physics(see e.g. the review by Chiosi 2007). Fortunately, among the richness of pulsating stars across the Hertzsprung-Russell diagram, main-sequence B stars can present β Cephei pulsations, which are excited by the κ mechanism (Moskalik & Dziembowski 1992). Such oscillations may present both a pressure and gravity mode character. Because these modes probe in part the layers at the border of the convective core, they offer strong constraints on the deep layers of β Cephei stars.

A goal of asteroseismology is to interpret the properties of these pulsations to deliver information on the internal conditions of stars. As a main success, the observation of rotational splitting in at least four β Cephei stars led to first constraints on the ratio of core to surface rotation rates (see e.g. the review by Goupil 2011). It also succeeded in retrieving estimates of the mass, radius and amount of core overshooting for about ten β Cephei (see review by Aerts 2013). These constraints on the core overshooting are of prime importance to get an insight on the physics underlying the extra-mixing processes. In particular, the convective core of B stars recedes during the main sequence and a chemical composition gradient ($\nabla\mu$) develops in the radiative layers at its border. The limits and shape of this gradient depend on the nature of extra-mixing: for e.g. overshooting (if described as instantaneous mixing) simply extends the limit of the central fully mixed region while mixing induced by rotation is thought to act as a diffusive process, smoothing $\nabla\mu$ (e.g. Meynet et al. 2013).

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The overshooting ($\alpha_{\rm OV}$) values determined with asteroseismology in most of the previous studies correspond to an instantaneous mixing prescription. As such, they give the limit of the extra-mixing region –i.e. the extra region fully mixed at the top of the formal boundary of the convective core– in terms of local pressure scale height (H_p). A step further, obtaining constraints on the shape and extent of $\nabla\mu$ would be very valuable for a better understanding of the extra-mixing processes. Dziembowski & Pamyatnykh (2008) attempted to determine the chemical composition transition in the overshoot region for the two β Cephei stars ν Eri and 12 Lac, but they could not draw a clear conclusion.

The quality and the nature of the constraints also depend on the seismic observables. Considering four β Cephei stars observed with intensive and coordinated follow-up, the number of frequency modes detected went from 6 to 13 (ν Eri – De Ridder et al. 2004 ; HD 129929 – Aerts et al. 2004 ; θ Oph – Briquet et al. 2007 ; 12 Lac – Desmet et al. 2009). In each case, the number of axisymmetric modes with a clear angular degree identification was limited to 3.

As a consequence, it is important to address the following questions concerning the study of β Cephei stars: i) what are the classical and seismic observables necessary for a good seismic modelling; ii) how do the seismic inferences depend on the stellar models and their input physics; iii) can we constrain further the $\nabla\mu$ region in these stars. To investigate these issues, we have carried out a series of hare and hound exercises (see also the previous work of Thoul et al. 2003). We consider as "observed" stars a set of stellar models (the hares) covering a wide range of parameters representative of β Cephei stars. We also vary the input physics of these models.

Section 2 briefly details our seismic modelling method while Section 3 presents the results of the hare and hound exercises, illustrated for one particular case. The paper ends with perspective and conclusion sections.

2 The modelling process

We use a forward approach scheme by comparing theoretical frequencies to those observed with the following merit function:

$$\chi^2 = \frac{1}{N_{\rm obs}} \sum_{i=1}^{N_{\rm obs}} \frac{(\nu_{\rm obs,i} - \nu_{\rm th,i})^2}{\sigma_i^2}. \tag{2.1}$$

$N_{\rm obs}$ is the number of observed frequencies ($\nu_{\rm obs,i}$), σ_i^2 the error on $\nu_{\rm obs,i}$, and $\nu_{\rm th,i}$ the theoretical frequencies. The theoretical frequencies come from a pre-computed grid of models initially presented in Briquet et al. (2009). The stellar models and their oscillation frequencies were obtained with the stellar evolution code CLES (Scuflaire et al. 2008b) and the oscillation code LOSC (Scuflaire et al. 2008a), respectively. The revised solar mixture from Asplund et al. (2005, ; AGS05, hereafter) and the OP opacities (Badnell et al. 2005) were adopted. Other details on the input physics are summarised in Briquet et al. (2009).

The stellar parameters of the grid cover the following ranges: M from 7.6 to 18.6 $\rm M_\odot$ by step of 0.1 $\rm M_\odot$ for the mass, X from 0.68 to 0.74 (step of 0.02) for the initial H mass fraction, Z from 0.010 to 0.018 (step of 0.002) for the metallicity and $\alpha_{\rm OV}$ from 0 to 0.50 (step of 0.05) for the instantaneous overshooting parameter. We use in this work the adiabatic oscillation frequencies, which were computed from angular degrees $\ell = 0$ to 3 for each of the models on the main-sequence phase.

In our approach, we determine the theoretical model of the grid that corresponds to the global minimum of Eq. 2.1. The stellar parameters of this model give the seismic inferred parameters of the observed star. To estimate the uncertainty on the solution, we introduce Monte Carlo simulations. We generate randomly new set of frequencies (pseudo-observed frequencies). For each of these sets, we determine a corresponding best-fit model (the one minimising Eq. 2.1 on the grid). We then gather the stellar parameters of each of the solutions and build distributions for every parameter. These distributions are next used to derive confidence intervals on the different inferred stellar parameters. The pseudo-observed frequencies are drawn from Gaussian distributions centered on the original observed frequencies. The values of the standard deviations reproduce the typical theoretical errors made on the computation of oscillation frequencies. We estimate it to be of the order of 10^{-2} c/d (see Salmon 2014). More details on this method are given in Salmon (2014) and will be the object of a forthcoming paper.

3 The hare and hound exercises

In Salmon (2014), we have presented the results of several hare and hound exercises dedicated to β Cephei models. We have checked different sets of seismic constraints to establish the minimum requirements allowing to derive accurate seismic inferences. We have as well estimated the impact of the physics on the seismic inferences. In that aim, the input physics of stellar models used as an observed star were set different from that of the models in the grid. For the target stars to be representative of typical β Cephei stars, we have selected models with stellar masses from 9 to 14 $M_\odot$ and at different evolutionary stages with X_c from 0.2 to 0.5 for the central H mass fraction. The role of the input micro-physics is tested by selecting either OPAL (Iglesias & Rogers 1996) or OP opacities and either GN93 (Grevesse & Noels 1993) or AGS05, for the chemical mixture.

Table 1. Stellar parameters of the target star t1 and solutions from the seismic modelling when 3 and 5 identified frequencies are considered, named t1-3freq and t1-5freq, respectively.

Model	M (in $M_\odot$)	R (in $R_\odot$)	$X_{\rm initial}$	$Z_{\rm initial}$	$\alpha_{\rm ov}$	X_c
t1	14	7.48	0.70	0.014	0.20	0.288
t1-3freq	15.6	10.18	0.70	0.018	0.45	0.237
t1-5freq	13.8	7.45	0.68	0.014	0.20	0.274

To illustrate the dependency of the seismic inferences on the available observational data, we present here one exercise where the target star is a stellar model (called t1, hereafter) with exactly the same input physics as in the theoretical grid. The frequencies of this model are computed with the non-adiabatic code MAD (Dupret 2001), which leads to frequencies different by about 10^{-4} (up to 10^{-2}) c/d from the adiabatic ones. In this way, we avoid any bias in the search of the solution. Indeed frequencies of the model t1 and those of the same model in the grid would have perfectly matched, in an unrealistic manner.

We have analysed different cases, considering that 3 to 5 frequencies were observed, with or without identification of the angular degree ℓ. In this paper we compare in particular a case with 3 identified frequencies (1 $\ell = 0$; 1 $\ell = 1$; 1 $\ell = 2$) to one with 5 identified frequencies (1 $\ell = 0$; 2 $\ell = 1$; 2 $\ell = 2$). The results of the modelling are given in Table 1, where t1-3freq and t1-5freq are the best-fit models for the 3 and 5 frequency cases, respectively. The solution with 3 observed frequencies shows that the mass and radius are overestimated, as well as $\alpha_{\rm OV}$ which is erroneously derived. Fig. 1 clearly illustrates in its top panel that the solution is not satisfactory in this case, with the global minimum of χ^2 failing to match the parameters of the target star. The ridges with lower values of χ^2 correspond to models with the same dynamical timescale, which is constrained by the presence of a radial mode ($\ell = 0$) in the set of observed frequencies. With the help of Monte-Carlo simulations, we are able to refine the solution but find large uncertainties at the 1-σ level of confidence of 22% and 31% respectively on M and R.

However, with additional constraints, the solution can reach a higher level of accuracy. In the bottom panel of Fig. 1, the χ^2 global minimum is indeed found very close to that of the target star when 5 identified frequencies are considered. This statement is confirmed by the Monte-Carlo simulations that indicate an uncertainty* of 1% on M and R at the 1-σ level. Not depicted, the limits of the central mixed region and the extent of $\nabla\mu$ are also inferred† with 1% precision in terms of mass fraction (m/M), quantity of prime importance for stellar physics.

We derived other results with the help of additional exercises (see Salmon 2014), which can be summarised as follows:

- for a given input physics and when the number of identified frequencies is < 5 or part or them are unidentified, the additional knowledge of classical parameters ($T_{\rm eff}$, $\log g$) can however result in an accurate seismic modelling,
- when the target star and the theoretical grid do not present the same chemical mixture, global parameters such as M and R are still retrieved, although with a lower precision. Yet, the location of the central mixed region can be poorly constrained in terms of m/M, even if the $\alpha_{\rm OV}$ seems correctly inferred. That is the models have the same $\alpha_{\rm OV}$‡ but the mass of their central mixed regions is significantly different,
- when the target star presents a different macro-physics (for e.g. turbulent mixing induced by rotational mixing), the modelling process is unable to make the distinction since it as it has not been designed for it (instantaneous mixing in the grid models).

*this value is probably underestimated since we are in an ideal situation where target star and models present the same input physics

†this is also probably underestimated

‡the other inferred parameters do not match exactly those of the target star

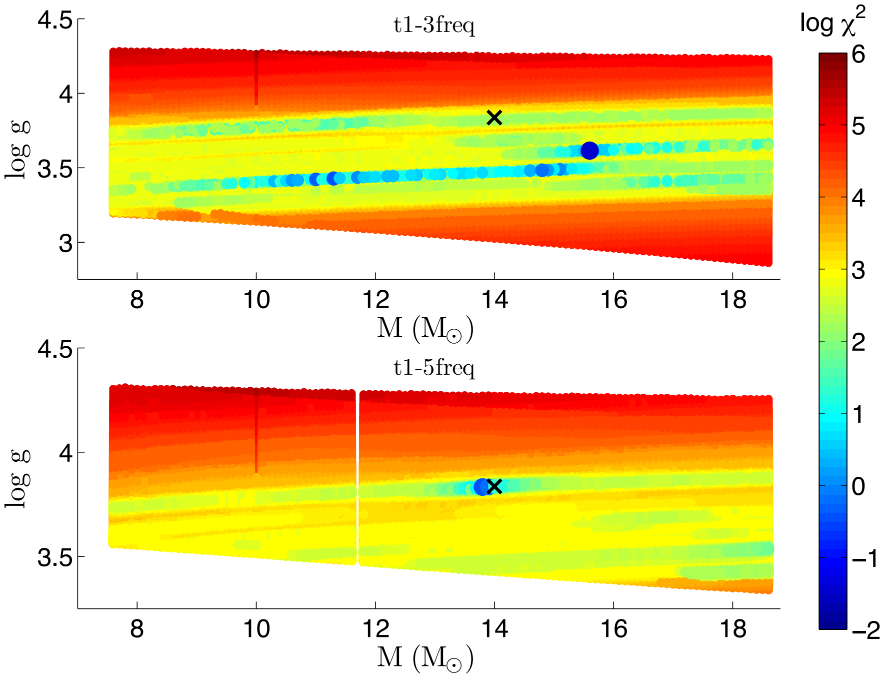

Fig. 1. Values of χ^2 as a function of M and surface gravity ($\log g$) for given $X_{\rm initial}$, $Z_{\rm initial}$, and $\alpha_{\rm ov}$, corresponding to those of the t1-3freq (top panel) and t1-5freq (bottom panel) best-fit models, respectively. The black cross indicate the M and $\log g$ values of the target star t1.

4 Perspectives: opacity and driving of the modes

In our hare and hound exercises, we have carried out a seismic analysis based on theoretical adiabatic frequencies. With the help of non-adiabatic computations, one can determine which modes are expected to be excited. The κ mechanism is known as the driving mechanism of oscillations in β Cephei stars (Moskalik & Dziembowski 1992). This latter works as a heat-engine mechanism, activated by the presence of the iron-group element peak of opacity at a temperature of about 200,000 K in β Cephei stars. The efficiency of the mechanism is very sensitive to the size and shape of the opacity peak (e.g. Pamyatnykh 1999), and so on the opacity data.

Several seismic studies of β Cephei stars were realised with help of a non-adiabatic approach. However requiring the excitation of the fitting theoretical frequencies revealed a discrepancy with observation. Dziembowski & Pamyatnykh (2008) could not find modes theoretically excited in the range of the low frequencies observed in the hybrid pulsators ν Eri and 12 Lac, whether the OPAL or OP opacities were used. Zdravkov & Pamyatnykh (2009) reached the same conclusion for the γ Peg star. More recently, observations of β Cephei candidates in the Magellanic Clouds presented a new challenge, since pulsations were not expected from a theoretical point of view at such low metallicities (see Salmon et al. 2012, and references therein).

All this suggests that current stellar opacities could be underestimated in the iron-group elements peak(Zdravkov & Pamyatnykh 2009; Salmon et al. 2012; Cugier 2014). Analysis of spectral opacity computations obtained with different opacity codes revealed differences with OP (OPAL could not be compared), in particular for Ni (Turck-Chièze et al. 2013). This could led to important changes in the Rosseland mean opacity values (Turck-Chièze & Gilles 2013). More recently, Bailey et al. (2015) found a large disagreement between the experimental measurement of Fe spectral opacity and theoretical computations, for conditions close to that of the solar base of the convective zone. These new issues have called for new opacity computations.

In the frame of the OPAC collaboration, solar models calibrated with the new OPAS opacities (designed for solar conditions of temperature and density, see Mondet et al. 2015) present a base of the convective zone and sound speed profile that reduce the disagreement with the helioseismic inferences (Le Pennec et al., submitted). Concerning B stars, more complete computations of iron-group elements have not led to major changes in the OPAL opacities (Iglesias 2015). However, the future release of ATOMIC opacities computed with new computer facilities by the Los Alamos team might help to solve part of the observational challenge of β Cephei stars(Walczak et al. 2015). With help of these new opacities, we plan to include in the future non-adiabatic approach in our modelling scheme. We suggest then to reanalyse well-known pulsators such as ν Eri and 12 Lac, and see what are the consequences of these new opacity data.

5 Conclusions

Asteroseismology of β Cephei stars is a powerful tool to derive constraints on their global parameters and the size of their central mixed regions, provided a sufficient number of seismic and/or classical observables are known. In particular, our study has shown the crucial need for the identification of angular degree of modes. Hence we suggest that new observing campaigns of β Cephei stars shall focus on this objective.

The Monte Carlo simulations we have introduced have revealed their potential to derive confidence levels. They could also help refining the solutions derived by the forward approach.

Considering the impact of the input physics selected in the theoretical models, it appears as essential to introduce a chemical mixture representative of the observed stars to get an unbiased estimate of the central mixed region extent. As a consequence it calls for additional observations when studying β Cephei stars, in particular the determination of individual element abundances.

Finally, we shall probably need to implement new parameters in the modelling process to access information on the shape and extent of the chemical composition gradient, and hence on the nature of extra-mixing in main-sequence B stars.

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