MODELING SMALL EXOPLANETS INTERIORS: A NUMERICAL SCHEME TO EXPLORE POSSIBLE COMPOSITIONS

B. Brugger¹, O. Mousis¹ and M. Deleuil¹

Abstract. Despite the huge number of discovered exoplanets, our knowledge of their compositions remains extremely limited. Modeling the interiors of such bodies is necessary to go further than the first approximation given by their mean density. Here we present a numerical model aiming at computing the internal structure of a given exoplanet from its measured mass and radius, and providing a range of compositions compatible with these data. Our model assumes the presence of a metal core surrounded by a silicate mantle and a water layer. Depending on their respective proportions, we can model various compositions, typically from terrestrial planets to ocean or Mercury-like planets. We apply this model to the case of CoRoT-7b, whose mass and radius values have recently been updated to $4.73 \pm 0.95 M_{\oplus}$ and $1.585 \pm 0.064 R_{\oplus}$, respectively. We show that these values are fully compatible with a solid composition, and find that CoRoT-7b may present a core mass fraction of 80% at maximum, or on the opposite, a maximum water mass fraction of 51%. If this latter composition is compatible with that of several icy moons in the solar system, a 80% core in mass is less conceivable and a lower limit can be placed from solar system formation conditions. These results confirm the Super-Earth status of CoRoT-7b, and show that an Earth-like composition may be obtained more easily compared to previous conclusions.

Keywords: Earth — planets and satellites: composition — planets and satellites: interiors — planets and satellites: individual (CoRoT-7b)

1 Introduction

The new exoplanet families now fill the gap existing between rocky and gaseous planets that compose the solar system. CoRoT-7b is considered as the first Super-Earth with known mass and radius. It was discovered by Léger et al. (2009), who detected a planet with a $1.68 \pm 0.09 R_{\oplus}$ radius orbiting the star CoRoT-7 with a period of $0.85359 \pm 5 \cdot 10^{-5}$ day. The mass of this planet was later obtained from HARPS radial velocity measurements, with a value of $4.8 \pm 0.8 M_{\oplus}$ (Queloz et al. 2009). Recently, Barros et al. (2014) and Haywood et al. (2014) performed new measurements of the radius and mass of CoRoT-7b, updating the values to $1.585 \pm 0.064 R_{\oplus}$ and $4.73 \pm 0.95 M_{\oplus}$, respectively. They also confirmed the semi-major axis of the planet's orbit at 0.0172 ± 0.00029 AU, corresponding to an equilibrium temperature of ~1750 K (Barros et al. 2014).

Knowing the mass and radius of an exoplanet provides its mean density, which gives a hint on its possible composition (mainly to differentiate rocky from gaseous planets). However this value is just a first approximation, and our knowledge regarding the composition of exoplanets remains extremely limited. Here we want to go further by modeling the interiors of these bodies by presenting a numerical model that computes the internal structure of a given exoplanet from its measured mass and radius. We apply our model to the case of CoRoT-7b, which has never been studied with its updated physical parameters, in order to explore the possible interiors of the planet, assuming that it belongs to the class of dense solid planet (rocky with possible addition of water). We excluded from this study the cases where CoRoT-7b could harbor a thick atmosphere, unlike Earth.

2 Model and parameters

Based on the one-dimensional approach described by Sotin et al. (2007), our model is capable of computing the internal structure of a given planet, assuming that its mass and composition are known. Earth is taken

¹ Aix Marseille Univ, CNRS, LAM, Laboratoire d'Astrophysique de Marseille, Marseille, France (e-mail: bastien.brugger@lam.fr)

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as a reference for several parameters and constants of the model, since it is the planet with the best known composition and internal structure. In our model, a planet can be made of up to five fully differentiated concentric layers (see Figure 1):

- 1. a metallic core made of pure iron and FeS iron alloy (Sotin et al. 2007);
- 2. the lower mantle, composed of a mixture of silicate rocks perovskite and magnesiowüstite;
- 3. the upper mantle, made from the same elements but in the form of olivine and enstatite;
- 4. a layer made of high-pressure water ices VII and X (Hemley et al. 1987);
- 5. a liquid water layer.

The mass of these layers – and thus their size – are variable, given as fractions of the total planet mass. This allows the model to simulate different planet compositions, from terrestrial (fully rocky) planets to ocean planets. Thus, we define two parameters of the model, the core mass fraction (CMF) and water mass fraction (WMF), forming a pair (CMF,WMF) to which the "composition" of a planet refers. The values of the CMF and WMF fix the locations of the core/lower mantle and upper mantle/water layer boundaries, respectively. The remaining boundaries (lower/upper mantle, water ice/liquid water) are directly computed from phase diagrams of the respective materials. The model also needs several compositional parameters, namely the fraction of alloy in the core and the relative distribution of the different types of silicate rocks in the mantle. Since we lack them in the case of CoRoT-7b, we use the Earth's values by default, as detailed in Figure 1.



Fig. 1. Internal structure of an ocean planet, with five concentric layers: metallic core, silicate mantles (lower and upper), and water layers (solid and liquid). Our model is able to handle any combination based on these layers.

In our model, the interior of a planet is described by a one-dimensional spatial grid, ranging from the planet center until beyond its surface. For each point of this grid, the model computes the gravitational acceleration g, pressure P, temperature T, and density ρ , by solving the differential equations verified by these quantities:

$$\frac{dg}{dr} = 4\pi G\rho - \frac{2Gm}{r^3},\tag{2.1}$$

$$\frac{dP}{dr} = -\rho g,\tag{2.2}$$

$$\frac{dT}{dr} = -g\frac{\gamma T}{\Phi},\tag{2.3}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho, \qquad (2.4)$$

with r the radius inside the planet, m the mass at a given radius, G the gravitational constant, and γ and Φ the Grüneisen and seismic parameters, respectively (Sotin et al. 2007). These quantities govern the internal structure of a planet, therefore the model iterates on the solution of Equations 2.1–2.4, computing new positions of the layer boundaries at each iteration, until convergence is reached. This occurs when the simulation matches the asked planet mass, CMF, and WMF, and when the boundary conditions are verified: no central gravitational acceleration, surface temperature and pressure fixed to the given values. Since the model is not yet able to handle layers of gaseous materials (mostly hydrogen and helium), we do not consider in this work the possibility that CoRoT-7b possesses a thick atmosphere made of gas, as it is the case for Uranus or Neptune. However, in order to allow the existence of liquid water on the surface of the planet, we assume that CoRoT-7b's surface conditions (pressure and temperature) are similar to those of Earth (1 bar and 288 K, respectively). This assumes the planet radius (since the mass of such an atmosphere would be negligible relatively to the planet mass, as it is the case on Earth). The physical and orbital parameters of CoRoT-7b and its host star CoRoT-7, are summarized in Table 1.

Table 1. List of physical and orbital parameters of CoRoT-7b and its parent star CoRoT-7

Parameter	Value	Reference
Planet parameters		
Orbital period (day)	0.853585 ± 0.000024	Queloz et al. (2009)
Orbital distance (AU)	0.0172 ± 0.00029	Queloz et al. (2009)
Planet mass (M_{\oplus})	4.73 ± 0.95	Haywood et al. (2014)
Planet radius (R_{\oplus})	1.585 ± 0.064	Barros et al. (2014)
Equilibrium temperature (K)	1756 ± 27	Barros et al. (2014)
Stellar parameters		
Effective temperature (K)	5259 ± 58	Barros et al. (2014)
Star mass (M_{\odot})	0.913 ± 0.017	Barros et al. (2014)
Star radius (R_{\odot})	0.820 ± 0.019	Barros et al. (2014)

To accurately simulate the behaviors of all materials used in the different layers, and therefore compute the density profile inside the planet, the model uses equations of state (EOS) for each of these materials. These equations are fitted to laboratory experiments that study the behavior of a material's density (or equally its volume) under variations of pressure and temperature. Here we choose to follow the work of Sotin et al. (2007) and Valencia et al. (2007), who use the third-order Birch–Murnaghan (BM3) and Vinet EOS, respectively. The BM3 EOS is particularly fast for computations, but diverges for high pressure values (>1.5–3 Mbar; Seager et al. (2007), Valencia et al. (2009)). We opted to use it in the upper layers (water ice and liquid water). For denser materials, namely the metals and rocks of the core and mantles, we switch to the Vinet EOS, that has been shown to better extrapolate at very high pressures (Hama & Suito 1996).

3 Results

We first consider the possibility that CoRoT-7b presents the same composition as Earth, corresponding to (CMF,WMF) = (0.325,0). Using the central mass inferred for this planet by Haywood et al. (2014), namely $M_P = 4.73 \ M_{\oplus}$, we compute its internal structure via the use of our model. As for Earth, the result is a planet composed of a metallic core surrounded by silicate mantles, but no significant water layer. The boundaries between these three layers are respectively located at 5018 km and 9384 km from the center, and the planet radius is 9704 km, i.e. $R_P = 1.523 \ R_{\oplus}$. This value lays within the range measured by Barros et al. (2014), meaning that an Earth-like composition for CoRoT-7b is fully compatible with the measurements.

However, CoRoT-7b may present a composition different from that of the Earth, following the variations of the CMF and WMF. Thus, we repeated the aforementioned simulation for every composition allowed by the variations of the CMF and WMF of the planet. The parameter space formed by these variations is represented by a ternary diagram, displayed three times on Figure 2. Each point of the ternary diagram corresponds to a unique composition, i.e. to a unique value of the pair (CMF,WMF). A planet located in one corner is fully

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composed of the corresponding compound (core, mantle, or water), whereas planets located on one side of the diagram do not contain any of the compound indicated on the opposite corner (as it is the case for Earth and Mercury, whose WMF is considered zero). Since the planet mass is fixed for the entire ternary diagram, each point of the diagram yields a corresponding planet radius (via the use of the model of internal structure). Thus, the ternary diagram can be seen as a heatmap of the planet radius, and we can draw curves on which the value of the radius is the same (named isoradius curves). Since the mass of CoRoT-7b is known with an uncertainty, we choose to plot a ternary diagram for three different values of the planet mass, namely 3.78, 4.73, and 5.68 M_{\oplus} (i.e. the minimum, central, and maximum values inferred for the mass of CoRoT-7b).



Fig. 2. Ternary diagrams displaying the investigated compositional parameters space: each point corresponds to a unique composition. From left to right, diagrams correspond to the minimal, central, and maximal masses inferred for CoRoT-7b. Each diagram shows a colored map of the planet radii obtained for the corresponding compositions. Also shown are isoradius curves denoting the minimal, central, and maximal radii measured for CoRoT-7b.

Since we know the radius of CoRoT-7b from Barros et al. (2014), we plot the isoradius curves corresponding to the minimum, central, and maximum values of its range of uncertainty, namely $1.585 \pm 0.064 R_{\oplus}$. As shown by Figure 2, these curves delimit an area on each of the three ternary diagrams. This is the domain of compositions that are allowed by the variations of the radius of CoRoT-7b within the measured range of uncertainty. It is interesting to follow the evolution of this area when the planet mass increases. For $M_P = 3.78$ M_{\oplus} , the domain of compositions allowed for CoRoT-7b lays in the center of the diagram, and we can deduce the limitations thus placed on the planet's CMF and WMF. The maximum CMF that can be reached with M_P = 3.78 M_{\oplus} is 66%, forming a 1.521 R_{\oplus} planet. On the opposite, the WMF can go up to 51%, and in this case the planet's radius is 1.649 R_{\oplus} . Note that for this value of the planet mass, an Earth-like composition cannot be reached. This is not true when using a planet mass of 4.73 M_{\oplus} , where a planet with the composition of the Earth has a radius of 1.523 R_{\oplus} . In general, as we increase the mass of the planet, the domain of compositions allowed for CoRoT-7b shifts to the right of the ternary diagram. This expresses the need to increase the planet's mean density in order to keep the same planet radius, when increasing the planet mass. Thus, we see that for $M_P = 5.68 \ M_{\oplus}$, the maximum CMF becomes 80%, and the maximum WMF 30%. Also, we see that for this maximum value of the mass, an Earth-like composition yields a radius of 1.586 R_{\oplus} . In any of these cases, CoRoT-7b may never present a composition corresponding to that of Mercury (CMF,WMF) = (0.68,0), since the corresponding point on the ternary diagram lies always outside of the domain of compositions allowed for this planet.

4 Conclusions

Assuming that CoRoT-7b is solid, we investigated the compositions this planet could present, taking into account the limits placed by the measurements of the planet's mass and radius. Via the use of a model of internal structure, we find that CoRoT-7b may present a CMF up to 80% (in the case where $M_P = 5.68 M_{\oplus}$ and $R_P = 1.521 R_{\oplus}$), whereas the WMF of the planet reaches 51% for a 3.78 M_{\oplus} planet with a radius of 1.649 R_{\oplus} . Interestingly, if the planet mass is set higher than the central measured value 4.73 M_{\oplus} , then the composition of the Earth (~33% CMF with no water) enters the domain of compositions allowed for CoRoT-7b. This confirms the Super-Earth status of the planet, that was suggested by the first measurements on the planet's radius.

Modeling Small Exoplanets Interiors

Valencia et al. (2010) investigated the possible interiors and compositions of CoRoT-7b, using the first measurements on the mass and radius of the planet. They also restrained their simulations to the cases of dense rocky planets (with possible addition of water). Their main conclusion was that CoRoT-7b probably was depleted in iron compared to the Earth. Indeed, in their results, an Earth-like composition was possible only for $M_P = 5.68 \ M_{\oplus}$ and $R_P = 1.521 \ R_{\oplus}$ (otherwise the composition of the Earth was never in the domain of compositions allowed for the planet). Here, using the updated values of CoRoT-7b's mass and radius by Haywood et al. (2014) and Barros et al. (2014), we show that there is no need for an iron depletion since an Earth-like composition can be reached more easily.

The case of CoRoT-7b shows some limitations of the model of internal structure in its current form. For instance, we have considered that the surface conditions on this planet are similar to those on Earth, in order to allow the presence of liquid water at the surface. Considering the equilibrium temperature inferred for CoRoT-7b (\sim 1750 K), it is probable that liquid water cannot exist at its surface. However, using a surface temperature of 1750 K would not be relevant either: because of its proximity to the parent star, CoRoT-7b is probably tidally locked in its orbit, always showing the same side to the star. To know what surface temperature has to be used, we would need a climatic model, that simulates the heat redistribution on the planet's surface.

We have used Earth values for the composition of CoRoT-7b as well. This assumes that the protoplanetary nebula that gave birth to CoRoT-7b was similar in composition to the protosolar nebula. The validity of this assumption could be constrained if we had measurements of the elemental abundances of the star CoRoT-7, that can be compared to the composition of the Sun.

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