SF2A 2016
C. Reylé, J. Richard, L. Cambrésy, M. Deleuil, E. Pécontal, L. Tresse and I. Vauglin (eds)

MODELS OF MAGNETIZED WHITE DWARFS

M. Oertel[1], D. Chatterjee[1,2], A.F. Fantina[3], N. Chamel[4] and J. Novak[1]

Abstract. To explain observations of overluminous Type Ia supernovae, strongly magnetized white dwarfs with masses above $2M_{\odot}$ have been proposed as their progenitors. This has interesting implications in high-precision cosmology, as Type Ia supernovae have been widely used as standard candles. We compute equilibrium configurations of magnetized white dwarfs, self-consistently determining the structure of a compact object in strong (poloidal) magnetic fields. Our results show that, although the magnetic field can support indeed these massive configurations, they might never be reached in nature since electron capture destabilizes the star already at lower masses. Hence strongly magnetized white dwarfs are unlikely to be the progenitors of overluminous supernovae.

Keywords: stars:white dwarf, magnetic fields, equation of state, methods:numerical

1 Introduction

White dwarfs (WDs) are the progenitors of type Ia supernovae (SNIa), which have been used as "standard candles" to measure cosmological distances assuming a unique astrophysical scenario for these events, the thermonuclear explosion of a Chandrasekhar mass WD. The picture of SNIa has much diversified recently, in particular with the discovery of overluminous type Ia supernova (SNIa) (Howell et al. 2006). The progenitors of such events are thought to be "super-Chandrasekhar" WDs with a mass > 2 $M_{\odot}$ (see, e.g., Hillebrandt et al. 2013), resulting either from the merger of two massive WDs, from rapidly (differentially) rotating WDs (Howell et al. 2006), or from strongly magnetized WDs (Kundu & Mukhopadhyay 2012; Das & Mukhopadhyay 2012a).

Therefore it is of utmost importance to determine the structure of magnetized WDs and their maximum mass. Hundreds magnetized WDs have been observed, with surface fields of up to about 10^9 G (Wickramasinghe & Ferrario 2000). The internal field, not directly observable, might be stronger and have a non negligible influence on the structure of the star. The study of the mass-radius relation of a magnetized WD has a long history and it was recognized early on that the impact of the magnetic field on both its radius and mass could be large. However, simplifying assumptions have been made for convenience. For instance, the pioneering work by Ostriker & Hartwick (1968) considers a vanishing magnetic field at the surface of the star and neglects any magnetic field effect on the euqation of state (EoS) as well as general relativistic (GR) effects and electrostatic interactions. The works of Adam (1986), Das & Mukhopadhyay (2012a), and Kundu & Mukhopadhyay (2012) focus on the effect of the magnetic field on the EoS, including the Landau quantization of the electron gas, but use a Newtonian description of the star's structure in spherical symmetry, i.e. neglecting the deformation of the star by the magnetic field. A similar approach is followed in Suh & Mathews (2000), where, however, the general relativistic (spherical) Tolman-Oppenheimer-Volkoff (TOV) equations are applied to solve for the star's structure. Recent attempts for more realistic WD models include equilibrium configurations in Newtonian framework (Bera & Bhattacharya 2014). Further, Das & Mukhopadhyay (2015a) and Bera & Bhattacharya (2015) computed the mass-radius relation of magnetic WDs for different field geometries in GR.

What all these works have in common is that for strongly magnetized WDs, masses are obtained well above the original Chandrasekhar limit, that are able to explain the overluminous SNIa. However, microscopic

[1] LUTH, Observatoire de Paris, PSL Research University, CNRS, Université Paris Diderot, Sorbonne Paris Cité,5 place Jules Janssen, 92195 Meudon, France
[2] Laboratoire de Physique Corpusculaire, ENSICAEN, 6 Boulevard Maréchal Juin, 14050 Caen Cédex, France
[3] Grand Accélérateur National d'Ions Lourds (GANIL), CEA/DRF - CNRS/IN2P3, Bvd Henri Becquerel, 14076 Caen, France
[4] Institut d'Astronomie et d'Astrophysique, CP-226, Université Libre de Bruxelles (ULB), 1050 Brussels, Belgium

processes like electron captures or pycno-nuclear reactions might induce instabilities and effectively limit the maximum mass (Chamel et al. 2013). Within this work we address these stability issues in a completely self-consistent setup: we solve combined Einstein+Maxwell equations including magnetic field effects in the EoS.

2 Model setup

2.1 *Equation of state*

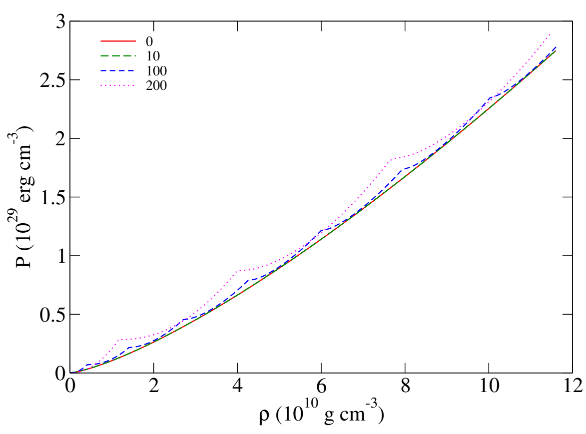
Our EoS follows the model developed in Lai & Shapiro (1991); Chamel et al. (2012), originally for the crust of strongly magnetized neutron stars. The interior of magnetized WDs is assumed to be composed of fully ionized atoms. Moreover, we assume that the internal temperature has dropped below the crystallization temperature so that ions are arranged on a regular crystal lattice and that we can neglect thermal effects. For simplicity, we shall consider crystalline structures made of only one type of ions ${}^A_Z X$, with mass number A and atomic number Z (^{12}C or ^{16}O). The EoS receives thus three contributions: from nuclear masses, from the relativistic Fermi gas of electrons and from the crystalline lattice, i.e. electron-ion interactions.

As in Lai & Shapiro (1991); Chamel et al. (2012), we neglect the change of nuclear masses in the presence of a strong magnetic field and take experimental masses (Audi et al. 2012). The lattice energy, evaluated for point-like ions arranged in a body-centered-cubic (bcc) lattice, is independent of the magnetic field. Magnetic field effects are thus considered only on the dense Coulomb plasma, i.e. Landau quantization of the electron gas is taken into account, see Chatterjee et al. (2016) for more details and explicit expressions. As an example, the EoS for a ^{12}C WD is shown in Fig. 1 for different magnetic fields strengths. De Haas-van Alphen oscillations are clearly visible for field strengths roughly above the critical field $b_{\rm crit} = \frac{m_e^2 c^3}{e\hbar} \approx 4.4 \times 10^{13}$ G. m_e denotes here the electron mass and e its charge, c the speed of light and $\hbar$ the reduced Planck constant.

At some mass density ρ_β (or equivalently at some corresponding pressure P_β), the nucleus ${}^A_Z X$ becomes unstable against the capture of an electron with the emission of a neutrino :

$$ {}^A_Z X + e^- \rightarrow {}^A_{Z-1} Y + \nu_e \, . \quad (2.1)$$

The daughter nucleus ${}^A_{Z-1} Y$ itself may be unstable. As electrons combine with nuclei, further compression of matter does not increase the pressure, thus leading to a global instability of the star. In the absence of magnetic fields, the onset of electron captures occur at mass density $\rho_\beta \simeq 4.16 \times 10^{10}$ g cm^{-3} (pressure $P_\beta \simeq 6.99 \times 10^{28}$ dyn cm^{-2}) for ^{12}C and $\rho_\beta \simeq 2.06 \times 10^{10}$ g cm^{-3} ($P_\beta \simeq 2.73 \times 10^{28}$ dyn cm^{-2}) for ^{16}O. In the presence of a strong magnetic field, the threshold density and pressure are shifted to either higher or lower values depending on the magnetic field strength Chamel & Fantina (2015).



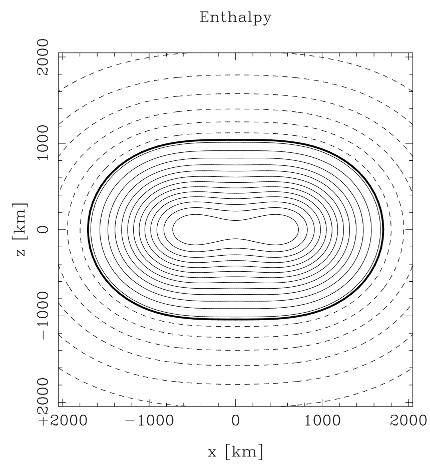


Fig. 1. Left: EoS (pressure P vs mass density ρ) for a ^{12}C WD, for different magnetic field strengths $b_\star = b/b_{\rm crit}$.**Right**:Enthalpy isocontours of a relativistic ^{12}C WD for $\mathcal{D} = 3\times10^{34}$ A m^2 resulting in a polar field of $B_p \sim 3\times10^{13}$ G.

2.2 *Stellar structure equations*

We compute the WD structure numerically assuming stationarity, axisymmetry, circularity and matter being a perfect conductor. The latter assumption implies that the electric field vanishes in the fluid rest frame.

Magnetostatic equilibrium equations are then solved combined with Maxwell equations for the electromagnetic field and gravity equations, either general relativistic or Newtonian, using the LORENE* numerical library. The magnetic field is purely poloidal by construction (Bocquet et al. 1995), which is not necessarily the most general one, but allows for an easy comparison with observed polar fields. In contrast to other works, our code allows to consistently include the magnetic field effects on the EoS in the structure equations, derived in a coherent way from the energy momentum tensor in presence of an electromagnetic field (Chatterjee et al. 2015). Note in particular that the equations for equilibrium do not contain any contribution from the magnetization. They thus differ from those given in Bera & Bhattacharya (2014), where the magnetization has been artificially included. More details can be found in a forthcoming publication (Chatterjee et al. 2016).

In Fig. 1, as an example of a stellar configuration we show the enthalpy isocontours of a non-rotating ^{12}C WD, taking a magnetic dipole moment, $\mathcal{D}$ of 3×10^{34} A m^2. This corresponds to a polar field strength of about $B_p \sim 3 \times 10^{13}$ G. The thick line indicates the stellar surface. The star's deformation due to the magnetic field is clearly visible. It is obvious that the star cannot be treated in spherical symmetry. Increasing further $\mathcal{D}$ (and thus the magnetic field), at some point the density at the center of the star vanishes and the star takes a toroidal shape (Cardall et al. 2001). This is not really a physical instability, although hardly imaginable astrophysically, but it cannot be treated by our code since a nonzero density is assumed at the center. We stop our calculations therefore at the maximally distorted configurations, before the star becomes toroidal.

3 Results and discussion

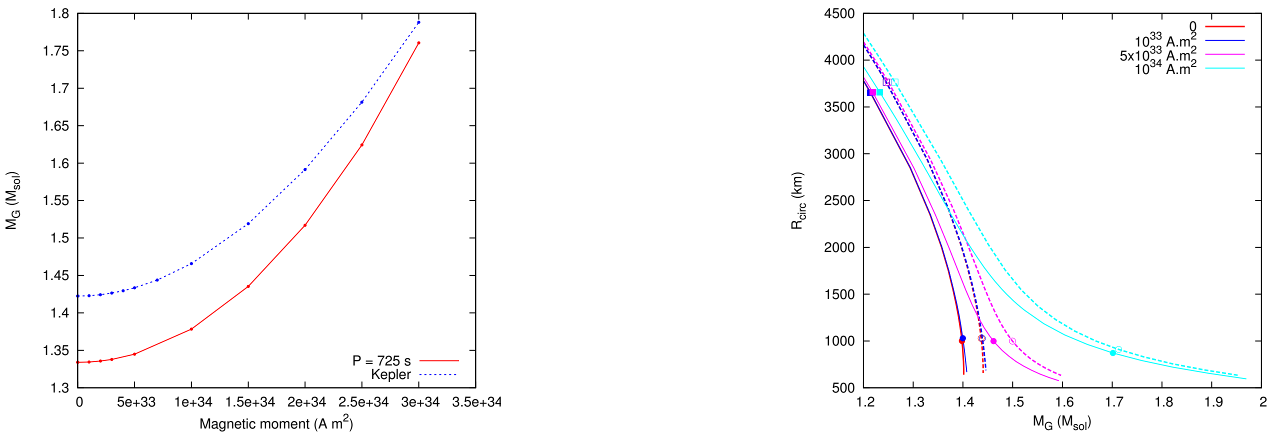


Fig. 2. Left: Mass for Newtonian magnetized ^{16}O WDs rotating at a period of 725 s (solid line) and at Kepler frequency (dashed line) for different values of $\mathcal{D}$. The central density has been chosen such that the mass of the non-rotating, non-magnetized WD is 1.34 $M_\odot$. **Right**: Mass vs radius for Newtonian ^{16}O WDs for different values of $\mathcal{D}$ with (solid lines) and without (dashed lines) lattice effects. The filled and empty dots mark the onset of electron capture on ^{16}O with and without lattice effects, respectively, whereas the squares indicate a lower limit for the onset of pycno-nuclear reactions.

Qualitatively the results for equilibrium ^{12}C and ^{16}O WDs are very similar and for most quantities the numerical values differ only slightly. Note that we do not consider sequences at fixed magnetic field strength, as in other works, e.g. in Bera & Bhattacharya (2014), since they would suggest artificially a gravitational instability, which disappears if the relevant quantity, $\mathcal{D}$, is kept constant throughout the sequences, see Chatterjee et al. (2016).

In Fig. 2 (right) the mass-radius relation for ^{16}O WDs is displayed for sequences –varying central density– at different values of $\mathcal{D}$. For the magnetized sequences, the curves end at the corresponding maximally distorted configuration. For even higher values of $\mathcal{D}$, the maximally distorted configuration is reached at approximately the same mass as for 10^{34} A m^2. Thus, in principle WDs with masses of the order $2M_\odot$ could exist. Accepting toroidal shapes, even higher masses could be reached. In addition, we consider here purely poloidal magnetic fields, which are known to be unstable on long time scales. Therefore a mixed poloidal-toroidal configuration

*`http://www.lorene.obspm.fr`

would be more realistic, which could again slightly increase the mass. Keep in mind, however, that the magnetic fields for the most massive configurations are orders of magnitude above currently observed values. And, as discussed below, electron capture (EC) and pycno-nuclear reactions might effectively be the main limiting factor.

3.1 Influence of different modelling parameters

Magnetic field dependence of the EoS : As mentioned in Sec. 2.1 and shown in Fig. 1 (left), the magnetic field dependence of the EoS starts to play a role for field values above roughly 10^{13}G. Even accepting that polar fields might be orders of magnitude above the currently observed values, such high magnetic fields can be hardly reached inside a WD with a poloidal field structure before the maximally distorted configuration, i.e. before the star takes a toroidal shape. Therefore the influence of the magnetic field dependence of the EoS on the results remains very small.

Lattice effects within the EoS : The electrostatic interaction between electrons and ions introduced by Hamada & Salpeter (1961) was found to lower the electron pressure resulting in a softer EoS and hence leads to slightly less massive configurations. We confirm this result. As can be seen from Fig. 2 (right), where results including lattice effects are shown as plain lines and those without as dashed lines, the effect on the radius can, however, be huge. The difference might be as large as factor of 2.

General relativity : It is well known that general relativity reduces the maximum mass of non-magnetic WDs by a few percent (see, e.g., Ibáñez 1984). For strongly magnetized WDs the difference between Newtonian and relativistic WDs becomes even smaller since magnetic energy increases and the deformation of the star renders it less compact, leading to less important relativistic effects.

(Uniform) rotation : In Fig. 2 (left) we display the masses of uniformly rotating magnetized WDs as function of $\mathcal{D}$. The period of 725 s has been chosen at the lower end of observed values (Ferrario et al. 2015). It is obvious that uniform rotation cannot considerably increase the mass, but that the main effect increasing the mass results from the magnetic field. Even a rotation at mass-shedding limit (Kepler frequency) cannot shift the mass to about $2M_\odot$ without magnetic field.

3.2 Instabilities induced by EC and pycno-nuclear reactions

The dots on the curves in Fig. 2 (right) mark the onset of EC reactions inside the star, i.e. more massive configurations become unstable. It is obvious that is it thus very improbable that the maximally distorted configuration is ever reached. The situation becomes worse if pycno-nuclear reactions are considered, see the squares in Fig. 2. Since the rates for pycno-nuclear reactions are very uncertain, the squares, however, represent only an estimate of the lower limit for the onset of these type of reactions. We show here results for ^{16}O, but it should be kept in mind, that threshold densities for the onset of EC and pycno-nuclear reactions are lower for ^{16}O than for carbon, such that the value of the maximum mass is lower for ^{16}O WDs than for ^{12}C ones. But, in most cases the maximally distorted configuration is not reached for ^{12}C WDs, neither.

We conclude that although the magnetic field in principle allows to support very massive WDs with masses of the order $2M_\odot$ or even slightly above, they might never exist as stable objects and are thus unprobable to be the progenitors of overluminous SNIa.

This work has been funded by the SN2NS project ANR-10-BLAN-0503, the "Gravitation et physique fondamentale" action of the Observatoire de Paris, Fonds de la Recherche Scientifique - FNRS (Belgium) and the COST action MP1304 "NewCompstar".

References

Adam D., 1986, A&A, 160, 95

Audi G., Wang M., Wapstra A. H., Kondev F. G., MacCormick M., Xu X., Pfeiffer B., 2012, Chin. Phys. C, 36, 1287

Bera P., Bhattacharya D., 2014, MNRAS, 445, 3951

Bera P., Bhattacharya D., 2016, MNRAS, 456, 3375

Bocquet M., Bonazzola S., Gourgoulhon E., Novak J., 1995, A&A, 301, 757

Cardall C. Y., Prakash M., Lattimer J. M., 2001, ApJ, 554, 322

Chamel N. et al., 2012, Phys.Rev. C, 86, 055804
Chamel N., Fantina A. F., Davis P. J., 2013, Phys. Rev. D, 88, 081301
Chamel N., Fantina A. F., 2015, Phys. Rev. D, 92, 023008
Chatterjee D., Elghozi T., Novak J., Oertel M., 2015, MNRAS, 447, 3785
Chatterjee D., Fantina A., Chamel N., Novak J., Oertel M., 2016, submitted to MNRAS.
Das U., Mukhopadhyay B., 2012a, Phys. Rev. D, 86, 042001
Das U., Mukhopadhyay B., 2015, JCAP, 5, 16
Ferrario L., de Martino D., Gänsicke B. T., 2015, Space Sci. Rev., 191, 111
Hamada T., Salpeter E. E., 1961, ApJ, 134, 683
Hillebrandt W., Kromer M., Röpke F. K., Ruiter A. J., 2013, Front. Phys., 8, 116
Howell D. A. et al., 2006, Nature (London), 443, 308
Ibáñez Cabanell J. M., 1984, A&A, 135, 382
Kundu A., Mukhopadhyay B., Mod. Phys. Lett. A, 27, 1250084
Lai D., Shapiro S. L., 1991, ApJ, 383, 745
Ostriker J. P., Hartwick F. D. A., 1968, ApJ, 153, 797
Suh I.-S., Mathews. G. J., 2000, ApJ, 530, 949
Wickramasinghe D. T., Ferrario L., 2000, Publ. Astron. Soc. Pac., 112, 873