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STEADY-STATE MODEL OF ACCRETION COLUMNS IN MAGNETIC CATACLYSMIC VARIABLES

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Abstract. The standard model of accretion columns in magnetic cataclysmic variables is investigated through semi-analytical solutions in the steady-state regime. The balance between bremsstrahlung and cyclotron cooling is studied and the effects of the white dwarf gravitational field are analysed.

Keywords: magnetic cataclysmic variables, radiative shocks, accretion processes, high-energy processes

1 Introduction

Among the different binary systems, cataclysmic variables provide the best environment to study high-energy radiation in the accretion processes. As possible progenitors of type Ia supernovae, understanding these complex systems is crucial to explain the initial conditions of these explosions (Maoz et al. 2014). Magnetic cataclysmic variables (mCVs) are close binary systems composed of a magnetised white dwarf accreting matter from a late type Roche-lobe filling companion star (Warner 1995). Depending on the white dwarf magnetic field, mCVs are classified into intermediate polars (IPs) when $B_{WD} < 10$ MG and polars when $B_{WD} \sim 10 - 230$ MG. In polars and some IPs, the accreting matter coming from the companion is channelled by the magnetic field onto the white dwarf magnetic poles. This leads to the creation of an accretion column (Cropper 1990; Wu 2000). The impact of the free-falling flow at supersonic velocities ($v_{ff} \sim 5000$ km.s^{-1}) creates a radiative reverse shock which heats the coming matter to typical temperatures of about 50 keV. This high-energy environment is structured by the cooling processes which shape the density and the temperature profiles of the post-shock region and produce strong gradients near the white dwarf photosphere. According to the current acknowledged model of accretion columns, the radiative shock is expected to reach an equilibrium height determined by the cooling processes. To determine the mCVs properties, particularly to infer the white dwarf mass, the knowledge of the spatial profiles of this high-energy radiative region is fundamental. Although integrated luminosities can be observed from mCVs, the spatial scales, associated with the accretion shock, are largely smaller than the white dwarf radius, which prevents direct observations. To obtain analytical and semi-analytical solutions from the radiation hydrodynamics equations which model the accretion column, a steady-state regime is frequently assumed. Steady-state models are crucial to forecast the column behaviour and to determine the white dwarf mass. Particularly to correctly model the accretion onto high mass white dwarfs ($M_{WD} > 0.8$ M$_{\odot}$), taking into account the white dwarf gravitational field is essential. In this paper, we present solutions of the radiative hydrodynamics equations with gravitation.

2 Steady-state model of the post-shock region

For mCVs, the main cooling processes in the post-shock region are bremsstrahlung and cyclotron emission (Wu et al. 1994). Other processes such as Compton cooling and thermal conductivity are negligible. In order to

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model the post-shock region, we assume a plane-parallel and collisional shock which can be described by a single temperature medium. In the one-temperature approximation and in the steady-state regime, the post-shock region is described by the equations of radiation hydrodynamics:

$$\frac{d}{dx}[\rho v] = 0 \tag{2.1}$$

$$\frac{d}{dx}[\rho v^2 + P] = \rho \frac{GM_{WD}}{(x_0 - x)^2} \tag{2.2}$$

$$v[\frac{dP}{dx} - \gamma \frac{P}{\rho}\frac{d\rho}{dx}] = -(\gamma - 1)\Lambda(\rho, P) \tag{2.3}$$

where x, ρ, v, P, G, M_{WD}, x_0, γ and Λ are respectively the spatial coordinate, the density, the velocity, the pressure of the flow, the gravitational constant, the white dwarf mass, a spatial constant, the adiabatic index and the cooling function. The constant x_0 is defined as $x_0 = x_s + R_{WD}$ where x_s is the steady-state height of the post-shock region and R_{WD} is the white dwarf radius. The latter is related to the white dwarf mass by the Nauenberg relation (Nauenberg 1972). The spatial axis has its origin at the shock front and its direction is the one of the accreting flow. In this model, the cooling function is expressed as the sum of the two radiative processes: $\Lambda = \Lambda_{brem} + \Lambda_{cycl}$. Each process can be described by a power-law function : $\Lambda_i = \Lambda_{0,i}\rho^{\alpha_i}P^{\beta_i}$ where $\Lambda_{0,i}$, α_i and β_i are three characteristic constants of the radiative mechanism. This form models the exact bremsstrahlung cooling ($\Lambda = \Lambda_0^{brem}\rho^{1.5}P^{0.5}$) and an effective cyclotron cooling function ($\Lambda = \Lambda_0^{cycl}(B_{WD})\rho^{-2.35}P^{2.5}$) (Wu et al. 1994).

3 Semi-analytical solutions of the post-shock region

To solve the equations (2.1), (2.2), and (2.3) in the steady-state regime, an intermediate variable η is introduced (Bertschinger 1989; Falize et al. 2009) defined by the following equations:

$$\rho(x) = \frac{\rho_0}{\eta(x)}, \qquad v(x) = v_{ff}\eta(x) \tag{3.1}$$

where ρ_0 and v_{ff} are respectively the density and the velocity of the supersonic-upstream flow. The upstream velocity is the free-fall velocity given by $v_{ff} = \sqrt{2GM_{WD}/x_0}$ and the upstream density is inferred from the accretion rate, $\dot{M}$, as $\rho_0 = \dot{M}/(Sv_{ff})$ where S is the column cross-section. At the shock front, the post-shock variables are related to the pre-shock variables by the Rankine-Hugoniot conditions. The white dwarf photosphere is assumed to be a cold and solid wall at which temperature and velocity tend to zero. After some algebraic manipulations, equations (2.1), (2.2), and (2.3) lead to a system of two coupled differential equations obtained on the intermediate variable η and the variable P:

$$\frac{dP}{dx} = \frac{\rho_0}{\eta}\frac{GM_{WD}}{(x_0 - x)^2} - \rho_0 v_{ff}^2 \frac{d\eta}{dx} \tag{3.2}$$

$$\frac{d\eta}{dx}\left[\gamma P v_{ff} - \rho_0 v_{ff}^3 \eta\right] = -(\gamma - 1)\Lambda(\eta, P) - v_{ff}\rho_0 \frac{GM_{WD}}{(x_0 - x)^2} \tag{3.3}$$

The transformation (3.1) allows us to simplify the reductions of the equations [(2.1), (2.2), (2.3)] in a more direct way than in previous studies (Cropper et al. 1999). This system [(3.2), (3.3)] is solved by a dichotomy method coupled with a fourth-order Runge-Kutta method. The shock height is extracted iteratively from the boundary condition: $v = 0$ at the white dwarf photosphere. In Fig. 1a and Fig. 1b, the spatial profiles of the post-shock region variables (density, velocity, pressure and temperature) are presented, in the case of the bremsstrahlung cooling, obtained with this method (filled color lines). When $x_s/R_{WD} << 1$, the gravitational field of the white dwarf can be assumed constant in the post-shock region and then it is negligible compared to the cooling processes. In this case, the system [(3.2), (3.3)] can be simplified in a single differential equation on η and an ordinary function for the pressure. The resulting differential equation admits analytical solutions when only one cooling process is taken into account (Falize et al. 2009). The inferred profiles from the analytical solution for the bremsstrahlung cooling are presented in dotted black lines. They are compared to the semi-analytical solution obtained from the system [(3.2), (3.3)] when gravitation is not taken into account (dashed color lines). In Fig. 1a, the post-shock region properties are calculated for a white dwarf mass of $M_{WD} = 0.4$

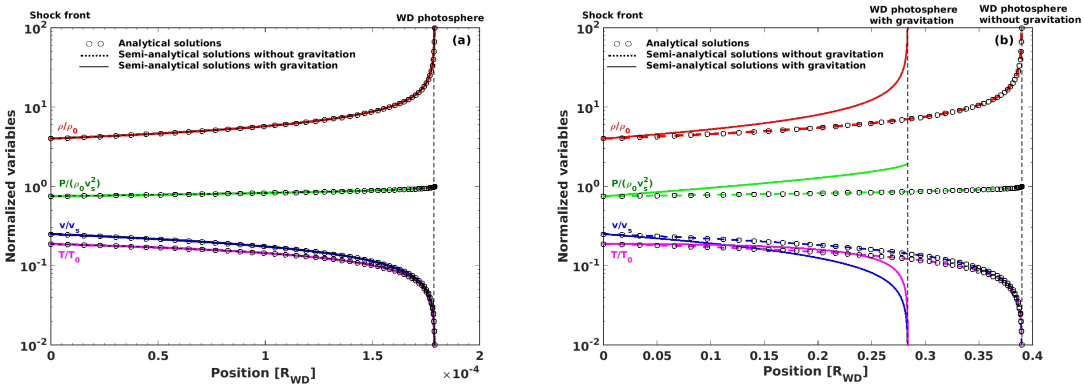

Fig. 1. Normalized profiles of the density, pressure, velocity and temperature of the post-shock region with only bremsstrahlung process. The shock front is located to the origin of the x-axis and the white dwarf photosphere is located at the dotted black lines. **(a)** Profiles when gravitation is negligible: $M_{WD} = 0.4\ M_{\odot}$, $\dot{M} = 10^{16}$ g.s^{-1}, and $S = 10^{14}$ cm^2. **(b)** Profiles when gravitation can not be neglected: $M_{WD} = 1\ M_{\odot}$, $\dot{M} = 10^{15}$ g.s^{-1}, and $S = 10^{15}$ cm^2.

$M_{\odot}$, an accretion rate of $\dot{M} = 10^{16}$ g.s^{-1} and a column cross-section of $S = 10^{14}$ cm^2. In this case, the white dwarf gravitational field can be neglected because $x_s/R_{WD} = 1.8 \times 10^{-4} << 1$ with $x_s = 2$ km. All the physical profiles perfectly match the analytical solutions which describe the high-energy region. In Fig. 1b, the spatial profiles are presented when the white dwarf gravitation strongly modifies the gradient of the post-shock region characterized by a white dwarf mass of $M_{WD} = 1\ M_{\odot}$, an accretion rate of $\dot{M} = 10^{15}$ g.s^{-1} and a column cross-section of $S = 10^{15}$ cm^2. When gravitation is not taken into account, $x_s/R_{WD} = 0.39$ with $x_s = 2130$ km whereas when gravitation is taken into account, $x_s/R_{WD} = 0.28$ with $x_s = 1550$ km. Gravitation tends to decrease the steady-state height due to the release of gravitational energy which is in agreement with the results presented by Cropper et al. (1999). In this case, gravitation is essential to correctly model the radiative region.

4 Steady-state shock height as function of mCVs parameters

As shown by the radiation hydrodynamics equations, the physics of the accretion column in the steady-state regime is strongly dependent on the four mCVs parameters: the white dwarf mass M_{WD}, the white dwarf magnetic field B_{WD}, the accretion rate $\dot{M}$ of the accretion flow and the column cross-section S. In Fig. 2a, Fig. 2b and Fig. 2c, the behaviour of the steady-state shock height is extracted as a function of the white dwarf mass for $M_{WD} = 0.2 - 1.4\ M_{\odot}$ and for different set of parameters [B_{WD}, $\dot{M}$ and S] whose values are taken in the observed parameters range (Bonnet-Bidaud et al. 2015). Each curve has a resolution of a hundred points. The filled lines correspond to the case with gravitation and the dashed lines without gravitation. In Fig. 2a, the variation of the magnetic field for different values (0, 30, 60, 90 MG) is shown assuming a representative accretion rate at 10^{15} g.s^{-1} and a cross-section at 10^{14} cm^2. In Fig. 2b, the evolution for different accretion rates (10^{14}, 10^{15}, and 10^{16} g.s^{-1}) is studied for an assumed magnetic field at 30 MG and a column cross-section at 10^{14} cm^2. Finally in Fig. 2c, the variation of different values of cross sections (10^{14}, 10^{15} and 10^{16} cm^2) is studied for an assumed 30 MG magnetic field and accretion rate value at 10^{15} g.s^{-1}.

An analytical expression of the steady shock height as a function of the mCVs parameters is possible only in the most simple case: with only bremsstrahlung cooling and no gravitation (Falize et al. 2009). When the model takes into account the cyclotron cooling and the gravitational field, there is no analytical solution of the steady-state height. For very low white dwarf masses, strong accretion rates and small column cross-section, the bremsstrahlung cooling is dominant and the shock height strictly increases ($x_s \sim S\dot{M}^{-1}M_{WD}^{3/2}R_{WD}^{-3/2}$) as a function of the white dwarf mass. When the cyclotron cooling starts becoming dominant, the shock height reaches a peak and decreases as the white dwarf mass increases. This maximum is due to the balance between the two radiative cooling processes and is the result of the inversion of the dominant cooling process in the post-shock region. The accretion rate and the column cross-section modify the position of this maximum by modifying the radiative balance. As shown before, x_s obviously decreases with the gravitational field. For very strong magnetic fields ($B_{WD} >> 30$ MG), the radiative processes, in particular the cyclotron cooling, is so

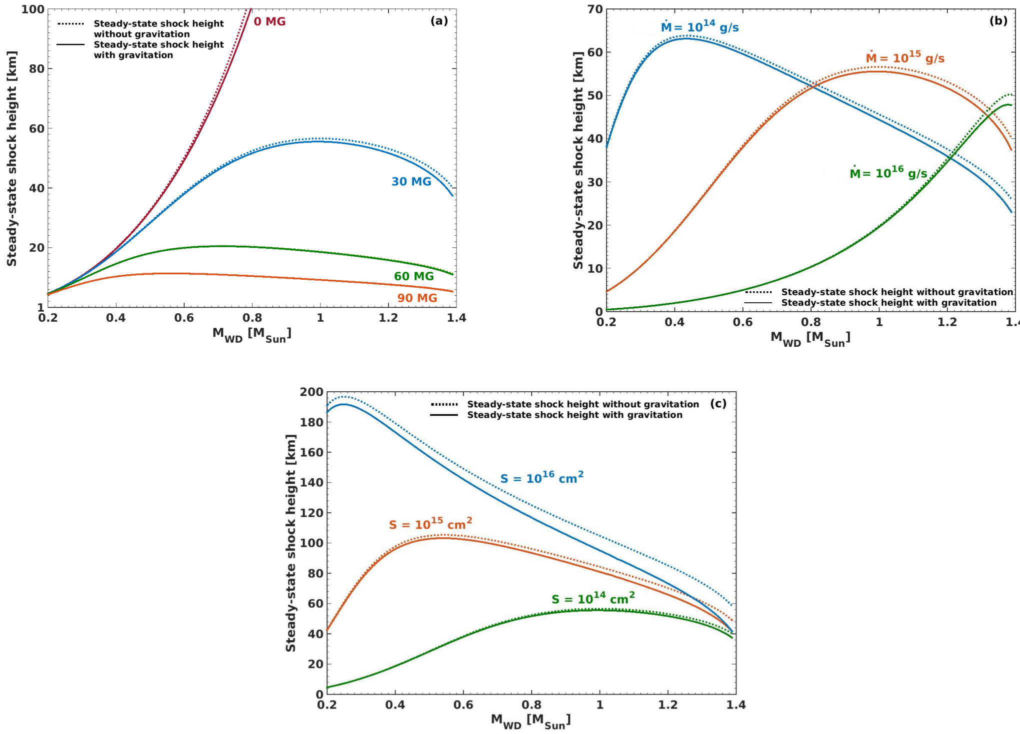

Fig. 2. Evolution of the steady-state shock height as a function of the white dwarf mass $M_{WD} = 0.2 - 1.4\ M_{\odot}$ for different mCVs parameters. The resolution without gravitation is given in dashed line and the resolution with gravitation in filled line. **(a)** An accretion rate of 10^{15} g.s^{-1} and a section of 10^{14} cm^2 are assumed. **(b)** A magnetic field of 30 MG with a cross-section of 10^{14} cm^2 is assumed. **(c)** A magnetic field of 30 MG with an accretion rate of 10^{15} g.s^{-1} is assumed.

strong that the shock height becomes very small ($x_s < 10$ km) and the gravitational field is saturated, not playing a role anymore.

5 Conclusions

We have presented solutions of the radiation hydrodynamics equations with gravitation which model the accretion column and particularly onto high mass white dwarfs. We have compared them with analytical and semi-analytical solutions which validates our resolution in the case where gravitation is negligible and in the case where gravitation is essential to correctly model the post-shock region. An extensive study of the steady-state shock height as functions of mCVs parameters has been presented. We have highlighted the wide range of shock height behaviours, and in particular the inversion of the balance between the two radiative cooling processes in this high-energy region generates a maximum of the shock height.

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