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HALL MAGNETO-HYDRODYNAMICS IN PROTOPLANETARY DISCS

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Abstract. Protoplanetary discs exhibit large-scale, organised structures. Because they are dense and cold, they should be weakly ionized, and hence concerned by non-ideal plasma effects, such as the Hall effect. We perform numerical simulations of non-stratified Keplerian discs, in the non-ideal magnetohydrodynamic framework. We show that the Hall effect causes self-organisation through three distinct stages. A weak Hall effect enhances turbulent transport. At intermediate strength, it produces magnetized vortices. A strong Hall effect generates axisymmetric zonal flows. These structures may trap dust particles, and thus influence planetary formation. The transport of angular momentum is quenched in the organised state, impugning the relevance of magneto-rotational turbulence as a driving mechanism of accretion in Hall dominated regions.

Keywords: protoplanetary discs, magnetohydrodynamics (MHD), turbulence

1 Introduction

Observations of young stellar objects reveal signatures of accretion at a rate $\dot{M} \sim 10^{-8} M_\odot\,\mathrm{yr}^{-1}$ onto the star. This calls for a mechanism to efficiently transport angular momentum through the protoplanetary disc. The magneto-rotational instability (MRI, Balbus & Hawley 1991) provides such a mechanism. It was originally studied in the context of fully ionised discs, threaded by a weak magnetic field. Under these conditions, it saturates in sustained turbulence, which exerts a net torque on the flow (Hawley et al. 1995).

However, protoplanetary discs are optically thick, so ionising stellar radiations and cosmic rays can hardly reach the densest regions. The ionisation fraction typically drops below $n_e/n \lesssim 10^{-10}$ within the disc (Fromang et al. 2002), whence non-ideal' magnetohydrodynamic (MHD) effects come into play. Ohmic and ambipolar diffusions can damp, and potentially quench the MRI (Jin 1996; Kunz & Balbus 2004). The Hall effect induces different behaviours depending on its intensity. A weak Hall effect can enhance or kill the turbulent transport, depending on the orientation of the large-scale magnetic field (Balbus & Terquem 2001; Sano & Stone 2002). A strong Hall effect was shown to generate self-organised flows in local simulations (Kunz & Lesur 2013).

Recent observations of the dust emission disclosed large-scale structures in discs, such as horseshoe-shaped traps (van der Marel et al. 2013) or axisymmetric rings (Brogan et al. 2015). It is tempting to ask whether these structures could emerge from Hall-MHD turbulence, without resorting to planet-disc interactions. Numerical simulations including the Hall effect were mostly carried in the local, shearing-box approximation. We address the ability of the Hall effect to produce self-organised structures in a global context (Béthune et al. 2016).

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2 Framework

2.1 Model

Protoplanetary discs obey the equations of non-ideal magnetohydrodynamics (MHD); the evolution of density, momentum and magnetic fields are thus given by (Balbus & Terquem 2001):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot [\rho \boldsymbol{v}] = 0, \tag{2.1}$$

$$\frac{\partial}{\partial t}[\rho \boldsymbol{v}] + \nabla \cdot [\rho \boldsymbol{v} \otimes \boldsymbol{v} + P\boldsymbol{I}] = \boldsymbol{J} \times \boldsymbol{B} - \rho \boldsymbol{\nabla} \Phi, \tag{2.2}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \left[\boldsymbol{v} \times \boldsymbol{B} - \eta \boldsymbol{J} - \frac{1}{n_e e} \boldsymbol{J} \times \boldsymbol{B} + \frac{1}{c\gamma \rho_i \rho} \boldsymbol{J} \times \boldsymbol{B} \times \boldsymbol{B} \right]. \tag{2.3}$$

The 'non-ideal' character comes from the low ionisation degree of the gas, and brings about the last three terms in equation (2.3). From left to right, they respectively correspond to Ohmic diffusion, the Hall effect, and ambipolar diffusion. Their relevance to protoplanetary discs is discussed by, for example, Wardle (2007).

We are primarily interested by the influence of the Hall effect. The intensity of this term is inversely proportional to the charge number density n_e. It follows that the Hall effect is strongest in the midplane. As a first step, we only consider the deepest layers of the disc, neglecting its vertical density stratification. The thermal stratification is ignored as well, the gas being considered as isothermal for simplicity.

2.2 Method

The PLUTO code (Mignone et al. 2007) is used to integrate the dynamical equations in time. We define a cylindrical computational domain, with the radius r extending from r_0 to $5r_0$, and the height z from zero to h. Periodic boundary conditions are applied in the vertical and azimuthal directions; outflow conditions are applied at the radial boundaries. The thin disk approximation requires h/r_0 to be small; we set it to $1/4$. The isothermal pressure scale height $H \equiv c_s/\Omega$ is made comparable to the geometrical height h, by setting the isothermal sound speed $c_s = 10\%$ of the inner Keplerian velocity $\Omega_0 r_0$. The initial density has a constant value. The initial magnetic field B_0 is axial and constant too, with a plasma $\beta \gg 1$, and with $\Omega \cdot B > 0$. The opposite polarity simply produces a stable, laminar flow (Balbus & Terquem 2001).

With $v_{\mathrm{A}} \equiv B/\sqrt{\rho}$ the Alfvén velocity, the Hall induction term in equation (2.3) can be written as

$$\frac{1}{n_e e} \boldsymbol{J} \times \boldsymbol{B} = \ell_{\mathrm{H}} v_{\mathrm{A}} \boldsymbol{J} \times \boldsymbol{e_B}, \tag{2.4}$$

where $\boldsymbol{e_B}$ is the unit vector along the magnetic field. The Hall length ℓ_{H} defines the scale below which the Hall effect becomes dynamically important (Kunz & Lesur 2013). We normalise it to the geometrical height of the computational domain, $\mathcal{L} \equiv \ell_{\mathrm{H}}/h$. This number is also set to a constant value in each simulation.

3 Results

3.1 Self-organisation

We perform a series of simulations, varying the magnitude of the initial magnetic field B_0 and Hall length ℓ_{H}. We show in Fig. 1 the result of three runs having the same initial magnetic field, but different values of $\mathcal{L}$.

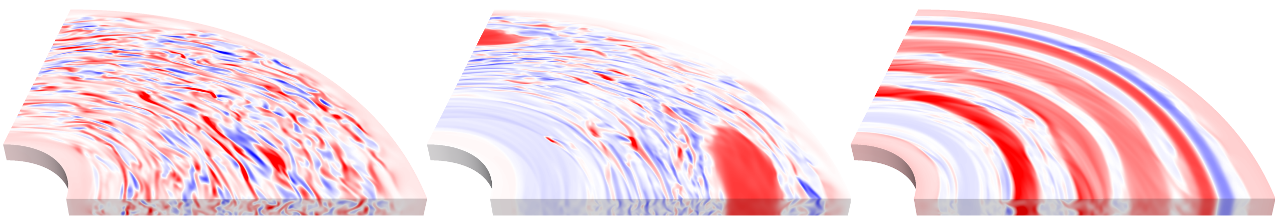

Fig. 1. Distribution of axial magnetic field B_z for three values of $\mathcal{L}$. **Left:** turbulence in the ideal MHD case $\mathcal{L} = 0$. **Center:** magnetized vortex in the intermediate case $\mathcal{L} = 0.4$. **Right:** zonal flows in the strong Hall regime $\mathcal{L} = 1$.

The case $\mathcal{L} = 0$ (*left panel*) corresponds to ideal MHD. Due to the weak, axial magnetic field, the magneto-rotational instability operates in this disc (Balbus & Hawley 1991). The instability saturates into turbulence, as known from local, shearing-box simulations (Hawley et al. 1995). When increasing $\mathcal{L}$ from zero to $\mathcal{L} = 0.4$ (*center panel*), the spatial scale of magnetic fluctuations increases, until leaving one single magnetic flux concentration. When increasing $\mathcal{L}$ beyond unity (*right panel*), the magnetic field gets confined into axisymmetric rings, with no flux between. For $\mathcal{L} \geq 1$, increasing B_0 or ℓ_{H} will increase the number of rings, their width remaining $\sim h$.

While causing these magnetic structures, the Hall effect also affects the velocity and pressure fields of the flow. The equations of incompressible Hall-MHD admit the conserved flux

$$\varpi \equiv \nabla \times \boldsymbol{v} + \frac{1}{\rho \ell_{\mathrm{H}}} \boldsymbol{B}. \tag{3.1}$$

As a consequence, a local increase in magnetic flux must come with a local decrease in vorticity flux, and vice versa. In the case $\mathcal{L} = 0.4$, the patch of accumulated magnetic flux has its vorticity lowered compared to the surrounds. In other words, it is an anti-cyclonic vortex in the disc. Vortices may play a role in the process of planetary formation, as they can attract dust particles (Barge & Sommeria 1995). The rings found for $\mathcal{L} = 1$ also induce radial oscillations of the orbital velocity, called zonal flows. The gas transits from super to sub-Keplerian rotation in these rings, making them potential dust traps too (Weidenschilling 1977).

3.2 *Transport of angular momentum*

The accretion of matter from a Keplerian disc onto its central star requires a removal of angular momentum. One way to transport angular momentum is given by turbulence, customarily described by its diffusivity $\nu = \alpha \Omega h^2$ (Shakura & Sunyaev 1973). We show in Fig. 2 how the Hall effect modifies the transport coefficient α.

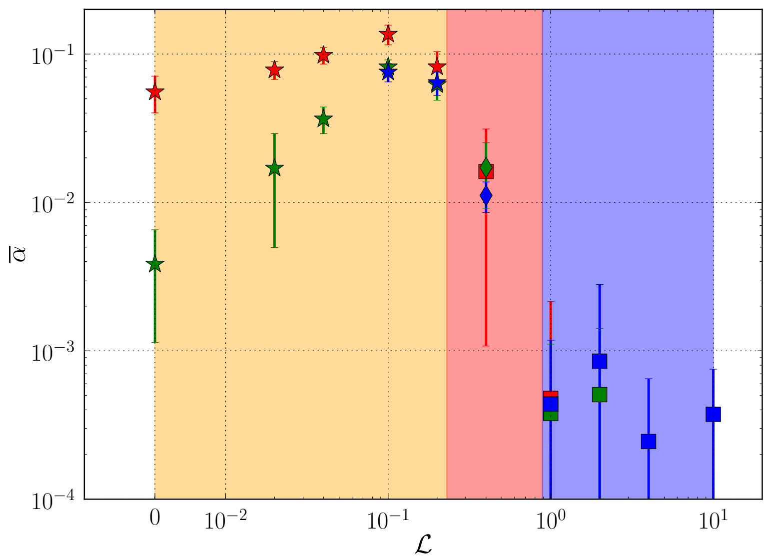

Fig. 2. Volume-averaged, normalised stress $\bar{\alpha}$ as a function of the Hall strength $\mathcal{L}$. The three coloured regions delineate the turbulence (orange), vortex (red) and zonal flow (blue) regimes. The symbols color corresponds to the initial magnetic field intensity B_0, being weak (blue), intermediate (green) or strong (red).

In the turbulent regime $\mathcal{L} \lesssim 0.2$ (orange area), the transport of angular momentum is increased by the Hall effect. As in ideal MHD, it also increases with the initial magnetic field (Hawley et al. 1995). The maximum $\alpha \approx 0.1$ is attained for $\mathcal{L} \approx 0.1$. The level of transport steeply decreases for $\mathcal{L} \in [0.1, 1]$, where vortices are observed. In the zonal flow regime $\mathcal{L} \geq 1$, the transport coefficient is stalled at $\alpha \lesssim 10^{-3}$. This stress is in fact caused by spiral density waves, excited near the inner radial boundary, where the sound speed $c_s < \Omega h$. The magnetic contribution alone would produce $\alpha \lesssim 10^{-5}$, insufficient to explain the observed accretion rates.

4 Conclusions

The Hall effect tends to organise the magnetic and vorticity fluxes threading a non-stratified Keplerian disc. When increasing its intensity, the flow transits trough three distinct structural states. A weak Hall effect $\mathcal{L} \ll 1$ enhances turbulent transport. For moderate strengths $\mathcal{L} \lesssim 1$, the disc features large-scale, magnetized vortices. A strong Hall effect $\mathcal{L} \gtrsim 1$ produces axisymmetric zonal flows, and kills angular momentum transport. These structures were found to hold when adding Ohmic and ambipolar diffusion with realistic intensities.

Both vortices and zonal flows induce local pressure maxima, which could attract dust particles and hence influence the course of planetary formation. In the strong Hall limit, the absence of turbulent transport raises questions regarding the driving mechanism of accretion. Stratified simulations by Lesur et al. (2014) suggest that the overall stress could be laminar in this regime. The self-organisation threshold depends solely on the ratio $\ell_{\mathrm{H}}/h \approx 0.1$, but the geometrical height h was introduced for modelling purposes. These results must therefore be confronted to global, stratified simulations.

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