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HOW BARYONIC FEEDBACK PROCESSES CAN AFFECT DARK MATTER HALOS: A STOCHASTIC MODEL

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Abstract. Feedback processes from stars and active galactic nuclei result in gas density fluctuations which can contribute to 'heating' dark matter haloes, decrease their density at the center and hence form more realistic 'cores' than the steep 'cusps' predicted by cold dark matter (CDM) simulations. We present a theoretical model deriving this effect from first principles: stochastic density variations in the gas distribution perturb the gravitational potential and hence affect the halo particles. We analytically derive the velocity dispersion imparted to the CDM particles and the corresponding relaxation time, and further perform numerical simulations to show that the assumed process can indeed lead to the formation of a core in an initially cuspy halo within a timescale comparable to the derived relaxation time. This suggests that feedback-induced cusp-core transformations observed in hydrodynamic simulations of galaxy formation may be understood and parametrized in relatively simple terms.

Keywords: Dark matter, Galaxies: halos, Galaxies: evolution, Galaxies: formation

1 Introduction

Despite its huge success at explaining the large scale structure of the Universe, the cold dark matter (CDM) model of structure formation faces different challenges at galactic scales. In particular, while CDM numerical simulations predict steep, 'cuspy' density profiles for dark matter halos, observations of dark matter dominated galaxies favor more shallower 'cores' (e.g., Moore 1994; de Blok et al. 2008; Oh et al. 2011).

Proposed solutions to this 'core-cusp' discrepancy and the related challenges of CDM cosmology, such as the 'too big to fail' problem, can be broadly divided into those considering fundamental changes in the physics of the model and those focusing on the baryonic processes at stake during galaxy formation and evolution. The first category of solutions comprises alternatives to CDM such as warm dark matter, self-interacting dark matter and models that fundamentally change the gravitational law like Mordechai Milgrom's MOND theory. Solutions invoking baryonic processes within the CDM framework are motivated by the fact that the discrepancies between model and observations precisely occur at the scale at which baryons start to play an important role, notably through powerful stellar and active galactic nuclei (AGN) feedback processes and outflows. Moreover, hydrodynamical simulations with different feedback implementations are able to reproduce dark matter cores (e.g., Governato et al. 2010; Teyssier et al. 2013). However, such complex simulations do not necessarily specify the physical mechanisms through which baryons affect the dark matter distribution.

Baryons can mostly affect the dark matter halo through their own gravity and by modifying the gravitational potential. Such is the case with adiabatic contraction (when the accumulation of cold gas at the center of the halo steepens its potential well and causes the dark matter to contract; Blumenthal et al. 1986) and with the dynamical friction through which a massive object such as a satellite galaxy or a clump of gas can transfer part of its kinetic energy to the dark matter background (Chandrasekhar 1943). This latter process can 'heat' the dark matter halo and remove the central cusp (El-Zant et al. 2001). Alternatively, repeated gravitational potential fluctuations induced by stellar winds, supernova explosions and AGN could also dynamically heat

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the dark matter and lead to the formation of a core (Pontzen & Governato 2012). In this case, variations in the baryonic mass distribution induce violent potential fluctuations which progressively disperse dark matter particles away from the center of the halo.

To isolate further the physical mechanism at stake during core formation, we present and test an *a priori* theoretical model in which the gravitational potential fluctuations leading to core formation arise from feedback-induced stochastic density variations in the gas distribution. Dark matter particles experience successive 'kicks' from the potential fluctuations, which cumulatively induce them to deviate from their trajectories as in a diffusion process or as two-body relaxation does for stellar systems. This work is described in more details in Freundlich (2015), Chapter 4, and El-Zant et al. (2016).

2 Theoretical model

2.1 Stochastic density fluctuations

We assume that the potential fluctuations leading to core formation arise from stochastic density perturbations in a gaseous medium of mean density ρ_0 confined within a sphere of radius d corresponding to the inner region of the halo. The density contrast $\delta(\boldsymbol{r}) = \rho(\boldsymbol{r})/\rho_0 - 1$ can be Fourier decomposed over $V = d^3$ such that

$$\delta(\boldsymbol{r}) = \frac{V}{(2\pi)^3} \int \delta_{\boldsymbol{k}} e^{-i\boldsymbol{k}.\boldsymbol{r}} d\boldsymbol{k}. \tag{2.1}$$

The perturbations are assumed to be isotropic, stationary and described by a power-law power spectrum

$$\mathcal{P}(k) = V \langle |\delta_k|^2 \rangle \propto k^{-n}. \tag{2.2}$$

Turbulent media such as the interstellar medium are indeed expected to display power-law power spectra as fluctuations initiated at large scale cascade down to the dissipation scale. For the sake of our calculations, we also assume minimum and maximum cutoff scales $2\pi/k_{\rm max} \ll 2\pi/k_{\rm min}$.

2.2 Repetitive kicks on the dark matter particles

Each perturbation mode $\delta_{\boldsymbol{k}}$ induces a small 'kick'

$$\boldsymbol{F}_{\boldsymbol{k}} = 4\pi i \; G\rho_0 \; \boldsymbol{k} \; k^{-2} \; \delta_{\boldsymbol{k}} \tag{2.3}$$

on the dark matter particles, the cumulative effect of these kicks leading the particles to deviate from their trajectories by a mean velocity variation after a time T such that

$$\langle \Delta v^2 \rangle = 2 \int_0^T (T-t) \, \langle F(0) F(t) \rangle \; dt. \tag{2.4}$$

This description is analogous to two-body relaxation in stellar systems, in which the kicks correspond to the successive interactions of the particles with one another.

2.3 Relaxation time in the diffusion limit

In the diffusion limit where $2\pi/k_{\rm min} \ll R$, i.e., where the density perturbations are small compared to the distance R traveled during T by the dark matter particles with respect to the fluctuation field, we analytically obtain a relaxation time

$$t_{\rm relax} = \frac{n v_r \langle v \rangle^2}{8\pi (G\rho_0)^2 V \langle |\delta_{k_{\rm min}}|^2 \rangle}, \tag{2.5}$$

where $v_r = R/T$ is the mean velocity of the dark matter particles with respect to the fluctuating field and $\langle v \rangle$ their initial orbital velocity. This expression assumes that the spatial statistical properties of the perturbations expressed through the force autocorrelation function $\langle \boldsymbol{F}(0).\boldsymbol{F}(r) \rangle$ can be transported into the temporal domain such that $\langle \boldsymbol{F}(0).\boldsymbol{F}(t) \rangle = \langle \boldsymbol{F}(0).\boldsymbol{F}(r = v_r t) \rangle$ (e.g., Wilczek et al. 2014, and references therein). The resulting relaxation time does not depend on the minimum and maximum cutoff scales, and only linearly on the power law exponent n. It mainly depends on the gas mass fraction through ρ_0 and the normalization of the power spectrum of the density fluctuations.

2.4 Application to a fiducial dwarf galaxy

We evaluate the relaxation time for a fiducial dwarf NFW halo, assuming orbital velocities $\langle v(l)\rangle \sim l\sqrt{G\rho(<l)}$ and gas movements dominated by those at the largest fluctuation scale, with $v_r \sim d\sqrt{G\rho(<d/2)}/2$, $\rho(<l)$ being the average density inside radius l. The halo is assumed to have a scalelength $R_s = 0.9$ kpc and a total mass $M_{\rm vir} = 2.26\ 10^{11}\ {\rm M}_\odot$ inside $R_{\rm vir} = 30$ kpc. The gas mass fraction inside $d/2 = 5$ kpc is $f(d/2) \equiv \rho_0/\rho(<d/2) = 0.17$ and we assume a power spectrum with $n = 2.4$ and $\langle|\delta_{k_{\rm min}}|^2\rangle \approx 0.005$. Eq. 2.5 yields a relaxation time of about 3.5 Gyr within $d/2$, decreasing towards the very center of the halo. This gives a timescale at which the density variations are expected to affect the trajectories of the dark matter particles, but does not specify the global response of the system.

3 Numerical test

3.1 Numerical test setup

In order to test the effects of power-law density fluctuations as in our theoretical model on the dark matter distribution of a galactic halo, we use the self-consistent field (SCF) method developped by Hernquist & Ostriker (1992). This algorithm was designed to describe the evolution of collisionless stellar systems by computing the gravitational potential at each time step and advancing the trajectories of the particles one by one accordingly. The density and the potential are expanded in a set of basis functions deriving from spherical harmonics with radial and angular maximal cutoff numbers $n_{\rm max}$ and $l_{\rm max}$.

We carry out such a simulation for the fiducial dwarf halo described in section 2.4, adding force and potential perturbations as in our theoretical model. The direction of each kick is random, and the total force is rescaled *a posteriori* to match the assumed power spectrum normalization. We further assume that the pulsation frequency associated to a mode k is either defined with a constant propagation velocity as $\omega(k) = v_r k$ or from Larson's relation (Solomon et al. 1987) as $\omega(k) = 2\sqrt{k}$, both choices yielding similar results.

3.2 Spherical case: a flattening of the cusp as expected from the theoretical model

To neglect non-radial modes and match more closely the analytical calculations, we start by considering the case where strict spherical symmetry is maintained by imposing $l_{\rm max} = 0$. The resulting evolution of the halo density profile is shown on the left panel of Fig. 1: the assumed stochastic density fluctuations do lead to the formation of a core in an initially cuspy configuration within a timescale comparable to the relaxation time derived analytically. As expected from Eq. 2.5, the effect mostly depends on the fluctuation level and the gas fraction, with a weak dependence in n and no variations with $k_{\rm min}$ and $k_{\rm max}$ (cf. El-Zant et al. 2016).

3.3 General case: an accelerated cusp-core transformation due to non-radial modes

In the general case, $l_{\rm max} \neq 0$ and no spherical symmetry is imposed on the system. An optimal choice for a simulation with $\sim 10^5$ particles is $n_{\rm max} = 10$ and $l_{\rm max} = 4$ (Vasiliev 2013). In this case, the cusp-core transformation is significantly faster than in the previous case but its parametrization remains unchanged, which can be seen on the right panel of Fig. 1. As the perturbations imposed on the halo particles are the same as when spherical symmetry is enforced, the difference must stem from how the imparted energy is transported and redistributed within the halo. This suggests that azimuthal modes significantly boost core formation and that the processes through which the energy stemming from the fluctuations is redistributed are largely non-isotropic. Such a conclusion is in agreement with Pontzen et al. (2015), who also show that the non-sphericity of dark matter haloes is a key ingredient for an efficient cusp-core transition.

4 Conclusion

We presented and tested through simple collisionless simulations an *a priori* theoretical model to describe the cusp-core transformation of dark matter haloes in which gravitational potential fluctuations arise from stochastic density variations in the gas distribution. Different stellar and AGN feedback mechanisms can account for such density variations. Their dynamical effects are modeled as a diffusion process in which repetitive kicks to the dark matter particles contribute to heating the halo and to forming a core. This model provides a relatively simple parametrization of the cusp-core transformation, mostly depending on the gas fraction and the fluctuation level.

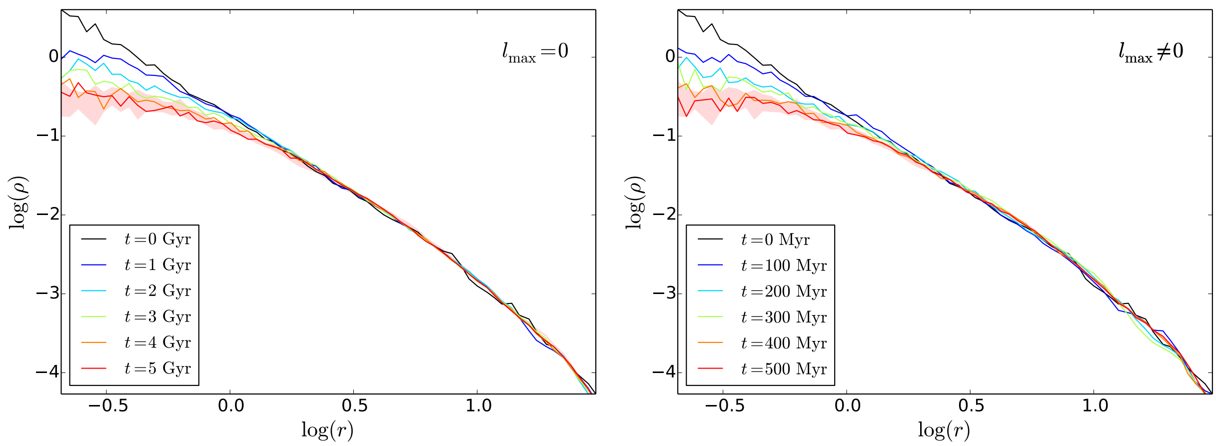

Fig. 1. Left: Evolution of the dark matter density profile for the fiducial halo described in section 2.4 with strict spherical symmetry imposed at each time step through $l_{\rm max} = 0$. The halo is submitted to a fluctuating gravitational potential stemming from power-law density fluctuations as in our theoretical model, presented in section 2, and forms a core from an initially cuspy NFW profile within a few Gyr. **Right:** A similar evolution is observed on a much smaller timescale when no strict spherical symmetry is imposed. On both sides, the shaded area highlights the scatter at $t = 500$ Myr between 10 random realization of the simulation with $l_{\rm max} \neq 0$.

A detailed comparison with hydrodynamical simulations is left to a future study. Different feedback implementations are likely to change the statistical properties of the fluctuating density field and hence the efficiency of the cusp-core transformation, which we could compare with our parametrization. Other theoretical models attempt at describing this transformation, with differences but also some similarities. Amongst them, Dutton et al. (2016) propose a spherical model based on a succession of global inflows and violent outflows, while Fouvry et al. (2016) focus on a diffusion mechanism not unlike ours but described by a dressed Focker-Planck equation. The predictions of these different models should be compared together with hydrodynamical simulations, the importance of non-radial modes and asphericity being one of the issues to be investigated more thoroughly.

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