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INFLUENCE OF PLUME-INDUCED INTERNAL GRAVITY WAVES ON THE ROTATION PROFILE OF LOW-MASS STARS

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Abstract. High-quality seismic data due to the space-borne missions CoRoT and *Kepler* provide precious information on the core rotation of thousands of stars from the subgiant to the red giant stages. We know today that current stellar evolution codes need for an additional physical mechanism to extract angular momentum from the core to the envelope of evolved low-mass stars and explain the low observed internal rotation. In this framework, internal gravity waves generated by penetrative convection at the top of the radiative region may play a role. In this work, we investigate whether the transport of angular momentum by plume-induced gravity waves may counteract the accelereration due the the strong contraction of the innermost layers. On the red giant branch, we find that the strong radiative damping near the H-burning shell prevents these waves from slowing down the core, so that another process should operate in these stars. Nevertheless, we show that plume-induced gravity waves are a good candidate to regulate the amplitude of the differential rotation in subgiant stars.

Keywords: stars: rotation - waves - convection - hydrodynamics

1 Introduction

Asteroseismology reveals the internal structure of stars and bring stringent constraints for stellar modeling. Since the achievement of the space-borne missions CoRoT and *Kepler*, a large amount of seismic data for stars from the subgiant to the red giant branches have been made available. Among the main scientific results, the detection of mixed modes, which are oscillation modes with amplitude both in the core and the envelope, made the measurement of the mean core rotation possible for thousands of stars. It thus provided a step forward towards a better understanding of the angular momentum redistribution through the post-main sequence evolution. From these observations, it turns out that the mean core rotation moderately increases on the subgiant branch (Deheuvels et al. 2012, 2014) and then strongly drops as soon as the beginning of the red giant branch, while the central layers are still contracting (Mosser et al. 2012).

Theoretical predictions made by the current stellar evolution codes including transport by meridional circulation and shear-induced mixing are far from reproducing the observations (e.g. Marques et al. 2013; Ceillier et al. 2013). In addition, we know for more than one decade that stellar modeling still fails to predict the quasi solid-body rotation measured in the solar radiative zone (e.g. García et al. 2007). All these discrepancies between theory and observations stress out the need for an additional mechanism to extract angular momentum from the core to the envelope of the stars.

In this framework, internal gravity waves (hereafter, IGW), which are buoyancy waves propagating through the radiative zone of the stars, may have a significative role to play. They are generated at the lower edge of the convective zone, either by turbulent stresses or by the penetration of convective plumes, as observed in geophysics or in numerical simulations (e.g. Dintrans et al. 2005). IGW can then travel in depth before being radiatively damped and thus deposit their angular momentum into the medium. In one hand, IGW generated by turbulent pressure, following the excitation model by Kumar et al. (1999), have already been shown to be able to explain the flat rotation profile observed in the solar radiative zone (Zahn 1997; Talon et al. 2002). However, they seem to rapidly decouple from the core as soon as the beginning of the subgiant branch because of an increasing radiative damping that prevents them from reaching the innermost layers (Fuller et al. 2014).

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In the other hand, a semianalytical estimate of the generation of IGW by penetrative convection is now available and has already demonstrated the ability of these waves to efficiently affect the rotation of the solar radiative zone (Pinçon et al. 2016). Nevertheless, a similar study on more evolved stars has not been undertaken yet.

In this work, we investigate the ability of plume-induced waves to modify a given rotation profile in subgiant and red giant stars. As a first step, our study is based on the comparison of the dynamical wave-driven timescale with the contraction/expansion timescale throughout a $1M_\odot$ star. Moreover, the role of the differential rotation amplitude on the transport by IGW is stressed out.

2 Characteristic timescale and plume-induced wave flux

2.1 *Wave-driven timescale*

Transport of angular momentum in stars is governed by an advection-diffusion equation. Therefore, the local timescale associated with the transport of angular momentum by IGW, given a rotation profile $\Omega(r)$ (within the shellular approximation), is equal to the ratio of the density of angular momentum in the star to the radial divergence of the angular momentum wave flux, i.e.

$$t_w(r) \sim \left| \frac{\rho r^2 \Omega}{\dot{J}} \right| \ , \tag{2.1}$$

with r the radius, ρ the density at the equilibrium and $\dot{J}$ the divergence of the mean radial wave flux of angular momentum. At this stage, the angular momentum extraction by IGW, in a shell at the radius r, will be said to be efficient compared to the acceleration caused by the core contraction if $t_w(r) < t_{cont}(r)$, with t_{cont} the local contraction timescale. As shown in Eq. (2.1), the computation of t_w requires the knowledge of the wave flux throughout the star via the term $\dot{J}$.

2.2 *Wave flux generated by penetrative convection*

To estimate the wave energy flux emitted at the top of the radiative zone, Pinçon et al. (2016) considered the pressure exerted by an ensemble of incoherent convective plumes in the penetration zone as the source term in the wave equation. By assuming a high Péclet number and a very sharp thermal transition at the base of the convective zone, they derived a simplified expression for the mean radial wave energy flux per unit of frequency, for an angular degree l and an azimuthal number m at the top of the radiative zone,

$$\mathcal{F}_{E,w}(r_t, \omega, l, m) \sim \frac{1}{4\pi r_t^2} \frac{\mathcal{A}\mathcal{S}_p}{2} \frac{\rho_b V_b^3}{2} F_{R,l} \frac{e^{-\omega^2/4\nu_p^2}}{\nu_p} e^{-l(l+1)b^2/2r_t^2} \ , \tag{2.2}$$

where r_t is the radius at the top of the radiative zone, $\mathcal{A}$ is the plumes filling factor in the excitation region, $\mathcal{S}_p = \pi b^2$ is the horizonthal area occupied by one single plume, with b the plume radius, ρ_b and V_b are respectively the density and the plume velocity at the base of the convective region, $F_{R,l} = \sqrt{l(l+1)} V_b / r_t N_0$, with N_0 the Brunt-Väisälä frequency at the top of the radiative zone, and $\nu_p = 1/\tau_p$, with τ_p the plume lifetime.

To go further, the total wave flux of angular momentum at each radius in the radiative zone is deduced from the sum of all the contributions to the wave energy flux emitted from the top of the radiative zone and modulated by a damping term (Zahn 1997)

$$\mathcal{F}_{J,w}(r) = \sum_l \sum_{m=-l}^{m=+l} \int_{-\infty}^{+\infty} \frac{m}{\omega} \frac{r_t^2}{r^2} \mathcal{F}_{E,w}(r_t, \omega, l, m) e^{-\tau(r,\hat{\omega},l)} \mathrm{d}\omega \ , \tag{2.3}$$

with

$$\tau(r, \hat{\omega}, l) = [l(l+1)]^{3/2} \int_r^{r_t} K \frac{N N_T^2}{\hat{\omega}^4} \left(\frac{N^2}{N^2 - \hat{\omega}^2} \right)^{1/2} \frac{\mathrm{d}r}{r^3} \ , \tag{2.4}$$

where N is the Brunt-Väisälä frequency, with its thermal part N_T, K is the radiative diffusion coefficient, and

$$\hat{\omega}(r, \omega, m) = \omega - m\delta\Omega(r) \tag{2.5}$$

is the Doppler-shifted intrinsic frequency. Note that $\delta\Omega(r) = \Omega(r) - \Omega_t$ where Ω_t is the rotation rate at the top of the radiative zone. Near a critical layer (i.e. where $\hat{\omega} = 0$), we will suppose that the considered wave component is totally dissipated and deposit all the angular momentum that they carry into the medium, so that they cannot go deeper in the star.

3 Ability of plume-induced waves to extract angular momentum from the core

3.1 Input physics and assumed rotation profile

We consider two $1M_\odot$ stellar models computed with the evolution code CESTAM (Marques et al. 2013) localized on the subgiant branch and at the beginning of the ascent of the red giand branch. The chemical composition follows the solar mixture as given in Asplund et al. (2009), with the initial helium and metal abundances $Y_0 = 0.261$ and $Z_0 = 0.0146$. We used the NACRE nuclear reaction rates and the OPAL2005 equation of states and opacity tables. The convection was modeled by the mixing-length theory parametrized with $\alpha_{MLT} = 1.75$. We did not consider microscopic diffusion, overshooting and rotation. To compute Eq. (2.2), we assume that the plume lifetime is close to the convective timescale at the base of the convective zone as given by the MLT, i.e. $\nu_p \approx \omega_{MLT}$. In addition, we fix the plumes filling factor at a reasonable value $\mathcal{A} \approx 0.1$, as observed in the uppermost layers of numerical simulations of the Sun (e.g. Stein & Nordlund 1998). All the other quantities are directly estimated using the equilibrium internal structure from the stellar models (see Pinçon et al. 2016, for details).

As seen in Eq. (2.3), the total wave flux depends on the differential rotation in the radiative zone via Eq. (2.4). As shown by Pinçon et al. (2016) in the solar case, its amplitude can have strong consequences on the transport by IGW. We will then assume a given rotation profile for each stellar model. Doing so, we assume that the rotation rate is low enough to have no effect on the internal structure of the stellar models. Nevertheless, little is known about the shape of the rotation profile in evolved stars. By assuming a priori, first, that an efficient mechanism prevents the core contraction from developing a strong differential rotation in the radiative zone, and second, that this latter forces a quite smooth profile, it leads us to consider a rotation profile in the form

$$\delta\Omega(r) = \Delta\Omega \cos^2\left(\frac{\pi}{2}\frac{r}{r_t}\right) \quad \text{for} \quad r < r_t \ , \tag{3.1}$$

with $\Delta\Omega$ the amplitude of the differential rotation between the center and the top of the radiative zone. We thus assume a decreasing rotation rate from the core to the envelope and the $\cos^2$ function ensures a smooth profile at the center and near the base of the convective zone. Using such a synthetic profile is questionable, but it will give us a first hint about the efficiency of the transport by IGW in subgiant and red giant stars while considering different values for $\Delta\Omega$.

3.2 Efficiency of the transport by plume-induced IGW

We now have all the ingredients to compute t_w and compare it to t_{cont} throughout the radiative zone of the stellar models. For each of them, we vary the amplitude of the differential rotation $\Delta\Omega$ in a range between 0 and 12 µrad s^{-1}, which is representative of the observations of evolved low-mass stars (Mosser et al. 2012; Deheuvels et al. 2012, 2014).

3.2.1 RGB stars

For red giant stars, all our computations show that the wave-driven timescale is well larger than the contraction timescale below the H-burning shell ($t_w \gg t_{cont}$). This is illustrated in Fig.1 *(left panel)* for a $1M_\odot$ model at the beginning of the ascent of the RGB with $\log T_{eff} = 3.68$ and $\log L/L_\odot = 0.55$. Since this result is conservative over a range that is representative of the observed values for $\Delta\Omega$, we can conclude that IGW alone are inefficient to slow down the core rotation and that another process should operate in these stars in order to explain the observations. Indeed, as pointed out by Fuller et al. (2014), the wave damping rate, which is locally proportional to N^3, strongly increases as the innermost layers contract. Near the H-burning shell, where N is maximum, IGW suffer a serious damping preventing them to go deeper and affect the core rotation in these stars. Nevertheless, we cannot exclude that damped IGW near and above the H-burning-shell, which have deposited their angular momentum in the medium (see the low values of t_w in this region), could play a role by interacting with meridional circulation and boosting the extraction of angular momentum from the stellar core to the envelope. This hypothesis will need to be checked using a more complete calculation.

3.2.2 Subgiant stars

On the subgiant branch, the situation is similar to RGB stars for very low differential rotations. Nevertheless, as $\Delta\Omega$ increases, the wave-driven timescale decreases. This is illustrated in Fig.1 *(right panel)* for a $1M_\odot$ subgiant

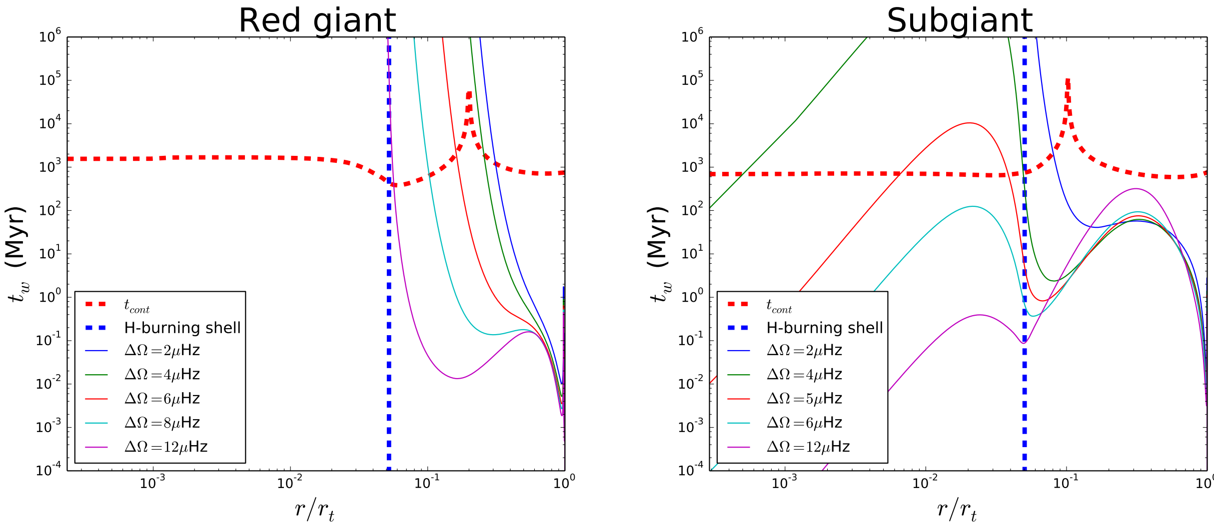

Fig. 1. Left: Wave-driven timescale, t_w, computed using Eq. (2.1) as a function of the normalized radius in the radiative zone, for a $1M_\odot$ model at the beginnng of the ascent of the RGB with $\log T_{eff} = 3.68$ and $\log L/L_\odot = 0.55$. The contraction timescale and the location of the H-burning shell are represented by the red and blue dashed lines, respectively. Different amplitudes for $\Delta\Omega$ are considered. **Right:** Same as the previous figure, but for a $1M_\odot$ subgiant model with $\log T_{eff} = 3.72$ and $\log L/L_\odot = 0.3$.

model with $\log T_{eff} = 3.72$ and $\log L/L_\odot = 0.3$. It even gets lower than the contraction timescale throughout the region below the H-burning shell as soon as $\Delta\Omega$ is larger than a threshold. This trend is mainly due to the asymmetry between prograde (m>0) and retrograde (m<0) components that is enhanced and the radiative damping of the retrograde waves that decreases as $\Delta\Omega$ increases. In the example of Fig.1 *(right panel)*, we can see that IGW are able to counteract the acceleration due to the core contraction as soon as $\Delta\Omega \gtrsim 6$ μHz. We note that this theoretical threshold value is consistent with the mean core rotation observed in subgiant stars. Therefore, we demonstrate here that IGW generated by penerative convection may be a major actor in the transport of angular momentum on the sugiant branch before the radiative damping becomes too strong as the stars evolve on the RGB. Obviously, they have to be taken into account in stellar modeling.

4 Conclusions

Internal gravity waves generated by penetrative convection cannot be responsible alone for the low rotation rates observed in the red giant stars. Indeed, the strong radiative damping near the H-burning shell prevents them from modifying the core rotation, as it has already been shown in previous works for turbulence-induced IGW. Nevertheless, in subgiant stars, these waves seem to be a good candidate to limit the amplitude of the differential rotation in the radiative zone. The results of this work are preliminary and have to be confirmed by a more thorough study. This will be subject to a near future work. Efforts will have also to be done to properly include transport by IGW in a stellar evolution code, which is a necessary step to study their interaction with the other transport processes and their effect along the stellar evolution.

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