SF2A 2016
C. Reylé, J. Richard, L. Cambrésy, M. Deleuil, E. Pécontal, L. Tresse and I. Vauglin (eds)

INTERNAL ROTATION OF γ DORADUS STARS

R.-M. Ouazzani[1], S. J. A. J. Salmon[3], V. Antoci[1], T. R. Bedding[1,4] and S. J. Murphy[1,4]

Abstract. Thanks to the exquisite *Kepler* data, resulting from four years of quasi-continuous photometric observations, we are now able to use g-modes in order to reveal the internal structure of γ Doradus stars. In particular, it is now possible to detect series of g-modes with non-uniform period spacing, which carry the signature of internal rotation. In a theoretical work published earlier this year, we have defined a new seismic diagnostic for rotation in the γ Doradus stars that are rotating too rapidly to present rotational splitting. It is based on a new observable that is the slope of the period spacing when plotted againt the period. Here we recall the one-to-one relation between this observable and the internal rotation rate. We explain how it can be used without any additional constraint in order to retreive the rotation rate in the cavity probed by the observed g-modes. Finally we evaluate the uncertainty induced by the use of the asymptotic formulation of the traditional approximation, and we give a word of caution concerning retrograde modes.

Keywords: asteroseismology, stellar interiors, stellar rotation, γ Doradus stars

1 Introduction

γ Doradus (γ Dor) stars are late A- to early F-type stars, of masses between 1.3 and 2 $M_\odot$, on the main sequence. Thanks to the four years of nearly continuous photometry from *Kepler*, we were finally able to measure their g mode pulsations to the precision level required to perform seismic modelling. These g modes probe the innermost regions and in particular the interface between the convective core and the radiative envelope, where transport of chemical elements and angular momentum is expected to occur. Whether this transport is caused by overshooting, shear induced turbulence or gravity waves is still matter of debate. In this context, the determination of internal rotation rates in γ Dor stars constitutes a valuable constraint.

These stars typically have projected rotation velocities of around 100 km s^{-1}, but that can reach up to 250 km s^{-1} (see for instance Abt & Morrell 1995; Royer 2009). Rotation lifts the degeneracy of pulsation frequencies. For slow rotation, g modes in γ Dor stars can exhibit splittings which can be used in order to determine the rotation rate in their propagation cavity (see Kurtz et al. 2014; Saio et al. 2015; Schmid et al. 2015; Keen et al. 2015; Murphy et al. 2016). In the case of moderate to rapid rotation, the structure of the frequency spectrum differs drastically. The prograde modes are shifted towards higher frequencies, whereas the retrogrades are shifted towards lower frequencies, to such an extent that they appear in the spectrum as clusters of modes, each with given angular degree (ℓ) and azimuthal order (m), and varying radial orders. Moreover, each of these clusters show a period spacing with a linear trend (Bouabid et al. 2013) which is related to the identity of the modes $\{\ell, m\}$, and the rotation velocity in their cavity.

Based on this and making use of an approximate treatment of rotational effects, the traditional approximation (TAR), Van Reeth et al. (2016) performed seismic determination of rotation in a sample of γ Dor stars observed by *Kepler*. We opted for the development of seismic diagnostics which are model independent, and are therefore not affected by the lack of knowledge of the stellar structure. To establish such diagnostics, we make use of non-perturbative calculations for the effect of rotation on γ Dors g modes (implemented in the ACOR code, see Ouazzani et al. 2012, 2015).

[1] Stellar Astrophysics Centre, Department of Physics and Astronomy, Aarhus University, Aarhus C, Denmark
[2] Laboratoire AIM, CEA/DSM-CNRS-Université Paris 7, Irfu/Service d'Astrophysique, CEA-Saclay, France
[3] Institut d'Astrophysique et de Géophysique de l'Université de Liège, Liège, Belgium
[4] Sydney Institute for Astronomy (SIfA), School of Physics, University of Sydney, Australia

2 A new asteroseismic diagnostic for internal rotation in γ Doradus stars

As mentioned above, in γ Dor stars that are moderate to rapid rotators, the modes gather by ℓ and m, and their period spacings (ΔP) vary linearly with modes periods. In Ouazzani et al. (2016) we have shown that the slope of this trend, which we named $\Sigma_{\ell,m}$ strongly depends on the rotation in the probed cavity, and marginally on other aspects of the stellar structure. This is illustrated in Fig. 1 (left), which represents the slope Σ as a function of the rotation rate, for dipolar ($\ell = 1$) modes. The solid lines are the average of Σ, calculated with the non perturbative method, for three representative models of γ Dor stars, which have been computed by the CLES code (Code Liégeois d'Évolution Stellaire, Scuflaire et al. 2008): the model 1z is 1.4 $M_\odot$ on the zero-age-main-sequence (ZAMS), the model 2m is 1.6 $M_\odot$ mid-way on the main-sequence (MS), and the model 3t, a 1.86 $M_\odot$ star on the terminal-age-main-sequence (TAMS). We used the asymptotic formulation of the TAR to evaluate the scatter caused by the difference in structure encountered in the γ Dor instability strip (see Ouazzani et al. 2016, for more details). This yields the dispersion areas in colour in Fig. 1 (left).

Hence, we established a one-to-one relation between the observable $\Sigma_{\ell,m}$ and the average rotation velocity in the cavity probed by the g modes of angular degree ℓ and azimuthal order m. The largest spread is obtained for the retrograde modes, this is treated is further details in Sect. 3.2. As a proof of concept, we chose four stars which have been quasi-continuously observed by observed by *Kepler* for 18 quarters (*Kepler* input catalog number KIC 4253413 and KIC 6762992), 17 quarters (KIC 5476299) and 15 quarters (KIC 4177905), respectively. Their measured oscillation periods were extracted using the classical pre-whitening procedure (Period04, Lenz & Breger 2005) and are plotted against their period spacings in Fig. 1 (right). We rely on the combined knowledge of the range of observed periods, their period spacings, as well as the slope Σ in order to identify the angular degree ℓ and azimuthal order m of the ridges. The four sequences of modes were identified as belonging to dipolar prograde modes. The slopes of the period spacings series were determined using a simple linear fit of the data points. By reporting these slopes in the diagram given in Fig. 1 (left), we directly retreive the average rotation frequency in the cavity probed by these modes. The results are the following: 7.1 ± 0.9 μHz for KIC 6762992, 9.8 ± 0.9 μHz for KIC 4177905, 10.7 ± 1.4 μHz for KIC 4253413, and 17.8 ± 2.9 μHz for KIC 5476299.

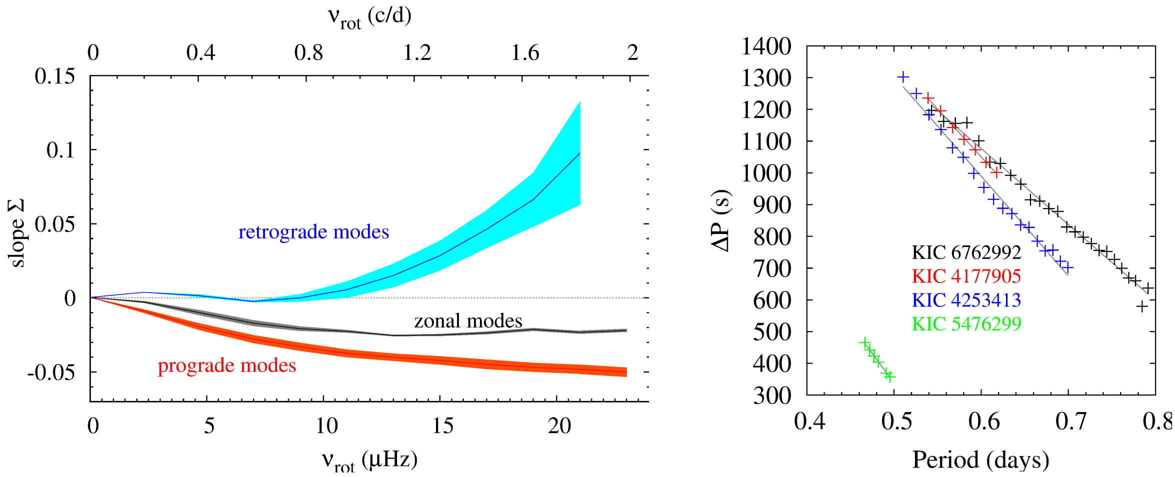

Fig. 1. Left: Diagram giving the one-to-one relation between the slope of the period spacing, i.e. the observable Σ, and the rotation frequency established as an average of the non-perturbative calculations for models 1z, 2m, and 3t. It is given here for dipolar modes: prograde modes in red, zonal modes in black and retrograde modes in blue. The dispersions correspond to the variations of Σ due to the mass, age on the main sequence, metallicity, and type of mixing on the edge of the convective core, computed using the asymptotic formula at each rotation rate for a grid of models covering the γ Dor stars instability strip. **Right:** Period spacing as a function of the period for four sequences of modes observed in four stars observed by *Kepler*: KIC 6762992 in black points, KIC 4177905 in red points, KIC 4253413 in blue points, and KIC 5476299 in green points. The grey lines correspond to the linear fits used to determine the slope of the respective ridge.

3 A word of caution

In the previous section, we give the one-to-one relation between the slope of the period spacing series, and the internal rotation rate in γ Dor stars, as introduced by Ouazzani et al. (2016). In this section, we evaluate the uncertainties induced by using the asymptotic formulation of the TAR for the determination of the rotation rate from Σ. Moreover, we investigate more closely the specificity of retrograde modes.

3.1 Uncertainties related to the use of the Asymptotic formulation of the TAR

In rotating stars, the equation system for pulsations is not separable in terms of the radial and horizontal coordinates, unlike for the non rotating case. The TAR is not a perturbative method. This is an approximate treatment which conserves the separability of the system by making specific assomptions. The first one is that the stars is rotating as a solid body. The centrifugal distortion is neglected, i.e. the spherical symmetry is assumed. Moreover, considering the properties of high radial order g modes, the TAR neglects the Coriolis force associated with radial motion, and radial component of the Coriolis force associated with horizontal motion. Finally, the Cowling approximation is made (Cowling 1941). Under the TAR, the simplification of the problem allows for an asymptotic formulation derived from the Tassoul (1980) formula for g-mode periods, where $\ell(\ell+1)$ is replaced by λ. This eigenvalue depends on ℓ, m and the spin parameter $s = 2\nu_{rot}/\nu_{co}$, ν_{co} being the frequency of the modes in the corotating frame. The asymptotic formula gives:

$$P_{\rm co}(n) = \frac{2\pi^2(n+\frac{1}{2})}{\sqrt{\lambda_{\ell,m,s(n)}}\int_{r_0}^{r_1}\frac{N}{r}\mathrm{d}r} \quad \text{and} \quad \langle\Delta \mathrm{P_{co}}\rangle \simeq \frac{2\pi^2}{\sqrt{\lambda_{\ell,\mathrm{m,s(n+1)}}}\int_{\mathrm{r_0}}^{\mathrm{r_1}}\frac{\mathrm{N}}{\mathrm{r}}\mathrm{d}r\left(1+\frac{1}{2}\frac{\mathrm{d}\ln\lambda}{\mathrm{d}\ln \mathrm{s}}\right)} \tag{3.1}$$

The asymptotic formulation requires very little computational resources and time. For that reason it is often used in order to study the pulsational properties of grids of stellar models. However, the aim here is to investigate its validity when used to model the diagnostic Σ, compared to the non-perturbative method. To do so, we computed the slopes Σ of dipolar modes period spacing series computed with the two methods for the three representative models 1z, 2m, 3t mentioned before. The largest uncertainties arise for the model 1z, which is chosen to be shown in Fig.2. The period spacing is plotted against the period for model 1z with a rotation frequency ranging from 11 to 23 μHz, computed with the non-perturbative method (lines with points), and with the asymptotic formulation (solid lines).

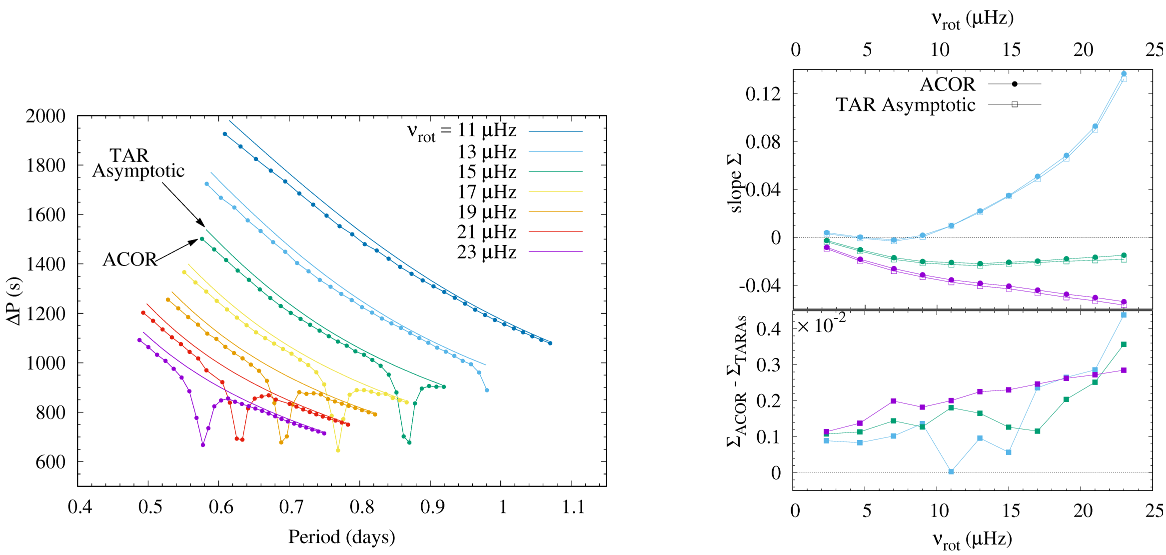

Fig. 2. Left: Period spacing as a function of the period for model 1z, computed with the asymptotic formulation (solid lines) and with the non perturbative method (ACOR lines with points) for rotation frequencies ranging from 11 to 23 μHz. **Right, top:** Slope Σ of the period spacing series as a function of the rotation frequencie of the model 1z, computed with the TAR asymptotic formulation (open squares), and with the non-perturbative method (ACOR, filled circles), for prograde (purple), zonal (green), and retrograde modes (blue). **Right, bottom:** Discrepancy between the two computations.

One striking difference between these two series of curves is the occurence of dips for the non-perturbative series, around the same period spacing value whatever the rotation rate. These dips occur preferentially at higher rotation frequencies (from $\nu_{rot} = 13\mu$Hz upwards). After exploring the pulsation spectrum of modes with higher ℓ values, it turns out that these features in the $\ell = 1$ series appear in the frequency range where g modes of $\ell = 3$ are increasingly numerous in the spectrum. Even if conserving a dominant $\ell = 1$ character, modes in this region are bumped into and perturbed by their $\ell = 3$ counterparts. These dips appear for the retrograde and zonal modes, because for prograde modes the shift in frequency due to rotation is such that g modes of different ℓ do not lie in the same frequency range at all.

These features are not at all reproduced by the asymptotic formulation, and they seem to modify the overall slope of the ridges noticeably towards negative values. However, let us compare the period spacing series, when the dips are removed from the ridges, i.e. far from the dips. The result is given in Fig.2 (right). Apart from an overall increasing trend with rotation, the discrepancy it is not negligible at low rotation frequency (around 10 % at $\nu_{rot} = 2.5\mu$Hz), and can reach up to approximately 20 % for zonal modes at high rotation.

3.2 The case of retrograde modes

In the inertial frame of reference, the effect of rotation of g modes is twofold: the intrinsic effect of the Coriolis and centrifugal forces, and the change of reference from the corotating frame to the inertial one. Zonal modes are not impacted by this change of frame, but retrograde modes are shifted towards shorter periods, whereas prograde modes are shifted towards longer ones respectively. For retrograde modes, this shift substracts from the intrinsic effect of rotation. The specificity of retrograde modes resides in the competition between these two effects. Although quantitatively inaccurate, the asymptotic for the period spacing helps understand this phenomenon. Following Eq. (3.1) the period spacing in the inertial frame can be written as:

$$\Delta P_{in} \propto \frac{1}{\sqrt{\lambda_{\ell,m,s}}\left(1 - m\frac{P_{co}}{P_{rot}}\right)}, \tag{3.2}$$

where the factor $\sqrt{\lambda_{\ell,m,s}}^{-1}$ comes from the TAR and stands for the Coriolis effect on the pulsations in the corotating frame, and $\left(1 - m\frac{P_{co}}{P_{rot}}\right)^{-1}$ for the change of reference frame. As rotation varies, three regimes can be identified for the behaviour of ΔP_{in}:

- *Slow rotation*: when the pulsation periods are significantly smaller than the rotation period, ΔP_{in} follows the behaviour given by the Ledoux (1951) perturbative formalism at first order. This is illustrated in Fig. 3, with the two green ridges, computed for models rotating at 2.33 μHz (4.97 days). The green open triangles are obtained with the Ledoux formula, whereas the green open circles are obtained with the non-perturbative calculations. For inertial periods shorter than $P_{rot}/4$ (straight dashed green line), the ΔP_{in} follows the first order perturbative formula.

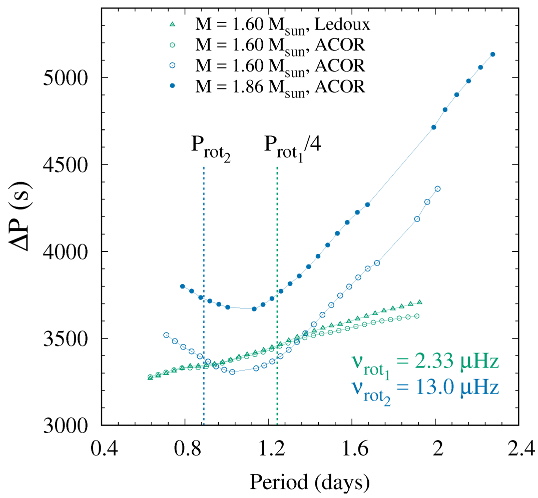

Fig. 3. Period spacing as a function of period in the inertial frame for retrograde modes computed for a 1.86 $M_\odot$ (filled symbols), and a 1.60 $M_\odot$ (open symbols) stellar models, with the ACOR code (circles), or with the Ledoux splitting (triangles), for slow rotation (ν_{rot1} =2.33 μ Hz, green) and rapid rotation (ν_{rot2} =13.0 μ Hz, blue). The two vertical dashed lines stand for the period in the inertial frame that equals the rotation period P_{rot2} = 0.89 days for the rapid rotation case, and when it equals a quarter of the rotation period for the slowly rotating case $P_{rot1}/4$ = 1.24 days. Some modes are discarded for not presenting a clear $\ell = 1$ character.

- For *inertial periods* P_{in} *longer than* $P_{rot}/4$, the rotational effect is determined by Eq. (3.2). For P_{in} shorter than P_{rot}, i.e. in the superinertial regime, the behaviour of ΔP_{in} is dominated by the factor $\sqrt{\lambda_{\ell,m,s}}^{-1}$, which causes a decreasing trend of ΔP_{in} with respect to P_{in}. This is illustrated in Fig. 3 by the part of the blue ridges which are leftward of the dashed blue line.
- For P_{in} *longer than* P_{rot}, i.e. in the subinertial regime, the effect of the change of reference frame dominates under the form of an asymptotic behaviour towards infinity.

The spread in Σ for retrograde modes, shown in Fig. 1, is explained by the difference between the two blue ridges (open and filled circles) in Fig. 3. The difference between the two ridges resides in the period of modes of given radial order: for the 1.6 $M_\odot$ model (blue open circles), the modes are more numerous on the decreasing part of the ridge than for the 1.86 $M_\odot$ model (blue filled circles). As a result, when performing a linear fit of these points, the slope of the ridge corresponding to the 1.6 $M_\odot$ model is smaller than for the 1.86 $M_\odot$ model. In other words, because these ridges are not linear, the periods change of the excited modes (radial orders n between -50 and -15) due to a change of the model's parameters impacts Σ. Therefore, should the diagnostic given in Fig. 1 be used on observed series of retrogrades modes, we would recommend a more detailed modelling accounting for the period range on which the parameter Σ is determined.

4 Conclusions

We have reported on the establishment of a new seismic diagnostic of rotation for g modes when the rotational splitting cannot be extracted correctly: Σ, the slope of the period spacing when plotted against the period. We give the one-to-one relation between Σ and the internal rotation rate. We then explore the relevance of using the asymptotic formulation of the TAR in order to establish the one-to-one relation mentioned above. It appears that the asymptotic method fails to reproduce features that are related to the multiple ℓ character of the modes. When these features are removed, the uncertainties can be significant, but lower than the spread in Σ due to the differences of internal structures encountered in the γ Dor instability strip. Finally we give a word of caution concerning the use of the Σ diagnostic with retrograde mode, and explain in details the peculiar nature of these modes.

Funding for the Stellar Astrophysics Centre is provided by The Danish National Research Foundation (Grant DNRF106). The research was supported by the ASTERISK project (ASTERoseismic Investigations with SONG and Kepler) funded by the European Research Council (Grant agreement no.: 267864).

References

Abt, H. A. & Morrell, N. I. 1995, ApJS, 99, 135

Bouabid, M.-P., Dupret, M.-A., Salmon, S., et al. 2013, MNRAS, 429, 2500

Cowling, T. G. 1941, MNRAS, 101, 367

Keen, M. A., Bedding, T. R., Murphy, S. J., et al. 2015, MNRAS, 454, 1792

Kurtz, D. W., Saio, H., Takata, M., et al. 2014, MNRAS, 444, 102

Ledoux, P. 1951, ApJ, 114, 373

Lenz, P. & Breger, M. 2005, Communications in Asteroseismology, 146, 53

Murphy, S. J., Fossati, L., Bedding, T. R., et al. 2016, MNRAS, 459, 1201

Ouazzani, R.-M., Dupret, M.-A., & Reese, D. R. 2012, A&A, 547, A75

Ouazzani, R.-M., Roxburgh, I. W., & Dupret, M.-A. 2015, A&A, 579, A116

Ouazzani, R.-M., Salmon, S. J. A. J., Antoci, V., et al. 2016, ArXiv e-prints

Royer, F. 2009, in Lecture Notes in Physics, Berlin Springer Verlag, Vol. 765, The Rotation of Sun and Stars, 207–230

Saio, H., Kurtz, D. W., Takata, M., et al. 2015, MNRAS, 447, 3264

Schmid, V. S., Tkachenko, A., Aerts, C., et al. 2015, A&A, 584, A35

Scuflaire, R., Théado, S., Montalbán, J., et al. 2008, Ap&SS, 316, 83

Tassoul, M. 1980, ApJS, 43, 469

Van Reeth, T., Tkachenko, A., & Aerts, C. 2016, A&A, 593, A120