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*C. Reylé, J. Richard, L. Cambrésy, M. Deleuil, E. Pécontal, L. Tresse and I. Vaughlin (eds)*

# SPIRALS, GAPS, CAVITIES, GAPITIES: WHAT DO PLANETS DO IN DISCS?

A. Crida[1 2]

**Abstract.** In this presentation, part of the "Observations of discs" workshop, I address the theoretical point of view of planet-disc interactions. In section 2, I will review the physics of spirals, and explain why the inner and the outer wake created by a planet in a gaseous keplerian disc look very different, but their shape is independant of the mass of the planet and almost only depends on the aspect ratio of the disc. In the third section, I discuss the axisymmetric features (gaps and cavities), and how they differ in the gas or the dust component.

However, to start with, some clarification of the nomenclature seems to be required, as observers and theorists may have a different idea of what a gap is. The definition of a "gapity" may help to clarify the situation of pre-transitional discs.

Keywords: Planet-disk interactions, Protoplanetary disks, Planets and satellites: formation

## 1 Gaps, cavities and terminology

Let me first take this opportunity to write down clearly a few definitions. Our community generally agrees on these terms, but to avoid any misunderstanding between observers and theorists, and to help students discovering our field, here is a little glossary.

**Cavity :** A *cavity* in a disc is an empty region inside a given radius $r_{\rm cav}$. Cavities are observed in a few discs, called *transition discs*. For the sake of clairity, a cavity should be completely empty, that is there is nothing between the star and $r_{\rm cav} > 1AU$, which is then the radius of the inner edge of the disc.

**Gap :** In contrast to a cavity, a gap is an empty region separating an inner and an outer disc. A gap has the shape of an annulus, while the cavity has the shape of a full circle. Giant planets open gaps in the gas disc. Those gaps have roughly the width of the horseshoe region of the planet, that is in total $\sim 3r_{\rm Hill}$, where $r_{\rm Hill} = r_p(M_p/3M_*)^{1/3}$ is the Hill radius. Hence, in general, the radius of the outer edge of the inner disc (which is also the inner edge of the gap) $r_{\rm in}$ and the radius of the inner edge of the outer disc (or outer edge of the gap) $r_{\rm out}$ differ by less than a factor 2.

**Gapity :** In some discs (so called *pre-transitionnal discs*) a small inner disc is observed inside a cavity. Or in other words, very large gaps are seen, where $r_{\rm out}/r_{\rm in} > 3$. Such structure can not be caused by a single giant planet, in contrast with the gaps described above. To account for a possible different physical origin, a different word is needed. In-between a narrow gap and an empty cavity, I suggest a *gapity*$^*$.

**Migration versus drift :** Finaly, I would like to remind that the radial motion of planets and dust inside a protoplanetary disc are not the same phenomenon. Planets exchange orbital angular momentum with the gas via gratity, while dust does so via friction. Hence, it is important to keep using the word *migration* for planets and gravitationnal phenomenons, and *drift* for dust, pebbles, and friction phenomenons.

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[1] Université Côte d'Azur / Observatoire de la Côte d'Azur / CNRS, Laboratoire Lagrange, Boulevard de l'Observatoire, CS 34229, 06300 Nice, FRANCE
[2] Institut Universitaire de France, 103 Boulevard Saint-Michel, 75005 Paris, FRANCE

*En français, ces structures intermédiaires entre sillons et cavités pourraient être appelées *cavillons.*

## 2 Spirals and wake

### 2.1 Spirals : some reminders

Consider a curve in polar coordinate $(r, \theta)$. The *pitch angle* is the angle between the curve and the azimuthal direction, so that :

$$\tan\alpha = \mathrm{d}r/r\,\mathrm{d}\theta\ . \tag{2.1}$$

The most famous spiral is *Archimede's spiral*, whose equation is simply $r = \lambda\theta$ with $\lambda$ any real constant. In this case, for every full rotation, $r$ increases by the same amount : $2\pi\lambda$. For this spiral, the pitch angle depends dramatically on $r$ : $\alpha = \arctan(\lambda/r)$. Hence, $\alpha = \pi/2$ at $r = 0$, where the spiral starts in the radial direction ; conversely $\alpha \to 0$ as $r \to \infty$, which means that the curve become more and more orthoradial and looks more and more like a circle when one goes far from the centre.

Imposing $\alpha$ constant, one gets $r = \exp(\lambda\theta)$ (with $\lambda = \tan(\alpha)$ ), that is the *logarithmic spiral.* This curves looks the same close to the centre and far from it, and crosses radii always with the same angle.

### 2.2 Few prerequisites on disc physics

We consider a protoplanetary disc made of perfect gas, with equation of state :

$$P = \left(\frac{c_s^2}{\gamma}\right)\rho\ ,$$

where $P$ is the pressure, $\rho$ the (volume) density, $c_s$ the sound speed, and $\gamma$ the adiabatic index, the ratio of specific heats. With this equation of state, one shows easily that the vertical hydrostatic equilibrium is fulfilled if the density follows (assuming $z \ll r$ and $c_s$ independent of $z$) :

$$\rho(r, z) = \rho_0(r) \times \exp\left(-\frac{z^2}{2\,H^2}\right) \tag{2.2}$$

where $H = c_s/(\Omega\sqrt{\gamma})$, with $\Omega = \sqrt{GM_* r^3}$ the Keplerian angular velocity ($G$ being the gravitational constant and $M_*$ the mass of the central star). This defines $H$, the scale height of the disc. We note $h = H/r$ the *aspect ratio* of the disc.

Last, using the perfect gases equation $P = (R/\mu)\rho T$ where $R$ is the constant of perfect gases, $\mu$ the molecular weight, and $T$ the temperature, one finds :

$$T = {c_s}^2\,\mu/(\gamma R) = h^2(GM_*/r)(\mu/R)\ . \tag{2.3}$$

With $h$ uniform, this leads to $c_s \propto r^{-1/2}$ and $T \propto 1/r$.

### 2.3 Wake of a planet

With these well-known prerequisites in mind, one can study the shape of the wake created by a planet in a protoplanetary disc. The wake is a pressure-supported wave, so it propagates radially away from the planetary orbit with a velocity $c_s$. The azimuthal velocity is the relative velocity between the planet and the gas, and is set by the Keplerian shear : $v_{\rm rel} = r(\Omega(r) - \Omega_p)$, where the index $p$ refers to the planet.

In the limit $r - r_p \ll r_p$, we can define $\Delta = (r - r_p)/r_p \ll 1$, and $v_{\rm rel} \approx \frac{3}{2} r\Omega_p\Delta$. Hence, the pitch angle of the wake follows :

$$\tan(\alpha) = c_s/v_{\rm rel} = \frac{3}{2}(c_s/\Omega_p r\Delta) = \frac{3}{2}\sqrt{\gamma}h_p/\Delta\ . \tag{2.4}$$

Close to the planet $\alpha \approx \pi/2$ and the wake is radial. The wake then bends as $\Delta$ increases.

In the inner disc, as $r \to 0$, $v_{\rm rel} \approx r\Omega(r) \sim r^{-1/2}$, and $\tan(\alpha) = \sqrt{\gamma}h$. With $h$ uniform, this is a logarithmic spiral. Taking $\gamma = 1.4$ and $h = 0.12$, one gets $\alpha = 8.5°$.

In the outer disc, as $r \to \infty$, $v_{\rm rel} \approx r\Omega_p \propto r$, and $\tan(\alpha) = c_s/v_{\rm rel}$ tends to 0 faster than in an Archimede's spiral. The outer wake bends a lot, and looks closer and closer to a ring.

As an illustration of how different the inner and outer wake look, the spiral structure observed by Benisty et al. (2015) in MWC758 could be satisfactorily reproduced by Dong et al. (2015a) only by placing a companion outside the disc, so that the spiral was the inner wake, not the outer one.

In all the above, the only paramater in the shape of the wake is $h$. The shape of the wake does not depend on the mass of the planet, nor on the viscosity of the gaseous disc. The larger $h$ (that is : the thicker the disc), the more open the wake is.

## 3 Gaps, cavities, gapities

It is well known that giant planets open gaps in a gaseous protoplanetary disc (Lin & Papaloizou 1986; Crida et al. 2006, 2016). If one puts a few giant planets in a proto-planetary disc, they may well have a convergent migration until they open a common gap and lock in a resonant chain. In this case, the large gap extends from inside the orbit of the innermost planet to outside the orbit of the outermost one, and could correspond to the definition of a gapity.

The edges of the gap open by a planet are pressure maxima in the gas component, therefore they behave as dust traps (dust always drifts towards the pressure maxima). Fouchet et al. (2010) "*show that the gap in the dust layer is more striking than in the gas phase and that it is deeper and wider for more massive planets as well as for larger grains*". In fact, Lambrechts et al. (2014) and Rosotti et al. (2016) found that even planets of mass $M_p > 20\, M_\oplus (h/0,05)^{1/3}$ are able to create a pressure maximum outside of their orbits, although they don't open a real gap, but barely carve a dip around their orbit.

Such planets can therefore create a gap in the dust component, which would not be detectable in the gas component. Rosotti et al. (2016) argue that measuring the distance between the pressure maximum (seen in sub-mm) and the middle of the gap (observed in scattered light) allows to derive the planet mass, as this distance should be 10 Hill radius.

Of course, dust grains of different sizes behave differently. More specifically, dust grains with a stopping time much shorter than the dynamical time (that is a Stokes number $St \ll 1$) are tightly coupled to the gas, and follow its distribution. As grains get larger and $St \to 1$, the pressure trap becomes more and more effective, and the dust gap becomes deep and sharp (e.g. Gonzalez et al. 2012; Dong et al. 2015b). Actually, when the pressure trap is so strong that no dust goes through, dust in the inner disc keeps drifting inwards whithout being replaced by drifting dust from the outer disc, and a gapity opens. Hence, a single massive planet can open a gapity in the dust component, but not in the gas component. Such gapities may transition to cavities once all the dust has drifted out of the inner disc.

Last, let us remind that if a giant planet is more massive than the disc, the latter can not push the former, and migration stalls (Crida & Morbidelli 2007). If the gap is so deep (or the planet accretes so fast) that the gas shall not pass from the outer to the inner disc, the inner disc will eventually spread into the host star and vanish, leading to the opening of a cavity in gas.

## 4 Conclusion

In this little review, I have discussed what planets can and can not do in discs. To interpret observations correctly, it is extremely important to be aware of these constains. Not all structures in discs can be due to planets, and structures like gaps, cavities or spirals could be created by other phenomenons, which are not the topic of this presentation. To date, there is no direct proof of a structure being created by a planet, but future observations will be extremely exciting. May this little contribution help bridging the gap between theory and observations of planet-disc interactions.

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